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Geometry Curriculum for High School Students During a Summer School Program

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Geometry Curriculum for High School Students

During a Summer School Program

by

Kyle E. Kucsmas

December 15, 2012

A thesis submitted to the Department of Education and Human Development of the State University of New York College at Brockport in partial fulfillment of the requirements for the degree of Master of Science in Education.

Submitted on December 15, 2012.
Dedication

The following thesis is dedicated to my wonderful wife and my six month old son. The immense love and continued support from Ann has greatly influenced the completion of my first graduate degree in Education and Human Development. Finn contributes to my goal of earning a future second graduate degree in Educational Administration. Thank you both. You provide loving support to make my ambitions become a reality.
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Abstract

This geometry curriculum project was designed to be used during a high school summer school credit recovery program. The National Council of Teachers of Mathematics (NCTM), New York State (NYS) Learning Standards for Mathematics, Science, and Technology (MST), and the most recent addition of the Common Core Learning Standards (CCSL) harbor the foundations for each lesson. The curriculum presented will provide teachers with a condensed standards based instrument that can be utilized during a twenty-two day summer school geometry program which contains mathematical content, appropriate rigor, and opportunities for student growth. The curriculum is engaging, obtainable, cyclical, and diverse enough to help many students overcome the challenges of summer school and achieve the credit recovery goal. The curriculum offers students the prospect of becoming more proficient with the content through the building of vocabulary skills, improving problem solving techniques, and memorization through repetition of key concepts. Researched based practices and learning styles of mathematics are incorporated into the literary review. The institution of summer school and specific geometric topics are both discussed in the review. Suggestions and recommendations are made regarding the efficiency and effectiveness of the curriculum. Future possible research is discussed based on the contents of the project.
Keywords: geometry, summer, school, curriculum, mathematics, credit, recovery, professional, development, project
Chapter One: Introduction

A typical high school year in New York State is approximately 180 days. Students are lead through courses with established curriculums and earn graduation credits for the appropriate completion of the requirements. There are times, however, in which credit obtainment does not occur during the typical ten-month school year. A number of students fail courses each year and may be required to attend summer school in an attempt to earn back missed credits. Credit obtainment is one of the several major factors for high school graduation.

Course offerings during the summer are determined by funding and student enrollment. A typical high school geometry course has a majority of tenth grade students, yet, some juniors and seniors are occasionally mixed into the course. Geometry has a state assessment called a Regents Exam administered at the conclusion of the course. Passing the exam could be another graduation requirement if the ninth grade Integrated Algebra Exam has not been previously passed.

A teacher is fortunate if the same topic is taught from one summer to the next. The greatest advantage to teaching a familiar course is the reduction of time needed to create the curriculum, which means more time can be spent with other endeavors. Teaching summer school students is plagued with the challenges of time constraints, low motivation, weak learning habits, and of course, the intense summer heat. Students in summer school have many rational and irrational reasons for being placed into the
program; however, many of them all share the common goal of regaining credits. A summer curriculum should be engaging, obtainable, cyclical, and diverse enough to help many students overcome the challenges of summer school and achieve the credit recovery goal.

State and national instructional standards for high school mathematics are the basis for any lesson. The National Council of Teachers of Mathematics (NCTM), New York State (NYS) Learning Standards for Mathematics, Science, and Technology (MST), and the most recent addition of the Common Core Learning Standards (CCSL) harbor the foundations for each lesson. It is challenging to take all of the standards from a ten month high school geometry course and compact those into a succinct curriculum intended to be delivered in 12% of the typical instructional time. The curriculum presented will provide teachers with a condensed standards based instrument that can be utilized during a summer school geometry program. This curriculum contains rich content, appropriate rigor, and opportunities for student growth. It also offers students the prospect of becoming more proficient with content through the building of vocabulary skills, improving problem solving techniques, and memorization through repetition of key concepts.

The literary review will support current teaching professional development and learning styles in mathematics, compare traditional and idealistic summer school models, and depict geometry topics for general and inclusive education students.
Chapter Two: Literary Review

Many factors contribute to the success of high school students in mathematics. High quality of teachers, obtainable curriculum, and individual learning skills all attribute to credit obtainment for graduation. Unfortunately, many students cannot fulfill the requirements for a particular course and therefore enroll in a summer school program to properly complete the course. The following literary review will discuss: (1) The level of education and training that teachers in New York State must complete; (2) Possible strategies that may be used in a geometry course; (3) The general and inclusive education students in mathematics; and (4) The summer school program itself.

