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Abstract

Using data from the first National study on high school preparation for college calculus success, the Factors Influencing College Success in Mathematics (FICSMath) project, this article connects student high school instructional experiences to college calculus performance. The findings reported here reveal that students were better prepared for college calculus success by high school instructional experiences that emphasized mathematical definitions, vocabulary, reasoning, functions, and hands-on activities. These findings serve to inform high school mathematics teachers about promising instructional practices. They can also inform teacher education programs about how to better prepare secondary mathematics educators to discuss conceptual understanding on the widely used Educative Teacher Performance Assessment (edTPA).

The challenge of preparing students for college level mathematics has led to a research focus on the transition from high school to college, often referred to as the school-to-college transition (Clark & Lovric 2008; 2009; Gueudet, 2008). There is no straightforward answer as to when this transition begins. However, research on the topic typically covers the period from two years before entering university to two years after (Gueudet, 2008). Student performance across the transition is viewed differently by different stakeholders. Teachers, administrators, and state legislators often consider students' performance from the perspective of how well they do on standardized tests, while college mathematics professors are most often interested in how well incoming freshmen are prepared for higher levels of learning (Pesek & Kirshner, 2000; Wade, Sonnert, Sadler, Hazari, & Watson, 2016).

In preparing students for the school-to-college transition, high school mathematics teachers must consider how students acquire knowledge, develop and change knowledge structures, and grow in conceptual understanding (Borko et al., 2000). Teaching for understanding in mathematics is a key notion; it has been defined as helping students develop a web of mathematical knowledge in which foundational mathematical concepts tie specific ideas and techniques together (Sweller et al., 1998; Wade, 2011). The National Council of Teachers of Mathematics (NCTM) has long focused on teaching for understanding and has stressed that procedural fluency builds on the foundation of conceptual knowledge (NCTM, 2000, 2014; NGA Center & CCSSO, 2010).

Mathematics teachers who invest in their field by working with future teachers, often referred to as preservice teachers, may recognize the renewed focus by teacher education programs on teaching for conceptual understanding. This most recent change, due to the implementation of the Educative Teacher Preparation Assessment (edTPA), was implemented in many teacher preparation programs about the same time when the Common Core State Standards (CCSS) were adopted across the U.S. As of 2016, there were 655 educator preparation programs in 36 states and the District of Columbia that required the edTPA for program completion or state teacher certification (edTPA, 2016). The edTPA requires preservice teachers to provide evidence of planning, teaching, and assessing for conceptual understanding.

Literature Review

Skemp (2006) distinguished two different types of understanding in school mathematics: relational and instrumental understanding. Relational understanding implies that students know what to do and why, whereas instrumental understanding indicates that students know rules without reason. Skemp (2006) claimed that high school teachers often adopt a two-track strategy of instruction where they spend some time on drill and practice, providing for skills and facts, and some time on developing and integrating understandings.

Mewborn (2007) stated that each student's mathematical understanding and problem solving ability is primarily shaped by the teaching experiences encountered in school. Most high school mathematics teachers are aware of the call to teach for conceptual understanding, but the administrative demands for standardized tests to provide "unambiguous documentation of learning" may inflate the drill and practice component in current classroom instructional experiences (Pesek & Kirshner, 2000, p. 524). As a consequence, students' development of conceptual understanding in high school mathematics may be undervalued. In addition to the pragmatic pressures toward drill and practice, teaching for understanding faces another challenge: No one knows exactly what instructional experiences promote understanding precalculus and calculus content, in preparation for the school to college transition.

The edTPA

Stanford University faculty and staff at the Stanford Center for Assessment, Learning, and Equity (SCALE) developed the edTPA from the National Board for Professional Teaching Standards (NBPTS), the standards of the Interstate Teacher Assessment and Support Consortium (InTASC), and the Performance Assessment for California Teachers (PACT) (Pearson Education, 2017). The edTPA is a performance-based, subject-specific assessment focused on three tasks: planning, instruction, and assessment. For high school mathematics, the planning task provides five rubrics, and four of them directly address how the teacher candidate plans to support the development of conceptual understanding. For the instruction task, three of the five rubrics address how to engage students in conceptually understanding mathematics. Finally, for the assessment task, three of the five rubrics address assessing students' conceptual understanding.

