Reviewing and Reflecting Skills During Mathematical Problem Solving

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Reviewing and Reflecting Skills
During Mathematical Problem Solving

by

William Michael Renick

APPROVED BY:

Dr. Conrad Van Voorst

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Date
Acknowledgments

I would never have succeeded in becoming a teacher or completing my Masters in Education without the love, encouragement and support of my precious wife Sarah. You listened to my rants and eased away my frustrations. You read countless papers and found non-threatening ways to let me know when my writing was unfocused and wandering. When I told you that I was ready to quit my job even though I had not found a teaching position, you held my hand, looked me in the eyes and told me that we would be okay. Thank you for everything, my love. You are Wonderful. <>>>you<<<>

Completing a classroom study when you are not a permanent teacher with your own students is very challenging. This paper would have never been possible without the help of established teachers willing to open their classroom doors to me. Patty, Rob, and Rob, I will be forever grateful for your willingness to take a chance on my work and me. Thank You.

There is always someone whose part is at the end of any great adventure. Being at the end does not indicate a lack of importance. David, thank you for your time and effort in reading this paper. Your effort has made it better. Thank You.
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I. Introduction

When a student graduates from school and prepares to become a productive member of society, he or she will be entering a technologically advanced society that is based on a global economy. As a second career mathematics teacher, I continually ask myself, “What can I do to best prepare my students to meet the challenges of our world?” When I look back at my first career as a software engineer and project manager, I always come up with the same answer, “Teach them to be problem solvers”. The problem solvers are the people on the team or project that do not need to be told what to do every step of the way. They know how to find solutions when the answers are not in the manual or do not fit the standard methods. Given the low number of good problem solvers I have known and worked with, there is a pressing need to include much more problem solving in our schools.

I believe that the mathematics classroom is one of the most appropriate places to teach our students problem solving skills, and I am not alone in this opinion. The National Council of Teacher of Mathematics (NCTM) has identified four key areas of problem solving that should be included in our schools’ mathematics instructional programs from prekindergarten through the twelfth grade: building new mathematical knowledge through problem solving, solving problems that arise in mathematics and other contexts, applying and adapting various appropriate strategies to solve problems; and monitoring and reflecting on the process of mathematical problem solving (NCTM, 2000). The importance of teaching problem solving skills
in the mathematics classroom is emphasized at the State level as well. The Department of Education in many States includes problem solving as part of their mathematical curriculum (AZ DE, 2008; CA SBE, 1997; NV DE, 2008; NYSED, 2005). Oregon’s Department of Education requires at least one local performance assessment of mathematical problem solving per year for grades three through eight and at least one in high school (OR DE, 2008). In New York State, the Mathematics Core Curriculum has only three instructional goals that are common to every grade level, of which problem solving is one (NYSED, 2005). There is a consensus at both the national and state level that problem solving should be a part of the mathematics curriculum.

While performing research for one of my education classes, I read an article that changed my viewing of problem solving and what it takes to teach problem solving. Wallace (2007), a mathematics teacher and educator with more than fifteen years of experience, explained that there is a difference between a student solving a problem and a student engaged in problem solving, and that it is not easy to recognize the difference. She describes how her students would try to figure out which method she wanted them to use so that they could follow known steps to solve the problem. Her students were reluctant to be problem solvers.

I realized that if a veteran mathematics teacher like Wallace was facing challenges teaching her student to problem solve, as a recently certified mathematics teacher, I would need to carefully define what I meant by problem solving and narrow the focus of my study. Alan Schoenfeld is a well known researcher of mathematical
education. Many researchers’ projects use his 1983 definition of problem solving which states that for an activity to be a problem, the person must not know how to go about solving it, and the problem must not be routine or familiar (Carlson & Bloom, 2005). Other studies use a variation of Schoenfeld’s definition. They define problem solving as a process utilizing non-routine, open-ended questions that require different problem solving strategies, such as drawing pictures, looking for patterns, guessing and checking, systematic listing or working backwards (Roberts & Tayeh, 2007; Jones, 2006). For my study, problem solving will be defined as a process for non-routine questions that cannot be solved by simply applying known formulas and mathematical procedures. However, the use of mathematical formulas and procedures may be required.

There are many different problem solving strategies one can teach in their mathematics classroom and each strategy is made up of multiple phases, steps or stages (Polya, 1990; Garofalo & Lester, 1985; Carlson & Bloom, 2005; UCLA, 1997). To narrow this study into a manageable project, I looked for an element that was common to all problem solving strategies. Every one of the problem solving strategies I have investigated or used has included activities for reviewing, reflecting on and verifying the problem, the work, or the answer. If my students can improve their reviewing and reflecting skills during problem solving, I will have made them more successful problem solvers regardless of which strategy they choose to use.

Including writing activities during mathematical problem solving has been shown to be beneficial, as writing about their mathematics helps students to organize
their thoughts, better understand concepts, and focus on difficult aspects of mathematics (NCTM, 2000). Can I include writing activities in my problem solving lessons in such a way that my students' ability to reflect upon the problems they are solving and to review their work improves? This study will investigate just that: Whether the use of writing during mathematical problem solving improves the quality of reviewing and reflecting which students perform.

II. Review of Related Literature

Problem Solving Methods

Schoenfeld (MAA, 1983) noted that the amount of problem solving literature is vast and at times overwhelming. Over the past twenty-five years that amount has only grown. One of the reasons for this immense amount of literature is the fact that there are many different disciplines, such as cognitive sciences, socio-culture studies and information processing, which are investigating problem solving, yet there is no widely accepted definition of problem solving (UCLA, 1997). However, there are defining moments in the history of problem solving. One just moment occurred in 1945, described as a year of great chaos for problem solving; G. Pólya’s *How to Solve It* has been called a demarcation line between two eras of problem solving (Schoenfeld, 1987; Michalewicz & Fogel, 2000). *How to Solve It* has gone through multiple publishing and copyright renewals since the original 1945 edition (Pólya, 1990). In addition to the number of printings, a web search from Google Scholar in the spring of 2010 showed that *How to Solve it* has been cited over 2,400 times. Pólya’s books and problem solving framework and work are well known within the
academic community (Garofalo & Lester, 1985). For this problem solving investigation, *How to Solve It* will be considered the starting point for the modern problem solving process.

Pólya (1990) defined problem solving as a four phase process. The first phase of the process is *Understand the Problem*. During this phase, the problem solver looks for what is unknown, what data are, and what the condition of the problem is. The problem is separated into its various parts, pictures are drawn and suitable notation is used to understand the problem. The second phase of the process is *Devises a Plan*. The problem solver looks for ways to connect and relate previous knowledge to the problem at hand. The third phase of the process is *Carrying Out the Plan*. The problem solver completes and checks each step of their plan. While checking a step, the problem solver verifies that it has been completed and that the results are correct. The fourth phase in Pólya’s problem solving process is *Looking Back*. The problem solver checks the overall results and asks if there are different ways to get the same answer, and whether their results answer the question. The four phases of Pólya’s problem solving process, *Understand the Problem, Devise and Plan, Carrying Out the Plan and Looking Back*, established a solid foundation for future problem solving processes to build upon.