A meaningful high school mathematics education experience begins with dedicated professionals who have studied and trained to become teachers. Effective and efficient teaching requires continual learning. According to the New York State Education Department (NYSED), a teacher must earn a Master’s Degree through an approved graduate program within a specific educational content area and obtain a Professional Certification from the state (http://www.highered.nysed.gov/tcert). Teachers must maintain proper status for the Professional Certification by completing 175 hours of professional development every five years after the issuance of the certification (http://www.highered.nysed.gov/tcert/resteachers/175.html). A recently qualified teacher could experience a minimum of 1050 hours completing professional
development requirements over the course of a thirty year career. Teachers spend entire careers as students of education and their chosen content areas.

Professional development can be perceived as a welcomed necessity that inspires and engages the participants to accept ongoing changes which can influence lessons, or it is may be viewed as a mundane requirement which occupies precious time that could otherwise could be filled with other activities. According to Gellert (2007), effective professional development delivers “collaboration and co-operation” (p. 93). Effective professional development issues specific benchmarks and goals to ensure the growth and learning can be achieved by all participants. Mathematical teachers using a holistic approach to learning mathematics are different than the teachers who rely heavily on rote memorization of algorithms. Professional development can bridge the gap between the two styles and provide teachers to become more proficient in the art of education.

Gellert (2007) writes,

The notions of ‘routines’ and ‘collective orientations’ are used as conceptual tools for analysis. It is supposed that routines and collective orientations are like double-edged swords where the professional development of mathematics teachers is concerned. On the one hand, routines and collective orientations might serve as backing and support; on the other hand, both might result in conservative forces impeding or even foiling the necessary change processes. (p. 98)
Gellert (2007) suggests that professional development for mathematic teachers can be successful if the participants are willing to accept the education and implement the changes within the classrooms.

Professional development addresses changes and educates the participants regarding shifts of previous mathematical teaching practices to current researched based methods. Effective teaching of mathematics relies on the continued growth of the education community. An example of effective professional development can be found in a program known as *From Another Perspective*, as mentioned by Gal (2011). The program, “…is a year-long course for teachers of mathematics that is designed to enhance teachers’ awareness of the way that their students think when they are experiencing difficulties in geometry” (p. 183). After completion of the training, Gal (2011) describes a female teacher’s achievement of:

1. Expanding and deepening her understanding of students’ ways of thinking;
2. Increasing her awareness of her students’ processes of thinking in order to identify their difficulties;
3. Equipping her with appropriate tools to analyze and cope with such difficulties; and
4. Enhancing her ability to retrieve and utilize this knowledge while making instructional decisions. (p. 183)

The above research is also generalized to male teachers as well.
Another professional development that has proven to be successful for teachers is the Science, Technology, Engineering, and Mathematic (STEM) programs. Nadelson, Seifert, Moll, and Coats (2012), researched the structure and outcomes of a four day professional development initiative during a summer institute for K-12 teachers. The authors state:

Our research sought to determine how the structure and content of the summer institute influenced the participants’ comfort with teaching STEM, efficacy for teaching STEM, content knowledge of STEM, inquiry implementation in STEM, and perceptions of STEM education (p. 69).

Analysis of pre and post surveys given to the participants revealed increases for each of the goals of the study. The article suggests that one of the most important times to implement STEM programs is during the middle school years. The authors mention:

Middle school has been identified as a critical juncture in which student motivation and performance is susceptible to decline…suggesting that teachers need to be at full capacity to ensure the highest levels of student engagement and achievement in STEM learning (p. 71).

A professional development program similar to STEM provides teachers an insight into the ways in which one could incorporate aspects of the vocabulary, practices, and strategies of the content into other courses. Presenting material across several content areas could enrich the learning and creates stronger student retention.
Recently, the Annual Professional Performance Review (APPR) process has charged school administration teams with the task of data collection of specific evidence for all teachers regarding professional and student growth. Teachers will be rated based on several aspects, one of which includes student performance on New York State Exams. Teachers will be given a score based on the percentage of their students that achieved previously determined goals on state assessments. The points are then added to other obtained points through multiple measures and a final score for the teacher is applied at the end of the school year. Teachers that have students not obtaining the goals and significantly lower scores will have improvement plans designed and set into place to help in the achievement of the students (http://engageny.org/search/site/appr).