In support of its focus on conceptual understanding, the edTPA document cites NAEP (2003) claiming that, among other things, students demonstrate conceptual understanding when they generate examples of concepts, model varied representations of concepts, know and apply facts and definitions, and compare terms used to represent concepts (Scale, 2016). The edTPA document, does not, however, relate students' understanding to their specific high school instructional experiences. Knowing what instructional practices support students' conceptual understanding of mathematics could help teacher educators to better prepare preservice teachers to perform well on the edTPA and likewise, to prepare them to eventually teach students for the transition to college calculus.

Challenges in the Transition

Part of the complexity of the school-to-college transition may be the discontinuity between high school mathematics preparation and college level mathematics expectations. When students experience discontinuities, they seek to assimilate new information into an existing framework. For example, the College Board Advanced Placement (AP) Calculus curriculum is considered by some mathematics professors to be so broad that students move through the course by learning procedures instead of concepts (Bressoud, 2009). This learning strategy will not assimilate well into college calculus, which is more theoretical and thus requires a new framework for learning.

According to Skemp (2006), if students fail to grasp mathematical concepts, or if they grasp concepts but cannot connect them to relevant procedures, flawed procedures develop into what Clark and Lovric (2009) referred to as a synthetic model. A synthetic model represents misconceptions of mathematics learned in high school that cannot assimilate well into college calculus. Although much is yet to be learned, it is known that many students moving from high school mathematics into college calculus find developing a new framework for higher levels of learning very challenging.

Teachers and Professors

High school mathematics teachers and college mathematics professors agree that rigorous instruction promotes mathematical understanding, but there is less agreement on how to actually implement such instruction (Harwell et al., 2009; Wade, Sonnert, Sadler, Hazari & Watson, 2016). Learning college calculus requires students to formalize, unify, generalize, and simplify, which can be difficult tasks for freshmen college students who are not prepared for the school-to-college transition. Gueudet (2008) and Moore (1994) noted seven difficulties that novice students experienced with proofs, which are very common in college level mathematics. Among these were that students had little intuitive understanding of concepts, did not know definitions, and were unable to use mathematical language and notation. (For a more detailed description of students' difficulties in college calculus, see Tall, 1993). Although some high school precalculus and calculus courses do require this level of understanding, many do not, and such understanding is indispensable for the transition to college level mathematics.

Research Questions

Using the Factors Influencing College Success in Mathematics (FICSMath) dataset, our research questions are: (1) Which instructional items (describing specific pedagogical methods and characteristics) correlate with conceptual understanding in high school mathematics, as reported by the students? (2) To what underlying constructs can these instructional items be reduced? (3) How do these constructs predict performance in college-level calculus?

Methods

The FICSMath Study

Funded by the National Science Foundation, the FICSMath study was the first national study of high school preparation for college calculus. It was carried out in the Science Education Department at the Harvard-Smithsonian Center for Astrophysics. The items on the FICSMath survey were created from three sources. The first source was a broad literature review of the school to college transition. The second was an online survey sent to precalculus and calculus teachers and professors, from across the nation, asking what they do, or believe what should be done, to prepare students for college calculus success (For a review of the findings see Wade, Sonnert, Sadler, Hazari & Watson, 2016). Lastly, a panel was created from mathematics educators, mathematicians, and researchers from various colleges and universities that met at Harvard University on two separate occasions to discuss the FICSMath survey items.

The FICSMath Survey

Along with many demographic items, the FICSMath survey contained five sections with 70 items about students' course and instructional experiences in their most recent mathematics course. The survey asked questions about: the organization and structure of the mathematics course (16 items); textbooks, homework, and in-class assignments (13 items); tests and quizzes given in the course (13 items); teacher characteristics (six items); and class time and methods used during instruction (22 items). The format of the items varied. Some were Likert scales, but others required marking all that applied, and still others were dichotomous questions. When appropriate, scales were linearized.