Garofalo & Lester (1985) defined a four phase framework for performing a range of mathematical tasks and did not limit the framework to problem solving. One of the purposes of their framework was to highlight where metacognitive decisions are likely to influence a person’s cognitive actions. The first phase of the framework
is Orientation: Strategic Behavior to Assess and Understand a Problem. During this phase, a person uses comprehension strategies, analyzes information and conditions, and assesses their familiarity, the level of difficulty and their chances of success with the problem. The second phase is Organization: Planning of Behavior and Choices of Actions. In this phase, a person performs global and local planning and identifies goals and sub-goals. The third phase is Execution: Regulation of Behaviors to Conform to Plans. During this phase, a person is completing tasks, monitoring progress of local and global plans and making trade-off decisions (e.g., speed vs. accuracy) as needed. The fourth phase of the framework is Verification: Evaluation of Decisions Made and of Outcomes of Executed Plans. During this phase, a person does not limit their reviewing and reflecting to their results and accuracy. They also reflect on their actions and the consistency of their work when compared to their plans. Garofalo & Lester’s framework is not a replacement of Pólya’s problem solving process. It is more of a broadening of Pólya’s problem solving model, focusing on the metacognitive processes involved during problem solving.

O’Neil & Schacter (UCLA, 1997) reviewed several problem solving frameworks from cognitive science literature for the purpose of creating a problem solving model for use with computer-based problem solving assessments. Their effort resulted in the CRESST model of problem solving (Figure 1). The CRESST model is a four element problem solving framework. The first element is Content Understanding. The second element is Problem-Solving Strategies. The third element is Metacognition. In this element, there is a self-checking component.
Self-checking looks both at verifying accuracy and completeness of work, and at self-monitoring tasks of how well the person is focusing on the problem solving task and how the overall work is progressing. The final element in the CRESST model is \textit{Motivation}. Although the CRESST model was derived for the purpose of computer-based problem solving assessment, the model includes many aspects that are found in previously mentioned problem solving methodologies.

Carlson \& Bloom (2005) approached the problem solving methodology differently than many researchers. Instead of creating a problem solving framework or modifying an existing one, Carlson \& Bloom observed expert problem solvers during problem solving and documented the processes they observed. Carlson \& Bloom observed a four phase problem solving cycle. The first phase of the problem solving cycle is \textit{Orienting}. During this phase, expert problem solvers' predominate behaviors include making sense of, organizing and constructing the problem. The
second phase is *Planning*. During this phase, the problem solvers consider multiple approaches and make conjectures about the possible success of each. The planning phase is a sub-process within the overall problem solving cycle. The problem solver identifies a possible approach for solving the problem, imagines how the approach would play out, and then evaluates the viability of the approach. If the problem solver thinks the approach has a low chance of success, the sub-process is repeated with a new approach. The third phase is *Executing*. During this phase, the expert problem solvers make constructions and complete computations. The final phase of Carlson & Bloom's problem solving cycle is *Checking*. During this phase, the problem solvers' efforts shift to verification activities. The problem solvers also contemplate their results and decide if they had completed the problem or if they needed to return to the planning phase of the cycle. The significant difference in Carlson & Bloom's four-phase problem solving methodology and the ones previously discussed is that it is a cycle, and the problem solver expects to repeat phases until an acceptable outcome is reached.

The problem solving frameworks of Pólya, Garofalo & Lester, O’Neil & Schacter and Carlson & Bloom are only a few of the problem solving methodologies available to the mathematics teacher. The other problem solving frameworks utilized in the studies reviewed for this investigation all included the main elements of problem solving, understanding the problem, creating a plan, completing the work and reviewing the results (Charles & Lester, 1984; Leitze & Mau, 1999; Thomas, 2006; Taylor & McDonald, 2007).
**Reviewing and Reflecting During Problem Solving**

Occasionally an exceptionally bright or lucky student may successfully jump to the correct solution of a problem without going through each of the problem solving phases - *Understand the Problem, Devise a Plan, Carrying Out the Plan* and *Looking Back* - but often undesirable results occur when problem solving steps are left out (Pólya, 1990). Even though all four phases of the problem solving process are important, there are some activities that occur more often than others. Most problem solving frameworks dedicate a complete phase to reviewing and reflecting activities and many include additional reviewing and reflecting activities within other phases.

It is easy to see that Pólya’s *Looking Back* phase is about reviewing and reflecting upon the work and results. However, a closer look at the *Understanding the Problem* and *Carrying Out the Plan* phases reveals that reviewing and reflecting are important elements within these phases as well. Pólya’s *Understanding the Problem* phase has two parts. The first part is ‘getting acquainted with the problems’. The problem solver works to understand the details of the problem. In the ‘working for a better understanding’ part, the problem solver reviews the details of the problem and reflects on their previous experiences with similar situations and information. Within the *Carrying Out the Plan* phase, Pólya challenges problem solvers to review their work and to prove to themselves the correctness of each step. For complex problems, Pólya recommends that both major steps and minor steps are reviewed and
verified. Reviewing and reflecting are important elements throughout Pólya’s problem solving framework (Pólya, 1990).

The importance of reviewing and reflecting throughout Garofalo & Lester’s problem solving framework follows Pólya’s model. Within the Orientation phase, the problem solver assesses their chance of successfully completing the problem. A good problem solver will reflect on previous experiences for an accurate assessment of their chance of success. In the Execution phase, the problem solver monitors the progress of both local and global goals. This task requires the problem solver to continually review their completed work and their plans. In Garofalo & Lester’s final phase, Verification, the reviewing and reflecting activities are not limited to the accuracy of the computations or the correctness of the problem solution. The problem solver also reflects upon the adequacy of their plans and actions as well as on the consistency of local and global plans with the actual work they performed. Reviewing and reflecting activities are used throughout the problem solving framework (Garofalo & Lester, 1985).

While observing expert problem solvers, Carlson & Bloom (2005) noted reviewing or reflecting behaviors in all of the problem solving phases. During the Orientation phase, Carlson & Bloom observed reflective behaviors in all of the problem solvers. The Planning phase has sub-cycles where the problem solvers consider and evaluate different approaches for solving the problem. Typically before problem solvers move into the Execution phase, they reflect on the problem solving methods considered and the decisions they make. During the Execution phase,
Problem solvers reflect on the progress for the approach they are attempting. When the problem solvers are done with the execution phase, they shift into the *Checking* phase and spontaneously assess the correctness of their computations and results. If the problem solvers are not satisfied with their results, they move back into the *Planning* phase and identify another approach to try. Reviewing and reflecting is not limited to computations and a final answer.

The mathematicians regularly engaged in metacognitive behaviors that involved reflecting on the effectiveness and efficiency of their decisions and actions. These reflections were exhibited frequently during each of the four problem-solving phases, and they appeared to move the mathematicians’ thinking and products in generally productive directions. (Carlson & Bloom, 2005, p. 64)

Through observing the problem solving behaviors of expert problem solvers, Carlson & Bloom have identified reviewing and reflecting to be an important activity throughout the entire problem solving cycle.