In order to avoid this consequence, teachers will need to seek quality professional development workshops to ensure current knowledge of pedagogies, standards, and curriculums. Mandates connected to the initial and future education of teachers, evaluation processes and rating systems, and the ever present pressures of global competition all strongly influence the practices and decisions of New York State teachers.

Despite the extensive training and accountability of the teachers, many students may not earn the credits necessary for graduation. Hampel and Menzer (2009) write about students who drop out of high school during their senior year because credit obtainment has not been reached. Hampel and Menzer (2009) track a high school near
Delaware and describe the data that was collected. The study found that the greatest population, (approximately half), that dropped out of high school were African American males. Hampel and Menzer (2009) state, “…they had exactly twice as many disciplinary referrals and they received almost three times as many suspensions. All of those figures fit the familiar attributes of “at-risk” students” (p. 661). Interventions needed to be in place to prevent similar future dropout rates. The school introduced after school credit recovery programs, online tutorials for additional support, and mentoring programs designed to monitor the progress and success of the students. The rate of dropout students reduced the following school year and the graduation rate increased. A school that can be forward thinking, utilizing the support from staff properly, and that has the resources to support preventative programs during the typical school year may prevent students from the option of dropping out of school.

Another category of students that can be “at risk” are the inclusive education students. A teacher may have just as many inclusive education students as general education students on the summer school roster. Browder, Courtade, Flowers, Jimenez, et al., (2012), conducted a study of a middle school in the southeast part of America which evaluated the strategies used to teach mathematics and science to inclusive education students. The main intervention implemented by inclusive education and general education teachers was the use of story-based problems with familiar context. Key vocabulary paired with picture symbols aided in the understanding of the content.
Graphic organizers and manipulatives helped to organize key facts and solve problems within the story. The article stated:

Training teachers to use story-based problems, task analytic instruction, and graphic organizers increased students’ acquisition of specific math skills. In earlier studies, benefits for using graphic organizers to teach mathematics have been demonstrated for students with mild disabilities. Graphic organizers have also been used with students with moderate and severe disabilities to support learning independent living skills. (p. 33)

The results of the study later revealed a correlation between inclusive education students performing better in the areas of mathematics and science and the teachers using the story based problems. The opportunity for inclusive education students to learn general education content can be hindered by a teacher’s weak understanding of the complexities of their learning styles and needs. A summer school teacher has a limited amount of time with the students to become proficient in delivering the content to the students, especially to those in which disabilities hinder the typical learning process.

Student mathematical achievement is linked to many factors i.e., the effectiveness of the teachers, student s’ learning abilities, and the learning environment. A student’s self- efficacy, as described by Bauman, Falco, and Summers (2011), also influence the
achievement. A team of counselors and mathematic teachers created a curriculum for middle school students which focused on, “…students’ attitudes, self-efficacy, and performance in mathematics” (p. 529). The curriculum was:

- Designed to increase students’ self-efficacy for learning mathematics by tapping into their cognitive capabilities using lessons designed to foster skills in planning, goal setting, and specific learning strategies. The four skills included in the curriculum are time-management, goal setting, mathematics study skills, and help-seeking skills. Because self-efficacy is a domain-specific construct in academic contexts the majority of the lesson activities are designed specifically to address mathematics learning (p. 533).

The evidence from the Bauman, Falco, and Summers (2011) study produced an improvement of more female students than male students in mathematics. Motivation, enjoyment, and confidence in mathematics increased for both genders in the experimental group. A mathematical curriculum with self efficacy elements is another key element for students to learn mathematics (Bauman, Falco, and Summers 2011).

Geometry can be broadly considered as the study of points, lines, shapes, and measurements. The applications of geometry have been an integral part of human achievements long before the building of Rome’s aqueducts and Egypt’s pyramids. Students of geometry learn from the teachers of geometry. A study conducted by Corey,
Jakubowski, and Unal (2008), linked the achievement of geometry students to their teacher’s ability to reason through spatial relationships. Spatial relationships refer to the ability to locate objects in a three-dimensional space by using visual or tactile cues (Corey et al. 2008). The study claims, “…those pre-service teachers with high or mid-range spatial ability scores showed both a greater beginning level of geometric understanding and a greater improvement in geometric understanding than the student with the lowest spatial ability score” (p. 1005). The spatial relationship aspect of teachers is primarily located in the right side of the brain. According to the authors, right brained people are more likely to teach through diagrams and pictures (p. 1006). The article claims that one innate ability or inability of the teacher can have a direct influence on the students’ capability to understand and apply the concepts of geometry. This one element is certainly not the only ingredient needed for student success in mathematics.