The FICSMath survey was administered to students in college calculus courses across the United States near the beginning of the fall semester of 2009. After the students completed the survey, their professors held the surveys until the end of the semester, recorded students' final grades, then returned the surveys to Harvard University where analysis began in the spring semester of 2010. For this research, the students' final grade in single variable college calculus is the dependent variable, and the independent variables of interest are students' instructional experiences in high school precalculus or calculus courses that significantly correlated with the conceptual understanding item.

Validity and Reliability

The literature review, responses from teachers and professors from the online survey, and the discussions by mathematics educators, mathematicians, and researchers of the FICSMath items were measures taken to assure content validity. To gauge reliability, we conducted a separate study in which 174 students from three different colleges took the survey twice, two weeks apart. Our analysis found that, for groups of 100, less than a 0.04% chance of reversal between the 50th and 75th percentiles existed (Thorndike, 1997, p. 117).

The Sample

The National Center for Education Statistics (NCES) kindly transmitted an Integrated Postsecondary Education Data System (IPEDS) table with enrollment numbers for two and four year degree-granting institutions. From the large, medium, and small colleges and universities across the nation, participants were recruited in a stratified random sample by contacting mathematics department heads, and they were requested to allow students in their college calculus courses to take the 20-minute FICSMath survey. Of the 276 institutions contacted, 182 (65.9 %) agreed to participate. From the 134 institutions that returned the surveys, a sample of 10,437 students was obtained. From this sample, 5,985 students had taken either precalculus (n=2,326), or any level of high school calculus (n=3,659) as their most advanced mathematics course in high school. This research limited the sample to include only students who moved from one of these high school courses directly into single variable college calculus. In the end, our sample included responses from 1,376 precalculus and 2,966 calculus students, for a total of 4,342 responses.

Concerns about combining student instructional experiences from high school precalculus and calculus courses were addressed by investigating the students' performance across high school mathematics and college calculus. The high school mean performance (measured by the grade received) was 89 for the precalculus group and 90 for the calculus group. This was different from the mean grade earned in single variable college calculus, which was 77 for the precalculus group and 85 for the calculus group. Because of the significant difference in performance in single variable college calculus between the high school precalculus and calculus groups, a variable was created to differentiate between these two groups, referred to as a calculus dummy variable (taking high school calculus was coded as 1, and taking high school precalculus was coded as 0).

Correlation of items with the Conceptual Understanding Item

The conceptual understanding item on the FICSMath survey read, "In terms of learning the material, the mathematics course required very little (coded as zero) or a lot (coded as five) of conceptual understanding." This 6-point rating scale item correlated positively with the students' performance (i.e., final grade) in college calculus (r=0.2; p=.001). However, the term "conceptual understanding" is fairly abstract, so it was unclear what it actually meant to the respondents. Thus, we tied this item to the more concrete instructional items to identify specific experiences that correlated to high conceptual understanding. Of the 70 instructional items, 33 had a Pearson coefficient of 0.20 or greater (indicating a positive relationship) with the conceptual understanding item.

Creation of the Constructs

The items with a correlation coefficient value of 0.2 or higher were then placed into factor analysis. The use of factor analysis allowed us to determine which items tended to go together so that we could combine them into composite items, also referred to as constructs. (See Costello & Osborne [2005] for factor analysis techniques). To create the composites or constructs, we standardized the pertinent items (mean of 0, standard deviation of 1), because they were not all on the same scale. We then summed them and, for ease of interpretation, standardized the result again. To make sure that the composites, or constructs, remained correlated with the conceptual understanding item, we computed the correlations again. The Mathematical Fluency Construct had the strongest correlation with the conceptual understanding item (r=0.402), and the Applications Construct had the weakest (r=0.207). The levels of correlation with the conceptual understanding item are presented in the discussion of each construct below.

