1-1-2006

When Does the Area of a Rectangle Equal the Area of a Circle?

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Abstract and Problem Definition

When presented with geometric shapes such as rectangles and prisms, many students readily gain a conceptual understanding of area ("*length* × *width*") and volume ("*length* × *width* × *height*") because they can "see" those dimensions.

However, when presented with shapes involving circles or spheres, this intuitive understanding disappears. Indeed, in urban environments, many students have difficulty recalling the correct formulas for circumference or area of a circle, despite teaching instruction involving diagrams, use of mnemonics, etc.

To try to deepen their understanding of the relationship between the circumference and area of a circle, we first explored whether circumference and area are proportional to each other. We did this through use of Geometer's Sketchpad and the graphing calculator.

Having done so, an extension of our study presented itself: if you wanted to create a rectangle with an area equal to that of a given circle, what could the dimensions of that rectangle be? Our previous exploration led us to that answer, and we were able to construct a model in Geometer's Sketchpad that dynamically confirmed our answers.

Modeling Software

This project used Geometer's Sketchpad and the graphing calculator for building and confirming our model. Geometer's Sketchpad was chosen because it not only

modeled our exploration, but could also measure components of the circle (radius, etc) and calculate metrics needed (circumference, area, etc).

The graphing calculator was used to record the data, graph it, and also to run a linear regression to highlight the relationship between circumference and area.

Project Development

I chose this particular topic for a number of reasons. These were students preparing for the Math A exam in January. I wanted to deepen their understanding of circumference and area of circles. Also, previously, they had never seen any problems relating the two metrics. I thought it could be of interest to the students.

In Geometer's Sketchpad, different sized circles were created, and measurements such as radius, circumference, and area were recorded, and the ratio $\frac{Area}{Circumference}$ was calculated. The radius and ratio for six circles were entered into lists on the graphing calculator, the graph was viewed, and a linear regression was performed. Taking into account rounding errors, a curious result was obtained. After finding the proportionality between area and circumference, we found that $Area = \left(\frac{1}{2} \cdot Radius\right) Circumference$.

With that, a natural extension presented itself. Given a particular circle, what would the related dimensions be for a rectangle which would produce the same area? We discovered that if $Width = \frac{1}{2} \cdot Radius$, and if $Length = Circumference$, then the areas would be equal. We were then able to model this dynamically using Geometer's Sketchpad.

Problems Encountered

The initial problem that I wanted to investigate (duplicating Geek, or should I say Greek, mathematicians' exhaustion methods for estimating pi through the use of perimeters of circumscribed polygons in circles) proved difficult in terms of the regression analysis. Rather than have the students work with a problem that may not be definitive, I decided on our current problem, with its higher degree of success.

The other problem seems inherent in Geometer's Sketchpad. I wanted to work with unit circles, but Geometer's Sketchpad's measurements are in terms of centimeters. Since this did not affect the results, I accepted things are they were.

Evaluation of Results

Not only did the model operate well, but it helped the students gain a deeper understanding of the relationship between circumference and area.

Most significant was the model's dynamic nature: Students were able to change the size of the circle and see how the rectangle would be changed correspondingly. Also, the model dynamically calculated various measurements and confirmed the equality of the areas.

Summary of Experience

The model was an appropriate and significant one in terms of increasing student understanding of circumference and area. Also, there was a feeling of accomplishment in

learning enough about the Geometer's Sketchpad software to overcome the initial problems so that the main features of the simulation became operable.

For one time use, Geometer's Sketchpad is too complex for students to build a model, especially since the school does not have the facilities for an entire class to work on a project together. However, as a model, it is easy to students to manipulate it and see dynamically the impact of that change. Overall, student comments regarding the model were very positive.

Curriculum Standards

This project addressed the following New York State standards for Math A:

- 4A. Modeling / Multiple Representations: Represent problem situations symbolically by using algebraic expressions and geometric figures.
- 5A. Measurement: Apply formulas to find measures such as length and area in real world contexts.
- 5F. Measurement: Apply proportions to scale drawings and direct variation.
- 5I. Measurement: Use geometric relationships in relevant measurement problems involving geometric concepts.