Creative and innovative instruction may be the factor that determines if learning and understanding of mathematics occurs for all students. Consider the immense number of geometry teachers, different individual teaching personalities, styles, and methods, and lastly, the personal strengths or weaknesses of a teacher, that could influence a high school geometry curriculum. Teaching the properties of a perpendicular bisector, for example, can be done using a kinesthetic approach. According to Touval (2011), an activity in which students walk away from a given point
on a line segment of the floor can help students understand the properties of equidistance and a perpendicular bisecting a segment. Touval (2011) states such activities, “…help students who encounter difficulties in differentiating a theorem and its converse” (p. 269). Posamentier (2009) suggests paper folding as a visual representation for proving the Pythagorean Theorem in right triangles. Posamentier (2009) also illustrates the importance of teaching less popular theorems and mathematics. Ceva’s Theorem is explored because it, “…proves the concurrence of medians, angles bisectors, and altitudes of a triangle” (p. 221). According to Posamentier (2009), students have a higher success rate of learning and remembering the definitions and properties of triangles through the pursuit of, “… above standard knowledge” (p. 222). Bucher and Edwards (2011) demonstrate how proofs of transformational geometry can be expanded to make connections to other content areas. Students that may not be strong in mathematics may connect more and remember an element of mathematics because it was linked to content that is one of their strengths.

The use of proof in geometry is an important aspect within the curriculum. Mathews and Reed (2008) write about the importance of proving techniques for students to, “…elaborate their reasoning in solving problems,” and create a, “…convincing mathematical argument” (p. 539 ). The article states that students should be able to, “…defend an algebraic solution, to affirm a trigonometric identity, and to justify an observed property in geometry” (p. 539). According to Mathews and Reed
(2008), this understanding can be achieved through technology. For example, Interactive Java Script® applications provide visual representations of the foundations of geometric theory which can easily be manipulated to demonstrate particular situations pertaining to a theorem or proof. Computer software including Geometer’s Sketchpad® allow students to create shapes by hand through midpoints, segments, parallel, and perpendicular commands that adhere to the properties and definitions of geometry. Such instructional techniques and strategies may aid teachers in fulfilling the tasks of student understanding. In order for the students to be successful in geometry, the teachers should have a variety of tools that can engage all students in the class.

During a typical school year, highly trained teachers using specific mathematical strategies and the school’s credit recovery interventions may not be enough for some students to complete a geometry course. Students not meeting the requirements of a geometry course during a typical ten-month school year could attend a summer school program to fulfill potential graduation requirements. Many opinions regarding traditional summer school can emerge from the students and teachers. Students may be reluctant, irritated, or not motivated to attend summer school. The prospect of regaining credits is the motivation for most to do well during the summer. Teachers may not be fully invested in the achievement of the students because little accountability has been associated with summer school. A typical credit recovery summer school program is composed of two hour courses for twenty-two days.
Students could attend two classes if necessary for a total of four hours each day. State exams are offered at the conclusion of the instructional days. Grades from these sessions are then sent to the students’ home school and it is at the discretion of the school whether or not sufficient requirements have been met to earn credit for the particular course. Summer school teachers may feel pressured to get through as much material as possible, and retention may not be their most important goal. The uneasy feelings from students and teachers regarding summer school may be justified because of the current nature and design of traditional summer school. A reform of the programs and structures of summer school may change perspectives and become more efficient for future teachers and students.

Many people may argue that the origins of summer school came from a time in which agriculture dominated the nation. Fairchild (2011) writes about an overlooked reason for the traditional school calendar.

At the turn of the twentieth century, there was a growing desire, particularly among the more affluent members of society, to flee cities during the summer months. The heat, threat of communicable diseases, and poor municipal sanitation during the early 1900s helped drive residents out of cities during the summer months, and city school calendars generally accommodated the needs and desires of wealthy families (p. 13).
Fairchild (2011) offers many reasons for the inability of American society to change the typical school calendar. Reasons such as: additional costs to schools, oppositions from businesses that rely on summer time vacations for third quarter profits, and the instilled idealistic time of social growth for the students. Despite these factors, Fairchild (2011) supports the need for academic interventions which encourage the retention of previously learned reading and math skills (p. 16).