No two problem solving frameworks are exactly the same. One framework is designed with a focus on metacognition (Garofalo & Lester, 1985). Another framework is designed for use with elementary or middle school students (Leitze & Mau, 1990; Thomas, 2006) while another framework is based on the problem solving behaviors of expert mathematicians and problem solvers (Carlson & Bloom, 2005). Yet regardless of the design or the purpose of the problem solving strategy, reviewing and reflecting are key elements.
Students’ Reviewing and Reflecting During Mathematical Problem Solving

During the course of this literature review, all the problem solving processes investigated include reviewing or reflecting activities. As discussed above, Carlson & Bloom (2005) observed expert problem solvers reviewing and reflecting through the entire problem solving cycle. However, my experience with students in the mathematics classroom tells a different story. I have watched students push the buttons on a calculator and write down an answer that does not make any sense for the question asked. I have seen students accurately complete a problem only to enter the wrong answer on a multiple choice on the answer sheet. Pugalee (2004) found that students checked computations less during the Verification phase than in earlier problem solving phases. In her research, Jones (2006) found that when a student was confident in their ability to solve a problem, they did not feel the need to reflect on or check their work. Schoenfeld (1989) noted that student can provide proof of understanding of the prerequisite mathematical knowledge required to solve a problem, and then proceed to completely ignore the necessary mathematics they know and not successfully solve a problem. Letize & Mau (1999) found the opposite was also true; “Many students have an ability to give the correct numeric answer without ever seeming to understand the problem” (p. 305).

Research has shown that reviewing and reflecting activities are beneficial to students. Reviewing and reflecting can help students understand what they know. Looking back and reflecting is the process that provides the context for students to
learn and understand mathematics (Cai & Brooks, 2006). Reviewing their work during problem solving can help students identify when a different problem solving strategy is needed. Good problem solvers regularly monitor their thinking and are aware when they should rethink the problem or switch their strategy (Robert & Tayeh, 2007). The more strategies attempted to solve a problem, the greater the likelihood of the problem being solved correctly and gifted students utilize more strategies than average or below average students (Pugalee, 2004; Montague & Applegate, 2000; Lesh, 1981). However, reviewing and reflecting is a problem solving activity that is available to all levels of learners. Through reviewing and reflecting on their own thinking, students significantly impact their ability to solve problems now and in the future (Roberts & Tayeh, 2007).

**Mathematics and Writing**

Including writing activities during mathematics and problem solving has been shown to be beneficial, as writing about their mathematics helps students organize their thoughts, understand concepts and focus on difficult aspects of mathematics (NCTM, 2000). Pugalee (2004) compared students who wrote about their thinking processes and the steps they took during problem solving with students who thought aloud while problem solving. He found that the students who wrote about their problem solving activities reread the problems twice as often as the students who used the think-aloud strategy. He also found that students who wrote about their thinking achieved more monitoring goals during the Execution phase and were significantly more successful with their problem solving than their non-writing peers.
Williams (2003) compared the problem solving performance of students who wrote about the Executive processes – their understanding of the information provided, the strategies they attempted and the difficulties encountered with students who only had to solve the problems. Both the students who wrote and the ones that did not write showed improvement in their problem solving skills. However, the students who wrote during problem solving showed greater improvement. When surveyed, eighty percent of the students stated that the writing activity made them better problem solvers, and seventy-five percent of them said they enjoyed the writing activity. The one concern with this study is that the teacher responded to all of the students’ writing, with no control group of students whose writing did not receive feedback. It was therefore not clear if the greater problem solving improvement came from students’ writing activities, the teacher’s feedback on the writing assignments, or a combination of both.

Taylor & McDonald (2007) developed a problem solving curriculum for first year university mathematics. The course included a problem solving workshop where the students were assigned to groups of three or four students. During the first offering of the course Taylor & McDonald realized that the process was not engaging all of the students. To correct this deficiency, the second offerings of the course included writing in the workshop. When the results from the first two offering are compared, there is evidence that the students who wrote during the second workshop performed better than the students in first offering. The inclusion of writing slowed down the entire problem solving process. Within their groups, the students spent time
reflecting on their work and verified their results and conclusions before being satisfied that the problem was complete. When surveyed, ninety-one percent of the students agreed that the workshop helped to develop mathematics communication, and eight-four percent agreed that the workshops assisted in developing problem solving skills.

Steele (2007) used writing assignments to understand students’ problem solving knowledge. The students were given eight problems to solve and were asked to write about how they arrived at their solution and to elaborate on and make generalizations from what they learned from solving the problem. Steele observed that detailed writing during problem solving helped the students to include all steps in the process and prevented gaps in reasoning.

Most of the research identified during this literature review dealt with students who wrote about their efforts to understand and solve problems, and all of the research showed positive results from the students who were writing while problem solving (Pugalee, 2004; Williams, 2003; Taylor & McDonald, 2007; Steele, 2007; Roberts & Tayeh, 2007). Carter (2009) had her second grade students write stories that would help other children understand mathematics. Her students became excited about writing and about mathematics, and they reread their stories to verify the correctness of the mathematics. Tong (2009) included a vocabulary activity and an acrostic poem to her mathematics class. Her results showed a positive correlation between the writing exercises and the understanding of mathematical concepts.
Conclusion

Both problem solving and writing are important aspects of New York State's Mathematics Core Curriculum (NYSED, 2005). Problem solving is one of the three foundational learning goals in mathematics for prekindergarten through the twelfth grade. From these three learning goals come five process strands and five content strands that weave together to build New York State's mathematics curriculum. Problem solving and communication are two of these process strands. As process strands, problem solving and communication impact each of the content strands. One of the goals within the problem solving strand is for the students to monitor and reflect on the process of mathematical problem solving. This study will focus on students' reviewing and reflecting during problem solving. Two of the goals for the communication strand are for students to organize and consolidate their mathematical thinking through communication, and to communicate their mathematical thinking coherently and clearly to peers, teachers and others. Although writing is not specifically called out at the process strand level, writing is specifically included in many grade levels starting with the second grade. Investigating the effectiveness of writing to improve student reviewing and reflecting during problem solving is beneficial to New York States' Mathematics Core Curriculum.

Educational researchers are constantly looking for a better way to teach students how to become successful problem solvers. Throughout my investigation of problem solving literature, I have only found problem solving processes that include reviewing and reflecting. It is a key element of problem solving. Additionally, I have
found many examples in which writing was included during problem solving and mathematics. The majority of the studies had the students write about their problem solving process from beginning to end. A few of the studies I found included writing stories or poems. However, I was not able to find any studies that focused exclusively on writing about the reviewing and reflecting which a student performs during problem solving. This investigation will study the impact of having students write about their reviewing and reflecting during problem solving.

III. Development of Hypotheses and Outcome Measures

Hypothesis

This study investigated the effectiveness of writing to improve reviewing and reflecting skills during problem solving. The investigation took place in two seventh grade classes. Both classes had received the same lessons from the same teacher for the 2009 – 2010 school-year. A baseline pretest was given at the start of the investigation. It was expected that both classes would have similar scores on this assessment. During the course of the investigation, both classes received the same problem solving and reviewing lessons, and the same homework problems. However, only the test class wrote about their reviewing and reflecting activities during the problem solving homework. The hypothesis was that the homework writing activity would improve the quality of the reviewing and reflecting the test class performs during problem solving. If this hypothesis was valid, the test class would score significantly higher when the concluding post test assessment was given.
Measures

The primary performance measures for this study were the baseline and concluding assessments. The baseline assessment established how effective the students were in solving four different types of problems, how many problems were answered incorrectly when the errors could have been found and corrected through reviewing and reflecting, and that the test and control classes statistically performed the same. The concluding assessment was used to measure each class’s problem solving performance improvement and to compare the test and control class’s overall problem solving performance.