Findings

In this section, we first present the items in each of the four constructs that summarized how students' instructional experiences correlated with conceptual understanding of high school precalculus or calculus content. Second, we present a main effects hierarchical linear model (HLM) that reveals if and how these constructs predicted performance in college calculus (grade on a 100-point scale). Lastly, we present an HLM interaction model, with graphs used for interpretation of the interactions.

Table 1.
FICSMath Survey Instructional Constructs, Displayed with Factor Loadings and Survey Items

Construct	Loadings	Survey Questions
1 Multiple Representations (5 Items)	.974	Highlighted more than one way of solving a problem
	.755	Explained ideas clearly
	.725	Used graphs, tables, and other illustrations
	.691	Presented various methods for solving problems
	.632	Teacher was enthusiastic about mathematics

2 Applications (4 Items)	.926	Connected math to other subject areas
	.857	Connected math to real-life applications
	.699	Connected math to everyday life
	.648	Examples from everyday world were used
3 Discuss (4 Items)	.843	Students questions and comments were valued
	.799	Class discussions were useful
	.719	Students were comfortable asking questions
	.653	Teachers' answers were valuable
4 Mathematical Fluency (5 Items)	.832	Emphasis on precise definitions
	.740	Emphasis on vocabulary
	.388	Emphasis on hands-on activities/labs
	.355	Emphasis on functions
	.319	Emphasis on mathematical reasoning

Multiple Representations Construct

The Multiple Representations Construct had a moderate positive relationship with the conceptual understanding item (r=0.301). This construct represented the use of multiple perspectives to teach mathematics. Mathematics education literature commonly describes using tables, graphs, and equations as multiple ways to present content (Elia, Panaoura, Eracleous & Gagatsis, 2007). Such methods provide connections across numeric, algebraic, and geometric concepts. Showing various ways to think about problem solving provides accessibility of content for heterogeneous groups of students.

Teacher enthusiasm is also part of this construct. Enthusiasm has been identified as a core quality of effective teaching (Kunter et al., 2008; Walberg & Paik, 2004). Although teacher enthusiasm is not immediately connected to multiple ways to teach content, this item was left in this construct because of the high factor loading (0.632). It could be either that enthusiastic teachers are more likely to present content in multiple ways, or that the very fact of showing multiple ways is perceived as a marker of teacher enthusiasm by the students.

Applications Construct

The Applications Construct had the weakest positive relationship with the conceptual understanding item (r=0.207). Real world or authentic problems have been defined as conveying contexts "for which there is no ready-made algorithm" (Kramarski, Mevarech & Arami, 2002, p. 226). Such problems typically connect to economic and societal issues and have appeared to improve student engagement in learning mathematics (Beswick, 2010). High school mathematics teachers often use such problems to address the question, "Why do I need to know this?"

Discuss Construct

The Discuss Construct had a weak positive relationship with the conceptual understanding item (r=0.254). This construct combined items representing teacher-student and student-student communication. The notion of mathematical discourse more precisely describes students engaging in meaningful problem solving, while conjecturing, scrutinizing, and defending problem solving ideas (Ball, 1993; Lampert, 1989). Although mathematical discourse may be a part of this construct, the teacher/student communication captured most of the variability, as evidenced by the high factor loadings. Hence, this construct was named discuss instead of discourse.

Mathematical Fluency Construct

The Mathematical Fluency Construct had a strong positive relationship with the conceptual understanding item (r=0.402). Mathematics is a language that includes definitions, vocabulary, numerals, symbols, and syntax that are at times interrelated and interdependent, and at other times disjointed and autonomous (Adams, 2003). Wakefield (2000) suggested that mathematical language includes abstractions, symbols, and expressions that improve with practice. The items “mathematical reasoning” and “hands-on activities/labs” also loaded into this construct. By definition, mathematical reasoning is observing generalizations, making connections between numbers and ideas, and drawing conclusions on the basis of evidence or stated assumptions (Martin et al., 2009). When students experience such instruction, hands-on activities are often integrated, as they work in groups and communicate ideas.