Reducing the academic loss that many students experience over summer is the leading factor for programs and initiatives offered by education programs. Wiley (2011), states, “Low income students lose more ground in reading, while their higher income peers may even gain, …rates of low-income and higher-income students contributes substantially to the achievement gap” (p. 25). Urban schools may have the greatest difficulties providing quality summer school programs. Keiler (2011) claims, “Remedial summer programs in urban settings…are designed to compensate for a deficit during the school year rather than prevent backsliding or provide enrichment” (p. 360). In order to transition away from traditional summer school programs, Keiler (2011) explains how a comprehensive change was made to an urban school in which the summer school program focused on active learning, mutual respect, and individual emotional support. Students and staff were initially reluctant to admit this change could produce higher levels of achievement. Through the perseverance and continued
support of the teachers and administrators, the program became successful. Keiler (2011) states:

The critical finding of this study is that students who have failed and been failed in their urban schools can thrive when offered high quality instructional experiences that (1) actively involve students; (2) focus on understanding of content; and (3) are facilitated by instructors who believe in students’ potential to succeed (p. 375).

Summer school programs have great potential to be successful for many students in areas of credit recovery, skill building, or content mastery. Programs devoted to mathematics can illustrate the applications and benefits of mathematical content to students. A program in Ireland called the Mathematics Applications Consortium for Science and Industry (MACSI) hosts an annual weeklong summer school for students (Charpin, Hanrahan, Mason, O’Brien, et al., 2012). Students were screened through an application process and selected by a committee. Students remained on campus during the summer and explored many topics through mathematical modeling and activities. “The MACSI summer school for teenagers is the first of its kind in Ireland, and among the first in Great Britain to focus specifically on the application of mathematics to real-world problems” (Charpin et al. p. 865). Daily items including cars, internet, mobile phones, satellites, banking, special effects, and many others have been studied through the lens of mathematics. Students exposed to the applications of mathematics had a better
appreciation for meaningful learning necessary to achieve in school. Students attended a workshop which focused on the variety of career options in mathematics unknown to the general public (Charpin et al. p. 874). Since the students lived on campus for the duration of the program, a majority of the cost was applied to room and board. In 2011, the cost to attend the program was 400 Euros ($521.89 U.S.) per student. Seventy five percent of the cost was funded by the Science Foundation Ireland and the remainder could have been paid by the students or awarded through scholarships (Charpin et al. p. 878). The MACSI program is one example of how some nations are embracing the potential of learning through summer school funding and endeavors.

Through many reforms to society’s notions towards traditional summer school, more funding and innovations could be applied to the eventual transformation of education. Mathematics remains as the leading universal language, and geometry has recognizable applications from many mathematical topics. Geometry is also studied in most high schools. Likewise, it may be the students proficient in mathematics had highly qualified teachers. New York State requires obtainment of graduate degrees and teaching certifications. Teachers must strive for continued education through professional development every five years. The geometry curriculum provided in this thesis was compiled over three years of teaching summer school to general education and special education students. Items from the literary review strongly influenced the structure, content, and rationale for creating a geometry summer school curriculum.
Chapter Three: Application

In an upstate New York high school, summer school instruction is typically completed in twenty-two days. Summer school has about 12% of the instruction time when compared to a typical school year, and at the end of summer school, two days are reserved for state examinations. A two hour summer class period needs to be split into several sections to maintain student focus and minimize unwanted behaviors. If a routine for the class is established early, then the accomplishment of the tasks most likely yields achievement and understanding. The standards covered are topics that are usually the most difficult for students during the typical school year, and are mostly likely to appear on the state exam. The inclusive education and general education students can benefit from the structure and rigor established within the summer school curriculum.

Upon entering the classroom on the first day of summer school, students are given a blank copy of the curriculum. The bound packet includes a title page, syllabus, reference formula chart as found in a state exam, and a grade recording form. Students are taught how to use the grade form correctly and should record their own grades daily to encourage ownership of the work completed. Each day, students can earn five points for completed classwork, ten points on daily quizzes, and fifty points on tests. Earned points divided by total possible points is the formula for grade calculations. The mathematical sections include warm up, proof, construction, vocabulary, and the topic
of the day. Repetition of key concepts is stressed within the curriculum. The majority of
the questions within the packet come from jmap.org, regentsprep.org, or old New York
State Geometry exams. Geometric graphics came from Clip Art in Microsoft Word.