The weekly homework assignments were used for secondary measurements. First, each class’s weekly performance was compared to its previous weeks’ to see if the percentage of correct answers improved. Then the class averages were compared to each other. For the test class, the students’ written responses were reviewed to determine if the amount of reviewing and reflecting increased and if the quality of the reviewing and reflecting improved.

The final output that was collected during the course of this study was my personal research notes. During each class I observed the students’ interactions with each other and the teacher, looked for insightful reflections from the students, and watched for questions that the teacher asked to promote deeper thinking in the students. Additionally, I made observations from the students’ homework and identified adjustments for the next problem solving lesson. These notes assisted me
in understanding how the students’ problem solving abilities changed during the
course of the problem solving unit.

IV. Methods and Procedures

Introduction

Problem solving continues to be an important element of our schools’
mathematics curriculum (NCTM, 2000; NYSED, 2005). Research on problem
solving has covered many different aspects ranging from methodologies to student’s
anxiety and self esteem (UCLA, 1997; Jones, 2006). The purpose of this study was to
investigate the effectiveness of writing during problem solving to improve the quality
of students’ reviewing and reflecting activities.

Participants

The subjects for this study were seventh grade students from a private
suburban school in Western New York. The seventh grade student body was divided
between two mathematics classes. Both classes had received the same lessons from
the same teacher for the 2009 – 2010 school-year. One class was randomly selected
to be the test class. The test class had eleven students; four were females and seven
were males. Two of the students were Asian-American. The control class had ten
students; six were females and four were males. One of the students was an African-
American. For both classes, English was the students’ first language and all students
were performing at grade level without any documented learning disabilities. The
test and control classes were similar without any notable difference.
Assessments

This study utilized a pretest-post test model for assessing the students. At the start of the study, both classes were given a forty minute class period to complete the baseline assessment, and the students were allowed to use calculators. The assessment contained eight problems covering four different problem solving strategies: drawing a picture, using matrix logic, making a list, and guessing and checking. The assessment had two problems for each strategy. This assessment was used to determine if the students understood how and when to use a given strategy. This information was used to guide the weekly problem solving lessons. A student did not have to use the intended problem solving strategy to receive full credit on a problem. The assessment was also used to measure the students reviewing and reflecting activities. If a student had an incorrect answer due to a calculation error or the misuse of information provided, the problem was scored as a reviewing and reflecting error.

At the end of the study, both classes were given a forty minute class period to complete the concluding assessment, and the students were allowed to use calculators. Like the baseline assessment, this test had eight problems covering the same four problem solving strategies with two problems per strategy. Some of the problems on the concluding assessment were similar in nature to problems from the baseline assessment or one of the weekly lessons. However, every problem throughout the study was unique. All the problems from both assessments can be found in the appendix.
In their book *How to Evaluate Progress in Problem Solving*, Charles, Lester & O'Daffer (1988) described four different techniques for evaluating problem solving: Observe and question students, Using student self-assessment data, Holistic scoring, and Multiple-choice and completion tests. Some of the advantages of a focused holistic approach are that it looks at the problem solving processes and not just final answers, it provides specific criteria to guide the scoring of the problem solution, and it provides one score to describe the student’s performance. Many research studies use a focused holistic approach for evaluating students' problem solving performance (Thomas, 2006; Williams, 2003; Leitze & Mau, 1999). For this study a focused

**Figure 2**

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<td><strong>4 Points</strong></td>
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holistic approach was used to create an assessment standard (Figure 2) that would highlight when an incorrect answer could have been identified and corrected by the student through reviewing and reflecting on their work. This standard was used for both the baseline and concluding assessments.

**Weekly Lessons**

This study had four weekly problem solving lessons. Each lesson focused on reviewing and reflecting activities and one of the four problem solving strategies: drawing a picture, using matrix logic, making a list, or guessing and checking. The weekly lessons were taught on Monday by the students’ mathematics teacher. All the lessons followed the same format. The lesson started with questions and comments about the previous week’s homework assignment, and an answer key was provided to the students. The new problem solving strategy was introduced and the baseline assessments were handed back to the students. The teacher worked with the whole class to complete the two problems from the baseline assessment that matched the lesson’s strategy. Once a problem was completed, the teacher had the students come up with a list of ways to review and reflect on the problem. Next, the students worked in pairs to complete a new problem and listed four things they had done to review and reflect on the answer and their work. When all the pairs had completed the problem, the whole class worked through the problem on the SMART™ Board and shared their list of reviewing and reflecting activities. At the end of the class, the baseline assessments were collected and two homework problems were assigned.
The homework problems were selected to reinforce the problem solving strategy taught during the lesson. The control and test classes were assigned the same problems, but the instructions for the test class included the additional step of writing up three different things the student had done to review and reflect on their work. The homework was due on the last day of the week, but was accepted on Monday before class began.

Throughout the course of the study, my role was limited to researcher and observer. The classroom teacher and I discussed the different options for teaching the weekly lessons and it was decided that the best way to maximize the students’ involvement in the lessons was for their teacher to deliver the problem solving and reviewing and reflecting lessons. This reduced the impact that a unfamiliar teacher would have had on the students’ performance and the study’s outcome. While observing the lessons, I clarified any questions that the students or teacher had about any given problem (e.g., is zero an odd or even number). I asked questions of the teacher and students to highlight important reviewing or reflecting activities that I felt were missed during the lesson. While the students worked in pairs, I walked around, watched and listen to the students solve the problems. There were times when I asked a student to reread a problem or praised them for an insightful reviewing strategy. By limiting my activities to researcher and observer, I believe that the students participated and learned more from the lessons while I was able to capture notes about classroom interactions.
For the homework, a focused holistic standard (Figure 3) similar to the assessment standard was created.

<table>
<thead>
<tr>
<th>Homework Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0 Points</strong></td>
</tr>
<tr>
<td>The problem had one or more of the following characteristics:</td>
</tr>
<tr>
<td>➢ The problem is blank</td>
</tr>
<tr>
<td>➢ Data from problem is written down, but there is no associated work.</td>
</tr>
<tr>
<td><strong>1 Point</strong></td>
</tr>
<tr>
<td>The problem had one or more of the following characteristics:</td>
</tr>
<tr>
<td>➢ There is an incorrect answer with no other work showing</td>
</tr>
<tr>
<td>➢ Inappropriate problem solving strategy that would most likely not provide the correct answer</td>
</tr>
<tr>
<td>➢ It appears that the student does not have or is not using required prerequisite knowledge</td>
</tr>
<tr>
<td>➢ Insufficient work to determine if the error is procedural or a calculation error.</td>
</tr>
<tr>
<td><strong>2 Points</strong></td>
</tr>
<tr>
<td>The answer was incorrect with one or more of the following characteristics:</td>
</tr>
<tr>
<td>➢ Calculation Errors</td>
</tr>
<tr>
<td>➢ Misuse of data provided</td>
</tr>
<tr>
<td>➢ Information included that was not provided by problem.</td>
</tr>
<tr>
<td><strong>3 Points</strong></td>
</tr>
<tr>
<td>The correct answer was given in terms of the problem.</td>
</tr>
</tbody>
</table>