The Hierarchical Linear Models

Because the students were in different calculus courses at different colleges and universities, hierarchical linear modeling (HLM) was used to account for the nested data structure. For both the main effects model and the interaction model, listwise deletion of subjects with missing data would have substantially reduced the original number of respondents (4,342). The main effects model (Table 2) would have included only 2,881 respondents, and the interaction model (Table 3) would have included only 3,123 respondents. To diminish data loss, multiple imputation was used (Horton & Kipsitz, 2001), which resulted in both models including 4,176 respondents.

The Main Effects Model

The controls at the student level in this model were: gender (male=1, female=0); SAT/ACT mathematics concordance score (Schneider & Dorans, 1999), which combines SAT and ACT scores by mapping the ACT scores onto the SAT scale; high school final grades for geometry, algebra-2, precalculus, or calculus; and a dummy variable to differentiate if students' most recent high school mathematic course was either precalculus or calculus. The controls at the course level differentiated between college calculus courses that were required for a science, technology, engineering, or mathematics (STEM) major (coded as 1), and all other types of calculus courses, i.e., business or other non-STEM major courses (coded as 0). A dichotomous variable differentiated between the type and size of post-secondary institution (2-year institutions coded as 0; 4-year institutions coded as 1). However, because this variable was not a significant predictor of college calculus performance, it was removed from the models. The main effects model captured 19.3% of the variability in the data.

The negative parameter estimate for gender indicated that females' expected performance was a little over two points higher, on average, than the males' performance in college calculus. The parameter estimate for the calculus dummy variable showed that, if students had taken calculus (instead of precalculus) in high school, their college calculus performance was almost six points higher. The students' grade in high school precalculus or calculus had the largest standardized coefficient, meaning that students' grade in their

Table 2.
Main Effect Hierarchical Linear Model of College-level Calculus Grade ($N=4176$, $R^2=0.193$)

Variable or Construct	Parameter Estimate	Standard Error	Standardized Coefficient	Min	Max	Mean
Gender	-2.129***	0.402	-0.077	0.000	1.000	-----
SAT/ACT Math	0.021***	0.002	0.137	200.000	800.000	633.710
Geometry Grade	1.498***	0.367	0.067	0.000	4.333	3.716
Algebra 2 Grade	1.647**	0.390	0.072	0.000	4.333	3.702
Precalculus or Calculus Grade	3.737***	0.284	0.204	0.000	4.333	3.496
Calculus Dummy	5.890***	0.427	0.204	0.000	1.000	0.603
Multiple Representations	-0.667**	0.221	-0.050	-3.539	1.211	0.054
Applications	-0.534**	0.207	-0.039	-1.116	2.792	0.034
Mathematical Fluency	0.698***	0.216	0.051	-3.425	2.362	0.036
Type Calculus Course	-4.314***	0.778	-0.078	0.000	1.000	0.934

**p<0.05; **p<0.01; ***p<0.001*

most recent mathematics course most strongly predicted their performance in college calculus. Lastly, college calculus course type was a significant predictor for college calculus performance with a -4.314 parameter estimate. This indicates that students in college calculus courses for STEM majors scored about 4 points lower than students who were in non-STEM major calculus courses. Thus STEM major college calculus courses appear to be more rigorous than non-STEM major courses.

The four composite items, representative of the constructs, were entered as main effects into the models. The Discuss Construct was not a significant predictor of performance, so it was eliminated from the models. Only the Mathematical Fluency Construct was a positive predictor of performance in college level calculus, while the "Applications" and "Multiple Representations" constructs were significant, but negative predictors of performance.

The standardized coefficient for the Mathematical Fluency Construct (0.051) indicates that a one standard deviation increase in the Mathematical Fluency Construct increased the college calculus grade by a little more than 5% of a standard deviation. With a standard deviation of 13.4, this means, on average, an increase of about two thirds of a point (0.698) in final college calculus grade. This may appear low, but, in light of the generally low long-term effects of pedagogical strategies, the Mathematical Fluency Construct is still the one with the largest effect size. If high school teachers knew that instructional practices associated with this construct better prepared students for the school-to-college transition, they might purposefully incorporate them into their precalculus and/or calculus courses. Moreover, if teacher education programs knew that instructional practices in this construct predicted performance in future learning, they could emphasize them in the preparation of teachers. That might, in turn, enhance teacher candidates' performance on edTPA in planning, instructing, and assessing conceptual understanding. The other two main effects in the model had smaller and negative standardized coefficients.