Each day, students entering the class complete a warm up question. The warm
up question is related to the previous day’s content. Students complete one proof and
one construction. Instead of covering these two topics in a day or two, students will
have the opportunity for multiple exposures. This will produce twenty-two
opportunities for students to become proficient in proofs and constructions, which are
typically difficult mathematical areas for them to master. The vocabulary discussed and
recorded is linked to the topic of the day and prepares students to properly solve
problems. Strategies and techniques are discussed and practiced within the topic of the
day section. The amount of time dedicated to the topic of the day resembles 25% or
more of the two hour class. The teacher can minimize transition time by numbering
each page of the packet, color coordinating each section, or adding the date to each item
in the curriculum prior to making copies for each student. These packets should remain
in the room to prevent lost materials.

In order to encourage focus and proper note taking, one open note quiz is
administered at the end of each day. Students should be allowed to use their own
curriculum but may not use another person’s. Memorization is encouraged through two
separate closed note tests throughout the summer. There is no work done outside of the
summer school class. Students feel a sense of gratitude with the remainder of the day dedicated to enjoying the summer. Students are then more likely to work diligently for two consecutive hours in class.

Students can earn up to five class work points each day by completing the particular warm up, proof, construction, vocabulary, and topic of the day at the designated times. Students are encouraged to attempt the problem on their own or with a partner prior to the teacher reviewing the solution. One point per category should be awarded for the attempt of the problem and not necessarily the correctness. Students should be given an opportunity to make corrections and ask questions prior to moving on to the next item in the packet.

Each open note quiz given separately at the end of class is worth ten points. Overall class grades are calculated by total earned points divided total possible points. Students will usually notice similar problems in their curriculum packet, but they must use proper mathematical reasoning to answer the quiz questions correctly. The quizzes are graded and returned to students the next class. A review of proper techniques and solutions is discussed by the teacher. The graded quizzes become a part of the open note policy for future quizzes. The quiz given on the last day of summer school will be scored, returned, and reviewed during that class time.
It is important to note the formatting and spacing of the actual curriculum given to students and the curriculum in this project may vary. Students may need more work space than indicated in this curriculum.
Lesson Plans

Materials: calculators, compasses, straight edges, curriculum packets, manila folders for returned assessments. One item for each student should be available. Smart Board files or overheads may be created, if available, by teacher to aid in proper note taking by the students.

Student Demographic: General education and inclusive education high school students.

Time Frame: Twenty-two days of two hour classes.

Performance Indicators: (http://jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#Geo)

- G.G.66 Find the midpoint of a line segment, given its endpoints.
- G.G.67 Find the length of a line segment, given its endpoints.
- G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas
- G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel
- G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal
- G.G.12 Know and apply that the volume of a prism is the product of the area of the base and the altitude
- G.G.13 Apply the properties of a regular pyramid, including lateral edges are congruent, lateral faces are congruent isosceles triangles, and volume of a pyramid equals one-third the product of the area of the base and the altitude

- G.G.14 Apply the properties of a cylinder, including bases are congruent, volume equals the product of the area of the base and the altitude, and lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base

- G.G.15 Apply the properties of a right circular cone, including lateral area equals one-half the product of the slant height and the circumference of its base, and volume is one-third the product of the area of its base and its altitude

- G.G.16 Apply the properties of a sphere, including the intersection of a plane and a sphere is a circle, two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles surface area is \(4\pi r^2\) and volume is \(\frac{4}{3}\pi r^3\)

- G.G.17 Construct a bisector of a given angle, using a straightedge and compass, and justify the construction

- G.G.18 Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction
G.G.19  Construct lines parallel (or perpendicular) to a given line through a
given point, using a straightedge and compass, and justify the construction

G.G.25  Know and apply the conditions under which a compound
statement (conjunction, disjunction, conditional, biconditional) is true

G.G.26  Identify and write the inverse, converse, and contrapositive of a
given conditional statement and note the logical equivalences

G.G.27  Write a proof arguing from a given hypothesis to a given conclusion

G.G.28  Determine the congruence of two triangles by using one of the five
congruence techniques (SSS, SAS, ASA, AAS, HL), given sufficient
information about the sides and/or angles of two congruent triangles

G.G.62  Find the slope of a perpendicular line, given the equation of a line

G.G.63  Determine whether two lines are parallel, perpendicular, or neither,
given their equations

G.G.64  Find the equation of a line, given a point on the line and the
equation of a line perpendicular to the given line

G.G.65  Find the equation of a line, given a point on the line and the
equation of a line parallel to the desired line

G.G.66  Find the midpoint of a line segment, given its endpoints

G.G.67  Find the length of a line segment, given its endpoints
- **G.G.69** Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas.