**V. Analysis of Data and Interpretation of Results.**

To compare the baseline and concluding assessment scores for the two classes, independent t-Test analysis was performed. The baseline assessments from the two classes were compared to verify that the test and control classes’ performance was the same. The t-Test showed that there was no significant difference in the way the two classes performed (Figure 4).
Since reviewing and reflecting activities were the focus of the study, only problems that the students attempted were used to calculate percentages. A blank
score for a question indicated that a student did not attempt the problem and the question was not used for calculating the percentage. Scores of zero indicated that the student attempted the problem and a score of zero was used in the percentage. The t-Test analysis of the concluding assessments also showed that there was no significant difference between the control and test classes (Figure 5). As with the baseline analysis, only problems attempted by the students were included in the percentages and analysis.

Figure 5

<table>
<thead>
<tr>
<th>Control Class</th>
<th>Concluding Post Test Scores by Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>Q1</td>
</tr>
<tr>
<td>C01</td>
<td>3</td>
</tr>
<tr>
<td>C02</td>
<td>1</td>
</tr>
<tr>
<td>C03</td>
<td>2</td>
</tr>
<tr>
<td>C04</td>
<td>3</td>
</tr>
<tr>
<td>C05</td>
<td>2</td>
</tr>
<tr>
<td>C06</td>
<td>3</td>
</tr>
<tr>
<td>C08</td>
<td>2</td>
</tr>
<tr>
<td>C09</td>
<td>3</td>
</tr>
<tr>
<td>C10</td>
<td>3</td>
</tr>
<tr>
<td>C11</td>
<td>4</td>
</tr>
<tr>
<td>Average Score</td>
<td>65%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Class</th>
<th>Concluding Post Test Scores by Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>Q1</td>
</tr>
<tr>
<td>T01</td>
<td>2</td>
</tr>
<tr>
<td>T02</td>
<td>2</td>
</tr>
<tr>
<td>T04</td>
<td>2</td>
</tr>
<tr>
<td>T05</td>
<td>2</td>
</tr>
<tr>
<td>T06</td>
<td>4</td>
</tr>
<tr>
<td>T07</td>
<td>3</td>
</tr>
<tr>
<td>T08</td>
<td>3</td>
</tr>
<tr>
<td>T09</td>
<td>3</td>
</tr>
<tr>
<td>T10</td>
<td>2</td>
</tr>
<tr>
<td>T11</td>
<td>3</td>
</tr>
<tr>
<td>T12</td>
<td>3</td>
</tr>
<tr>
<td>Average Score</td>
<td>65%</td>
</tr>
</tbody>
</table>

26
Both classes improved throughout the study. However, there were two exceptions. First, one of the students in the test class had the same score for both the baseline and concluding assessments (Figure 6). This was unexpected because the student had completed three of the four weekly homework assignments and had a combined homework score of eighty-eight percent.

Figure 6
The other exception was the Making a List problem strategy taught during week three. This was the only time during the study when the students failed to score higher than the previous week’s homework (Figure 7). The students in both classes were not as familiar with the Making a List strategy as they were with the other problem solving strategies, and they appeared reluctant to use the strategy. A common question in both classes was “Do we have to make a list?” The fact that this lesson was taught the week before spring break could have contributed to the lower scores. Once again, a blank score for a question indicated that a student did not attempt the problem and the question was not used for calculating the percentage. Scores of zero indicated that the student attempted the problem and the score of zero was used in the percentage.
Analysis of the test class' homework identified seven different types of reviewing and reflecting activities that the students wrote about. Checking calculations and verifying work were considered to be one type. The other activities were rereading the problem, checking to see if results answered the question, doing the problem again the same way, checking the problem's set-up, thinking about the
problem, and doing the problem again a different way. The reviewing and reflecting activities the students wrote about varied from problem to problem (Figure 8).

**Figure 8**

<table>
<thead>
<tr>
<th>Student</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Total Reviews &amp; Reflections</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
<td>Q1</td>
<td>Q2</td>
<td></td>
</tr>
<tr>
<td>T01</td>
<td>RR, RD, CQ</td>
<td>RR, RD</td>
<td>CC, RR, RR</td>
<td>CC, RR, CC</td>
<td>RR, RR, CC, CS</td>
</tr>
<tr>
<td>T02</td>
<td>RR, RD, RR</td>
<td>CC, RR, RR</td>
<td>CC, RR, CC</td>
<td>CC, RR, RR</td>
<td>RR, RR, CC, CS</td>
</tr>
<tr>
<td>T04</td>
<td>T, CC, RR</td>
<td>CC, RR, RR</td>
<td>CC, RR, RR</td>
<td>CC, RR, RR</td>
<td>RR, RR, CC, CS</td>
</tr>
<tr>
<td>T05</td>
<td>RR, CC, CC</td>
<td>RR, CC, CC</td>
<td>RR, CC, CC</td>
<td>RR, CC, CC</td>
<td>RR, RR, CC, CS</td>
</tr>
<tr>
<td>T06</td>
<td>T, DW, RR</td>
<td>DW, RR</td>
<td>RR, CC, RD</td>
<td>RR, RD, CC</td>
<td>RR, RR, RR, CC, RR, CC</td>
</tr>
<tr>
<td>T07</td>
<td>RR, DW, RR</td>
<td>RR, RD, RR</td>
<td>CC, CC, RR</td>
<td>CC, RR, RR</td>
<td>RR, RR, CC, CS</td>
</tr>
<tr>
<td>T08</td>
<td>RR, DW, RR</td>
<td>RR, RD, CC</td>
<td>RR, RR, CC</td>
<td>RR, RD, CC</td>
<td>RR, CC, CC, CC, CC</td>
</tr>
<tr>
<td>T09</td>
<td>RR, DW, RR</td>
<td>RR, RD, RR</td>
<td>RR, RR, CC</td>
<td>RR, RD, CC</td>
<td>RR, RR, CC, CC, CC</td>
</tr>
<tr>
<td>T10</td>
<td>RR, RR</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CC</td>
<td>RR, RR, CC, CC, CC</td>
</tr>
<tr>
<td>T11</td>
<td>RR, RR</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CC</td>
<td>RR, RR, CC, CC, CC</td>
</tr>
<tr>
<td>T12</td>
<td>RR, RR</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CQ</td>
<td>RR, CC, CC</td>
<td>RR, RR, CC, CC, CC</td>
</tr>
</tbody>
</table>

**Legend**

- CC: Checked Calculation / Verified work
- RR: Reread the problem
- CQ: Checked to see if the results answered the question
- RD: Did the problem a second time the same way
- CS: Checked the problem set up
- T: Thought about the problem
- DW: Did the problem a second time a different way

Checking calculations and verifying work and rereading the problem were the most frequently written about strategies (Figure 9). After some reflection on my part, this was not a complete surprise. When a mathematics teacher notices that a student has an error in their work, the teacher gives the student direction so that the student can find the problem on their own. If the error is in the student’s calculation, the teacher asks the student to check or verify their work. When the error is a result of the student misinterpreting the problem, the teacher asks the student to reread the problem. If the student’s work is correct but incomplete, the teacher asks the student to verify that they have answered the question. During the course of this study, the students in both classes were taught to use many different reviewing and reflecting strategies that included checking work, rereading the question and verifying that the
question was answered. When the students wrote about their reviewing and reflecting, I had hoped that they would include the insightful reviewing and reflecting that they talked about in class. In some cases, the students’ homework showed the same thoughtful reviewing and reflecting, but in the majority of the cases, the students wrote what they were taught by all of their previous mathematics teachers and what they thought I wanted to see.