The Interaction Model

There were two significant interactions between constructs and high school performance measures at the student level, as seen in Table 3. The parameter estimates and significance levels for the controls at the student level are similar to the variables in the main effects model (Table 2). The interaction coefficients between the Application Construct and precalculus or calculus grade and between the Multiple Representations Construct and the calculus dummy variable are shown in the table, with interpretations supported by Figures 1 and 2, respectively.

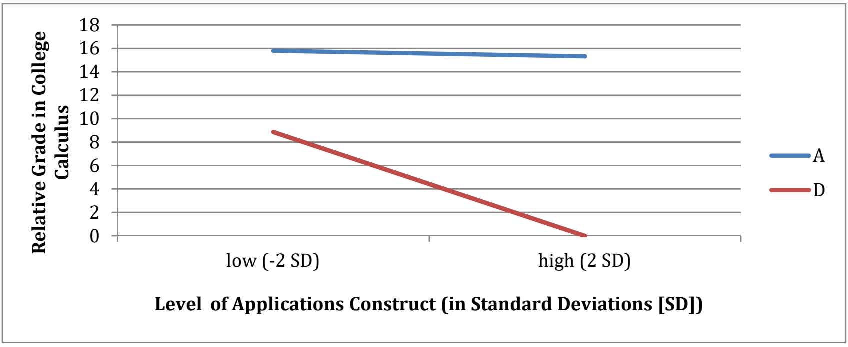
Table 3.
Interaction Hierarchical Linear Model of College Calculus Grade (N=4176, R^2=0.193)

Variable or Construct	Parameter Estimate	Standard Error	Standardized Coefficient	Min	Max	Mean
Gender	-0.228***	0.401	-0.081	0.000	1.000	-----
SAT/ACT Math	0.021***	0.002	0.139	200.000	800.000	633.710
Geometry Grade	1.534**	0.366	0.069	0.000	4.333	3.716
Algebra 2 Grade	1.680***	0.390	0.073	0.000	4.333	3.702
Precalculus or Calculus Grade	3.714***	0.286	0.212	0.000	4.333	3.496
Calculus Dummy	5.981***	0.427	0.207	0.000	1.000	0.603
Multiple Representations	-0.981**	0.347	-0.072	-3.539	1.211	0.054
Applications	-2.918**	0.937	-0.216	-1.116	2.792	0.034
Mathematical Fluency	0.679**	0.216	0.050	-3.425	2.362	0.036
Tertiary Calculus Course	-4.392**	0.778	-0.079	0.000	1.000	0.934
Interaction: Application and Precalculus or Calculus Grade	0.699*	0.257	0.186	-4.800	12.090	0.160
Interaction: Multiple Representations and Calculus Dummy	0.866**	0.412	0.052	-3.540	1.210	0.068