- **G.G.70** Solve systems of equations involving one linear equation and one quadratic equation graphically.

- **G.G.71** Write the equation of a circle, given its center and radius or given the endpoints of a diameter.

- **G.G.72** Write the equation of a circle, given its graph Note: The center is an ordered pair of integers and the radius is an integer.

- **G.G.73** Find the center and radius of a circle, given the equation of the circle in center-radius form.

- **G.G.74** Graph circles of the form $(x - h)^2 + (y - k)^2 = r^2$. **G.G.44** Establish similarity of triangles, using the following theorems: AA, SAS, and SSS.

- **G.G.45** Investigate, justify, and apply theorems about similar triangles.

- **G.G.46** Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle.

- **G.G.47** Investigate, justify, and apply theorems about mean proportionality the altitude to the hypotenuse of a right triangle is the mean proportional.
between the two segments along the hypotenuse, the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that leg

- G.G.49 Investigate, justify, and apply theorems regarding chords of a circle perpendicular bisectors of chords, and the relative lengths of chords as compared to their distance from the center of the circle

- G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle a perpendicular to the tangent at the point of tangency, two tangents to a circle from the same external point, and common tangents of two non-intersecting or tangent circles

- G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is inside the circle (two chords), on the circle (tangent and chord), outside the circle (two tangents, two secants, or tangent and secant)

- G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines

- G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle along two tangents from the same external point, and along two secants from the same external point
- G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem
- G.G.33 Investigate, justify, and apply the triangle inequality theorem
- G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle

**Day 1 (2 hours):** “Get to know you activity” (15 minutes). Students will state three true and one false statement about their life. Others in the class will guess the false statement. Distribute curriculum packets (2 minutes). Review syllabus, student code of conduct, collect medical forms, collect parking pass forms, assign calculators if needed, and lastly review fire safety procedures and classroom expectations (15 minutes). Construction (10 minutes). Proof (10 minutes). Vocabulary (10 minutes). Topic of the day (40 minutes). Quiz 1 (20 minutes).

**Days 2-7 (2 hours each day)** Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (15 minutes). Proof (15 minutes). Vocabulary (10 minutes). Topic of the day (40 minutes). Quiz (20 minutes).

**Day 8 (2 hours)** Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (15 minutes). Proof (15 minutes). Test 1 (1 hour).
Days 9-15 (2 hours each day)  Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (15 minutes). Proof (15 minutes). Vocabulary (10 minutes). Topic of the day (40 minutes). Quiz (20 minutes)

Day 16 (2 hours) Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (15 minutes). Proof (15 minutes). Test 2 (1 hour)

Days 17-20 (2 hours each day) Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (15 minutes). Proof (15 minutes). Vocabulary (10 minutes). Topic of the day (40 minutes). Quiz (20 minutes)

Day 21 (2 hours) Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (10 minutes). Proof (10 minutes). Multiple choice section of old NYS Geometry exam (60 minutes). Quiz (20 minutes).

Day 22 (2 hours) Return and review previous quiz (10 minutes). Warm up (10 minutes). Construction (10 minutes). Proof (10 minutes). Quiz 20 (15 minutes). Free response sections of an old NYS Geometry exam (50 minutes). Return and review Quiz 20 (15 minutes).
Name:________________________________________________________

School:_______________________________________________________
### Reference Sheet

| Volume           | Cylinder                             | $V = Bh$
|------------------|--------------------------------------|---------------------
|                  |                                      | where $B$ is the area of the base |
|                  | Pyramid                              | $V = \frac{1}{3}Bh$
|                  |                                      | where $B$ is the area of the base |
|                  | Right Circular Cone                  | $V = \frac{1}{3}Bh$
|                  |                                      | where $B$ is the area of the base |
|                  | Sphere                               | $V = \frac{4}{3}\pi r^3$ |

| Lateral Area ($L$) | Right Circular Cylinder              | $L = 2\pi rh$ |
|--------------------|--------------------------------------|---------------------
|                    | Right Circular Cone                  | $L = \pi rl$
|                    |                                      | where $l$ is the slant height |

| Surface Area       | Sphere                               | $SA = 4\pi r^2$ |

---

Geometry – August ’12

Figure 3.1 from http://www.nysedregents.org/geometry/812/geom82012-examrev.pdf
Summer School Geometry Syllabus

Course Objectives:

- understand the concepts of and become proficient with the skills of mathematics.
- communicate and reason mathematically.
- become problem solvers by using appropriate tools and strategies.
- educate students according to NYS and national standards.