Figure 9

<table>
<thead>
<tr>
<th>Student</th>
<th>CC</th>
<th>RR</th>
<th>CQ</th>
<th>RD</th>
<th>CS</th>
<th>T</th>
<th>DW</th>
<th>Number of Different Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>T01</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>5</td>
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<tr>
<td>T02</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>T04</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>T05</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td></td>
<td>3</td>
<td>2</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>T06</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>T07</td>
<td>6</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>T08</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>T09</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>T10</td>
<td>11</td>
<td>6</td>
<td>5</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>T11</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>T12</td>
<td>11</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>76</td>
<td>57</td>
<td>31</td>
<td>13</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>31</td>
</tr>
</tbody>
</table>

Legend
CC: Checked Calculation / Verified work
RR: Reread the problem
CQ: Checked to see if the results answered the question
RD: Did the problem a second time the same way
CS: Checked the problem set up
T: Thought about the problem
DW: Did the problem a second time a different way

The analysis of the data collected throughout the study did not support the hypothesis that writing about reviewing and reflecting during problem solving would improve problem solving performance scores.
VI. Discussion, Summary and Reflections

While researching problem solving, I became interested in the fact that reviewing and reflecting was an important component of many problem solving methodologies. While examining the research of Pugalee (2004), Williams (2003), Taylor & McDonald (2007) and others, I developed an appreciation for the effectiveness of students’ writing during the problem solving processes. I wondered if writing would be effective in improving students’ reviewing and reflecting skills, and if that in turn would improve the students’ problem solving abilities. To answer this question, I designed this study to focus the students’ writing on their reviewing and reflecting activities during problem solving. The problem solving test scores for the students who wrote about their reviewing and reflecting activities did improve, but the improvement was not any better than their non-writing peers’ improvement.

Although a little disappointing, it was not wholly unexpected to find a lack of data to support my hypothesis that writing about reviewing and reflecting during problem solving is beneficial. For the past thirty years, problem solving has been an extensively researched topic, and it is easy to find research that shows that a student’s ability to problem solve is improved by writing about the problem solving process. However, no research was found that showed writing to be an effective tool for improving reviewing and reflecting activities. At the same time, no research was found that discredits the use of writing to improve reviewing and reflecting activities. These facts indicate that it might be very difficult to successfully connect writing, reviewing and reflecting, and improved problem solving ability.
Each of the weekly problem solving lessons focused on one problem solving strategy and reviewing and reflecting strategies. The combination of teaching problem solving with reviewing and reflecting successfully raised both classes’ test grades. Additionally, the homework grades for three of the four lessons showed improvement over previous weeks. However, there was no evidence that indicated that the test class’ writing was responsible for the improved grades.

Observations from both the control class and test class showed improvements in reviewing and reflecting during problem solving. During the first few weeks, the students needed prompting to review and reflect on their work. Students needed to have obvious mistakes pointed out, or would have to be prompted to reread the problem before the error was identified. During the last week’s lesson, student pairs in both classes were still making mistakes in setting up or solving the problem, but in all cases, the students reread the problem and found the error without prompting. The students’ reviewing and reflecting skills had improved.

In reviewing the test class’ homework, I do not believe that the students’ writing accurately captured their reviewing and reflecting activities or that the writing encouraged the students to review and reflect more. During the classes, I observed the students verbally sharing very insightful reviewing and reflecting. However, the majority of the reviewing and reflecting that the students wrote about was short and simple. It was not uncommon for a student to write; “I reread the question, checked my calculation and made sure I answered the question.” The writing did not match the words the students used in the classroom. There were several cases where the
students would write that they had checked their calculations but obvious errors were not found. In one example, the student wrote that they recounted the number of combinations they had found. That meant the student had miscounted their list of ten combinations twice. I am not convinced that the students always wrote about the reviewing and reflecting that they performed or that they actually performed all the reviewing and reflecting activities that they wrote about.

I still strongly believe in the importance of improving our students’ reviewing and reflecting activities during problem solving. This study’s failure to show that writing is an effective tool for developing reviewing and reflecting skills opens up two different paths for future work. First, it is unknown if the problem solving lessons or the reviewing and reflecting lessons were responsible for the students’ improved grades. I would like to repeat this study where one class receives the problem solving lessons and the other class receives both problem solving and reviewing and reflecting lessons. This would help quantify the effectiveness of the reviewing and reflecting lessons. The second path of investigation would focus on accurately capturing and understanding the reviewing and reflecting activities that the students perform. A study could be set up to compare students who write about their reviewing and reflecting while they are doing the problem solving with students who write about their reviewing and reflecting once the problem has been solved. Once the effectiveness of the reviewing and reflecting lessons are known and it is determined how best to capture students’ reviewing and reflecting, the original study
could be modified and repeated to test the effectiveness of writing in improving reviewing and reflecting during problem solving.

VII. References

http://www.ade.state.az.us/standards/math/Articulated08/default.asp

http://www.cde.ca.gov/ci/ma/cf/


http://etd.lib.fsu.edu/theses/available/etd-07072006-195753/


VIII. Appendices

Pretest Baseline Assessment

1) A snail wants to climb to the top of a 12 foot wall. Every day the snail climbs up 3 feet, but during the night, he slides down 2 feet. How many days does it take the snail to reach the top of the wall?

2) How many 9-inch-square floor tiles are needed to cover a 12 feet by 15 feet floor?

3) There are three men at a table. Their names are Mr., Brown, Mr., Black and Mr. White. The color of the men’s hair is brown, black and white. However, the color of their hair did not match their names. Find the color of each man’s hair.

4) Four women (Ann, Phyllis, Riana and Wendy) live in four different cities (Charleston, SC., Gainesville, FL., Provo, UT. and San Francisco, CA.). From the following clues, determine which city each woman lives in:
   1. Riana, the woman from Charleston, and the woman from Gainesville are not related.
   2. Wendy and the woman from Provo are cousins.
   3. Neither Phyllis nor Wendy is from the West Coast.
   4. Anna and Riana are from coastal cities.

5) How many ways are there to add 4 nonzero even numbers and get the sum of 24?

6) A rectangle has an area measuring 120 square centimeters. Its length and width are whole numbers of centimeters. What are the possible combinations of length and width?

7) Annette has $3.30. She has five more dimes than quarters. How many of each coin does she have?