$*p<0.05$; $**p<0.01$; $***p<0.001$

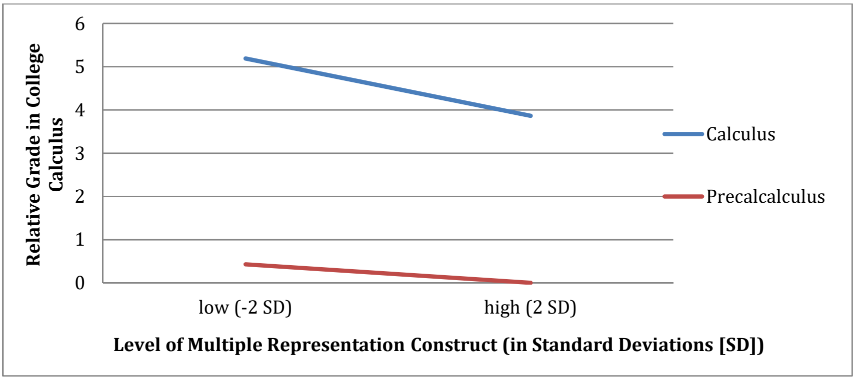
Figure 1 shows the interaction between the precalculus or calculus high school grade and the Applications Construct. This interaction suggests that the college calculus grade for students who had received an A in secondary mathematics was hardly affected by the amount of application focus they received through instruction in their high school class. By contrast, the weaker students (e.g., those with a D in their high school mathematics class) were more sensitive to the amount of application focus: The stronger the focus on applications, the lower their college calculus grade. The negative effect of the Application Construct found in the main effects model thus appears to be concentrated among the students with weaker preparation.

Figure1: Interaction: High school mathematics grade and the Application Construct



The second interaction was between the calculus dummy variable, differentiating between students who had precalculus or calculus as their most advanced mathematics course in high school, and the Multiple Representations Construct (Figure 2). This interaction shows that the level of multiple representation pedagogy impacted college calculus performance more for students who moved from high school level calculus than for those who moved from high school level precalculus. Thus, the negative effect on college calculus performance of the Multiple Representation Construct was concentrated among the calculus group. This appears to align with Bressoud's (2010) expressed concerns about high school calculus.

Figure 2: Interaction: Calculus dummy variable (differentiating between students who had calculus their senior year and those who had precalculus) and Multiple Representations Construct.



Limitation and Future Work

While there are many factors that influence the school to college transition, the focus of this research was students' instructional experiences in their high school precalculus or calculus course. One useful aspect of this research was that it empirically examined the constructs underlying conceptually understanding mathematics, as perceived by the students. On the other hand, this may be a limitation because how students recall their instructional experiences may not align with their teachers' views of what they actually did during instruction. More research is needed to understand teachers' views of their pedagogical practices and their experiences of preparing students for college calculus. That research should focus on the Mathematical Fluency Construct because it was a positive predictor of student performance in college calculus.

Discussion and Conclusion

Research about the transition to college calculus has addressed the instruction and learning of mathematics in terms of conceptual understanding (Bressoud, 2010; Clark & Lovric, 2008, 2009; Gueudet, 2008). In line with this emphasis, the conceptual understanding item in the FICSMath research was a positive significant predictor of performance in college calculus. However, because the students' ideas about what constitutes conceptual understanding in mathematics might have been somewhat nebulous and varied, we endeavored to ground the students' reports of conceptual understanding in their observations of more concrete instructional practices. Our initial investigation showed that 33 of the 70 instructional items on the FICSMath survey significantly correlated with the conceptual understanding item. After factor analysis and inspection of the correlation table, the number of items dropped from 33 to 18.

Because the Mathematical Fluency Construct was a positive and significant predictor of college calculus performance, we believe that this construct aligns with relational understanding, or knowing what to do and why (Skemp, 2006). The Mathematical Fluency Construct may advance understanding of what it means to have the "opportunity to learn" (Kilpatrick, Swafford, & Findell, 2001, p. 334) in high school precalculus and calculus courses. The opportunity to learn was described as "circumstances that allow students to engage in and spend time on academic tasks" and was found to be the single most important predictor of student performance (Kilpatrick et al., 2001, p. 333).

The Mathematical Fluency Construct had the largest effect size among the constructs in the main effects model, which suggests that when teachers emphasized definitions, vocabulary, functions, mathematical reasoning, and hands-on activities, students had instructional experiences that had a positive impact on *future* learning. In the end, this construct shows that an instructional emphasis on mathematical fluency helped students transfer their learning from high school mathematics to college calculus. The other three constructs either had no effect or small negative effects on college calculus performance. The Discuss Construct was not a significant predictor of performance in college calculus. For an explanation, we might look to Clark and Lovric (2008) who stated that part of the transition to college mathematics includes a change in the social climate of the classroom. Discussion and discourse have been investigated in secondary mathematics research, but little, if any, research has focused on that topic at the college or university level. This may be owed to the fact that students are more likely to have discussions with their teachers in high school classrooms than in large college or university auditoria (Clark & Lovric, 2009).