8) Find the dimension of a rectangle that has the area equal to twice the perimeter. Both the length and width are whole numbers.
Posttest Concluding Assessment

1) You and your friend are building a pyramid out of two-by-two square Legos. The top of the pyramid is only one Lego. The next layer is four square Legos which leaves a strip one bump wide, showing all the way around the outside. This pattern is continued for a total of 5 layers. If you looked down at the top of the pyramid, how many bumps will you see?

2) You have a rubber ball that bounces up $\frac{1}{3}$ the distance it falls. For example, when the ball is dropped from 3 feet, it will bounce up 1 foot. The ball is shot up in the air 9 feet, and you stop the ball when it touches the ground the third time. What is the total distance up and down the ball traveled?

3) Two women (Rachel and Brittany) and two men (Alex and Josh) each like different types of music (Jazz, Classical, Country-Western and Rock). From the following clues, determine who likes each type of music:
   1. The ladies knew the country-western fan in high school
   2. The classical-music lover said that she would teach Brittany how to play the piano.
   3. Alex and the man who likes rock work in the same office building.

4) Four high school friends are attending four different colleges. The friends’ last names are Burbank, Collins, Gunderson and Williams. From the following clues determine each person’s first and last name:
   1. The girls’ names are Cathy and Gladys.
   2. No student’s first name starts with the same letter as their last name.
   3. Neither Hank nor Williams is going to the community college.
   4. Alan, Collins and the student who is going to the university all live on the same street.
   5. No student’s first name ends with the same letter as their last name.
5) The product of two whole numbers is 360 and the sum of the two numbers is less than 100. Find 5 possibilities.

6) Samantha scored 10 points in the basketball game. She could have scored with one-point free throws, two-point field goal, or three-point field goal. In how many different ways could she have scored her 10 points?

7) Jordan mailed a package to her brother. The package needed $1.29 in postage. She used only 16 and 7 cent stamps. How many of each type of stamp did she use?

8) Austin’s free-throw percentage this season is .875. If he makes only 13 of his next 20 free-throws, his percentage drops to .860. How many free-throws has Austin made this season?
Weekly Lessons

Week One – Drawing a Picture

Class Example Problem:
Three adults and two kids want to cross a river in a small canoe. The canoe can carry two kids or one adult. How many times must the canoe cross the river to get everyone to the other side of the river?

Home Work Problems:
1) A man is trying to cross a river with a wolf, a goat and a box of cabbages. The boat for crossing the river is old and leaky. The boat can only take the man and one other thing across the river at a time. He can’t leave the wolf alone with the goat, and he can’t leave goat alone with the cabbages. The wolf will eat the goat and the goat will eat the cabbages. How can the man get the wolf, goat and cabbages safely across the river?

2) You have a rubber ball that bounces up half the distance it falls. For example, when the ball is dropped from 2 feet, it will bounce up 1 foot. You drop the ball from 16 feet and it bounces up and down. Your friend stops the ball when it touches the ground for the third time. What is the total distance up and down the ball traveled?

Week Two – Using Matrix Logic

Class Example Problem:
A group of four friends went fishing and you are trying to figure out who caught the biggest to the smallest fish. You ask each one of them to give you a clue, and each of them gives you one clue. Afterwards you remember that people who fish never tell the truth about fishing. From each of these false statements, figure out who caught each fish:
1. Mark: Luke was first
2. Luke: Wendy was second
3. Wendy: I beat Sherry
4. Sherry: Mark beat Wendy

Home Work Problems:
1) Three friends, Elaine, Kelly and Shannon all play on the volleyball team. Each girl plays a different position: setter, middle blocker, outside hitter. Using the following clues, find the position each girl played:
1. Elaine is not the setter
2. Kelly has been in school longer than the middle blocker
3. The middle blocker has been in school longer than the outside hitter
4. Either Kelly is the setter or Elaine is the middle blocker
2) Ted, Ken, Allyson and Janie make up two married couples. Each have of them have a favorite sport, but none of them share the same favorite sport. The sports are running, swimming, biking and golf. Using the following clues, determine each person’s favorite sport:

1. Ted hates golf
2. Ken would not run around the block if he didn’t have to and neither would his wife
3. Each woman’s favorite sport is part of a triathlon
4. Allyson bought her husband a bike for his birthday to use in his favorite sport

**Week Three – Making a List**

Class Example Problem:
You and three friends are all going to lunch. How many different ways can you line up to enter the cafeteria?

Home Work Problems:
1) If you have lots of nickels, dimes and quarters, how many different ways can you make 50 cents?
2) Scott likes archery. His favorite game is to shoot 4 arrows and determine his score. With each arrow he can score 25 points, 10 points, 5 points, 1 point or zero points for a miss. If he shoots and hits the target will all 4 arrows, how many different total scores are possible?

**Week Four – Guessing & Checking**

Class Example Problems:
1) There is a two digit number that if you take the digits and reverse them to make a second number, and add the first and second numbers together their sum is 132. What was the original number? (Multiple correct answers)
2) Jose’ has twice as much money in nickels as he does in quarters. When he counts all his nickels and quarters, he has a total of 33 coins. How much money does Jose’ have?

Home Work Problems:
1) There are nine boys to every ten girls in a school. There are 2622 students at the school. How many girls are there in the school?
2) Find three different ways to add four nonzero even numbers and get a sum of 36?
Baseline Testing

Test Class

➢ Two students were absent. They took the test during the first week of lessons.
➢ At the end of the test, the students asked if they thought the pretest was hard, average or easy. All 10 students said that it was average. 6 said that they could have done more if they had more time. One student said more time would not have help because her brain was tired.
➢ Test duration
  o 30 minutes: 1 student
  o 34 minutes: 2 students
  o 35 minutes: 3 students
  o 36 minutes: 1 student
  o 37 minutes: 1 student
  o 38 minutes: 2 students

Control Class

➢ Two students were absent. They took the test during the first week of lessons.
➢ At the end of the test, the students asked if they thought the pretest was hard, average or easy. 8 students said that it was average and one said it was easy. 2 said that they could have done more if they had more time.
➢ Every student in the class asked at least one question. Every student asked if they could use coins besides dimes and quarters for problem number 7. They were told to do what they thought the problem was asking.
➢ Test duration
  o 30 minutes: 1 student
  o 31 minutes: 1 student
  o 34 minutes: 3 students
  o 38 minutes: 4 students

Week One – Draw a Picture

Test Class

➢ While doing going over problem one from the baseline test, the class agreed that the answer was 12 (the correct answer is 10). While the class was doing the problem together using the draw a picture approach, one student said “Oh wait”. He had realized the mistake he made during the test. When the class was done with the problem, one student (who had the correct answer) said that he had 10 for the answer but did not want to speak up because everyone else had said the answer was 12.
➤ Time ran out before the class had a chance to do the in class problem for the week. The classroom teacher did the problem with the student on Tuesday. I was not there.

Control Class
➤ This class seems to be less focused than the test class
➤ Time ran out before the class had a chance to do the in class problem for the week. The classroom teacher did the problem with the student on Tuesday. I was not there.

Notes from Homework
➤ Two of the student from the test class wrote about how to solve the problem, not what they did to review and reflect. Normally, there teacher has them write about how to solve the problem.
➤ For problem # 2, the bouncing ball, 6 of the students included the distance the ball traveled up after the third bounce. Of these students, 2 of them caught their error and corrected it.
➤ Things to bring up during lesson two in both classes
  o When you find a mistake in your work, think about what you were doing when you found it. It could be a good reviewing skill for making sure your work is correct.
  o When you are rereading a problem, what are you focusing on?
➤ Things to bring up during lesson two for the test class
  o The paragraphs need to be what you are doing to review and reflect on your work and the problem, not how you solve it.

Week Two – Matrix Logic
General
➤ The results from the pre-test show that both classes are very comfortable with matrix logic.
➤ None of the student correctly answered #4, what city did the ladies live in, on the pretest. The issue was not with matrix logic. The student had Gainesville has a coastal city and Charleston as non coastal.
➤ Today’s lesson will go over and answer problems 3 & 4 from the pretest. For the in class work, the students will work with a partner to solve the problem and together they will come up with 4 things they did to review and reflect on the problem.

Test Class
➤ When asked how to solve problem 3, Men’s color of hair, one student said guess and check and a second student said to make a chart.
➢ While going over problem 4, an error was made on one of the clues. Before the end of the problem, one of the students stopped the class and pointed out that there was an error. The error was investigated, the matrix was corrected and the problem was completed.
➢ All the pair were able to correctly solve the in class problem and come up with 4 ways to review and reflect on the problem.

Control Class
➢ When asked how to solve problem 3, Men’s color of hair, one student said make a chart and a second student said to make a T-table.
➢ The teacher tried to introduce the same error from the test class, but the students immediately saw the issue with how the clue was being used and the error did not occur.
➢ The class became a little goofy while going over the second matrix problem.
➢ All the pair were able to correctly solve the problem and come up with 4 ways to review and reflect on the problem.

Notes from Homework
➢ More students did not correctly solve problem 1 than problem 2. I am wondering if it is clue 4 with the either or statement.
  o 14 of the 15 students correctly had Kelly as the setter
  o 8 of the 15 students had Kelly as the setter and Elaine has middle blocker
  o Only 6 of the 15 students had Kelly as the Setter and Elaine has outside Hitter/Blocker
➢ Some of the students check off the clues as they use them. Some of the students circle or underline parts of the clue and make comments or questions.

Week Three – Make a List
General
➢ Either / Or logic for matrix problems will be reviewed with both classes before the make a list lesson will start
➢ The results from the pre-test show that most of the students in both classes took a guess and check. Very few students had a systematic approach to solving the problems.
➢ Over half the students did not turn in the week 3 home work on Friday. The classroom teacher expressed his disappointment with the class not doing the homework. Before the end of the day, three students completed and handed in the homework. After spring break, additional students turned in the homework.
While going over HW problem 1 with the either or statement, the students did catch the typo of outside hitter vs. outside blocker.

To check the matrix problem for HW 2, a student said to take one person’s result and see if it contradicts any of the clues.

For the in class problem, “how many ways can you and 3 friends enter the cafeteria”, two pair started the problem with only 3 people and did not check the error until prompted by the teacher to reread the problem.

One pair had two different answers on the in class problem. The student with the correct answer told their partner that they were missing one of the groupings when person 2 was in line first. She explained that all the other groupings had six options and that the 2 group had only 5 options.

Control Class

While going over HW problem 1, the teacher had to lead the class into seeing the type for the outside hitter vs. outside blocker. Even then the class was casual about it.

For verifying the volleyball matrix problem, one student suggested substituting the players’ positions for their names in the clues and see if the clues still made sense.

For the in class problem, how many ways can you and 3 friends enter the cafeteria, two pair started the problem with only 3 people and did not check the error until prompted by the teacher to reread the problem.

At least two of the pairs noticed that there were 6 options for arranging 3 people with the fourth person being the first one. They wanted to multiply 6 times 4 instead of making a list.

Today’s class seemed less focused than the test class, and the least focused of the first three lessons.

The teacher had to prompt the class multiple times to get them to give examples of what they did to review and reflect on the problems.

Week Four – Guess & Check

General

The students were less familiar and comfortable with the make a list strategy. Several students tried to do the homework without a list.

The longer list problem seemed to discourage the students from using the strategy.

Make a list HW problem to list zero points for a miss and then stated that all the arrows hit the target. Some of the students still used zero in their list. This will be used for an example for using rereading to catch an error.
Test Class

- After reviewing why zero should not be included in the list, one student felt very strongly about including zero.
- Students asked if there was a shorter way to do the problem without making a long list.
- Teacher: These problems look similar to last week’s problems. When do we want to use guess and check and when do we want to make a list?
  - Student: A list is used when physical observations are being made.
  - Student: When the problem is looking for all the possibilities, we want to make a list so we don’t miss any.
- A couple of the students working on the in class coin problem solved the problem with the incorrect understanding that there is twice the number of nickels as quarters. Without prompting the students reread the problem and realized that the problem said that the amount of money in nickels was twice the amount of money in quarters.
- Teacher: Why do we have you review and reflect on your work?
  - Student: To make sure we interpret the information in the problem correctly.
  - Student: So that we can self check our work.
  - Student: So we don’t have to ask you if the answer is right.

Control Class

- A student asked if they had to make a list to solve the problem. The teacher said, “No, but the students who did use the list had the right answer or close to the right answers. All the students that did not use a list had the wrong answer”.
- Teacher: Where do we start when we are using guess and check to solve a problem?
  - Student: Your first guess is a random but reasonable guess and the answer. If it is not correct, you adjust your answer based on the previous answer.
  - Student: When the problem is looking for all the possibilities, we want to make a list so we don’t miss any.
- Teacher: These problems look similar to last week’s problems. When do we want to use guess and check and when do we want to make a list?
  - Student: Guess and check is only looking for one answer and we don’t have to make a list to make sure we have a right answer. We only need to check it. Last weeks problems were asking for how many different ways. They wanted all of the possibilities.
- Like the test class, a couple of the students working on the in class coin problem solved the problem with the incorrect understanding that there is twice the number of nickels as quarters. Without prompting the students reread the problem and realized that the problem said that the amount of money in nickels was twice the amount of money in quarters.
Teacher: Why do we have you review and reflect on your work?
Student: We don’t always do it right.
Student: So we can catch our mistakes before we turn the work in and get it back and have to do it again.
Teacher: We should be able to check ourselves.

Concluding Test
Test Class
➤ One student was absent. There was no opportunity for the student to take the test before data analysis started.
➤ One student asked a question during the test.
➤ At the end of the test, the students asked if they thought the pretest was hard, average or easy. All students said that it was average. 4 said that they could have done more if they had more time.
➤ Test duration
  o 32 minutes: 1 student
  o 33 minutes: 2 students
  o 34 minutes: 1 student
  o 35 minutes: 1 student
  o 36 minutes: 1 student
  o 38 minutes: 4 students

Control Class
➤ All students were present. One student had to leave early.
➤ At the end of the test, the students asked if they thought the pretest was hard, average or easy. 9 students said that it was average. None of the students said that they could have done more if they had more time.
➤ Five students asked questions and one student asked two questions. Test duration
  o 20 minutes: 1 student – Had to leave early
  o 26 minutes: 2 students
  o 30 minutes: 1 student
  o 36 minutes: 3 students
  o 38 minutes: 3 students