The Multiple Representations Construct was a significant and negative predictor of performance in college calculus. The interaction of this construct with the calculus dummy variable revealed the use of multiple representations as a negative predictor of college calculus performance, particularly for the high school calculus group. Bressoud (2010) noted that many colleges and universities now teach calculus in a more theoretical way, indicating professors are most likely to focus on theories that develop mathematical concepts instead of making connections using multiple representations. Although high school teachers may use multiple representations to present mathematics to heterogeneous groups of students, this instructional experience may be rare in college calculus. Clark and Lovric (2009) referred to disconnects between instructional styles as stemming from the “far from satisfactory” communication between high school teachers and university mathematics professors (p. 762).

The Applications Construct was also a significant and negative predictor of performance in college calculus. The use of real life problems has been found to align more with motivation to learn mathematics than with student performance (Beswick, 2010). Enthusiasm for the use of contextual problems appears to be in advance of the evidence for their effectiveness in increasing student performance (Beswick, 2010). Another hypothesis regarding the negative association between application problems and student performance in college calculus is that students may view application problems as ‘just hard mathematics problems.’ This would align with high school students’ general dislike for word problems.

Lastly, professors may not focus on contextual problems as secondary mathematics teachers do, which may reveal a disconnect in the school to college transition. The interaction of this construct with the grade from students’ last high school mathematics class, either precalculus or calculus, revealed that lower performing students in precalculus and calculus were more sensitive to low or high levels of application pedagogies. This again points to the challenges that application problems present to students learning mathematics. If students move through mathematical content applying instrumental reasoning, i.e., rules without reason (Skemp, 2006), then solving application problems may be especially challenging.

It is important to recognize that creating a high school classroom that fosters the understanding of mathematical concepts is a difficult undertaking that requires explicit effort on the part of the teacher (Yackel & Hanna, 2003). Preparing students for success in college calculus is clearly not the only goal of high school teachers of precalculus or calculus. Not all of their students will attend college, and, of the college-bound students, a sizable fraction will go into fields far removed from mathematics. For these groups of students, an appropriate goal is to instill a certain appreciation of mathematics concepts and a basic facility in mathematical thinking. However, when it comes to the future STEM workforce, it is known that single variable college calculus is a major gatekeeper for those pursuing degrees in this area. A poor performance in this course can prevent aspiring STEM professionals from realizing their career plans.

Preparing students to do well in college mathematics is one of the primary goals for high school mathematics teachers, but senior level mathematics teachers in high schools do not have the convenience of discussing with their colleagues how well prepared their students were for subsequent mathematics courses. This information may come to teachers if some of their previously graduated students communicate how well prepared they perceived they were for college calculus. However, such anecdotal feedback may be severely biased, because subsequent successes will be more likely than subsequent disasters to be reported back to the teachers. Our findings enable teachers to consider how to integrate the specific instruction from the Mathematical Fluency Construct into their precalculus and calculus courses so as to better prepare students for the school-to college-transition.

The Mathematical Fluency Construct also reveals which instructional practices should be modeled for preservice teachers who must successfully navigate the edTPA in order to complete a teacher educator program and receive state certification. This study apprises teacher educators who seek to prepare teacher candidates for the edTPA of instructional practices that correlate with conceptual understanding of mathematics. The description of how students demonstrate conceptual understanding in the edTPA documents indicates *assessing* conceptual understanding, but does not identify specific *instructional experiences* that support students learning concepts. It is these experiences that should be the focus of preservice teachers since they must plan and instruct for conceptual understanding before they can assess it.

We hope these findings contribute to smoothing the discontinuity between high school mathematics preparation and college mathematics expectations. Lastly, we hope that our results help mathematics professors become better informed about the impact of the instructional experiences that their students bring with them into college calculus.

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