Class dates:

- Session A: July 11, 12, 13, 16, 17, 18, 19, 23 test day, 24, 25, 26
- Session B: July 30, 31
- August 1, 2, 6 test day, 7, 8, 9, 13, 14, 15
- Regents Exam August 16 @ 8am

Grading:

- Class work (warm up, notes/practice, vocabulary, proof, construction) 5 points per day.
  Total of 110 points.
- Quizzes 20 @ 10 points each = 200 points.
- Two tests @ 50 points each = 100 points.
• Typically, summer school is divided into 2 sessions.

• Session A has **205 points**

• Session B has **205 points**

• **If you are absent (unless previously discussed with me) you will get ZERO points for that day.**

<table>
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<th>A = 205-184.5 pts</th>
<th>B = 184.4-164 pts</th>
<th>C = 163.9-143.5 pts</th>
<th>D = 143.4-133.25 pts</th>
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Classroom Procedures:

• See Summer School Student Handbook

I encourage you to contact me if a problem should arise. I will do my best to educate you.

*Will you do your best to learn?*

Let’s have a good summer together!
<table>
<thead>
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***********************************************************************************Session A Ends***********************************************************************************
**Session B Starts**

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******************Session B Ends*******************************
Warm Up Problems

7/12 Find the midpoint, slope, and distance of the following point Show all work! (-4, 5) and (10, -15)

<table>
<thead>
<tr>
<th>Midpoint</th>
<th>Slope</th>
<th>Distance</th>
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7/13 Warm Up
Give a line that is parallel to 4y - 6x = 20 then give a line that is perpendicular to the original.

<table>
<thead>
<tr>
<th>original</th>
<th>parallel</th>
<th>perpendicular</th>
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7/16 Warm Up Which of the following is a solution to the following equations? -20x + 4y = 12 and x^2 + 4x + 1 = y

[1] (1, -2)
[2] (-1, 2)
[3] (-1, -2)
[4] (1, 2)
7/17 Warm up  What is the volume of the figure below?

7/18 Warm up  If a sphere has a volume of 2144.7 in$^3$, then what is its surface area to the nearest tenth? Use the closest whole number for the radius.

7/19 Warm up  If the endpoints of a diameter of a circle are (4, 8) and (4, 2), then what is the equation of this particular circle?

   **Hint: Find the distance, then cut it in half to know the radius. Use midpoint formula to get the center.**

7/23 Warm up  A cylinder has a volume of 351.9 cm$^3$ and a height of 7 cm. What is the radius?

7/24 Warm up test review

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Give the image of the following points with the given transformations.

A = (-3, 4)  B = (5, -7)  C = (-8, -2)  D = (9, 6)

1. B' = \(R_{y=\frac{1}{2}}\)
2. D' = \(R_{\alpha=270}\)
3. C' = \(D_{-\frac{1}{2}}\)
4. A' = \(T_{(-5, 6)}\)

Give the final coordinate for the following compositions

(-3, -9)  (4, -7)  (-1, 7)  (2, 5)

\(R_{180^\circ}, D_0, \quad R_{90^\circ}, T_{(-1, 0)}, T_{150^\circ}, R_{90^\circ}, \quad G_{x=1}, G_{x=2}\)

Solve for DC, when CB = 6, AB = 15. Round to the nearest tenth if needed.

What is the height of a cylinder with a volume of 456 cubic inches and a diameter of 7 inches?

What is the midpoint of (-8, 4) and (10, 12)?
8/6 Warm up
What is the equation of the line that passes through (4, 9) and has a slope of 3/2?

8/7 Warm up
Write the negation of “I do not believe in dragons.”

8/8 Warm up
What is the distance between (-6, 2) and (4, 12) ?

8/9 Warm up
Give three numbers that could not be the side lengths of a given triangle.

8/13 Warm up
Write the equation of a circle with a diameter of 10 inches and center at (5, -7)

8/14 Warm up
Two planes intersect to form what? Provide your best sketch.

8/15 Warm up
What is the lateral area of a pyramid with a height of 11 inches and a square base of 12 inches?

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Proofs

7/11 Given parallel lines $l$ and $m$ cut by transversal $t$. Prove angles are congruent.

7/12 Prove the triangles are congruent

7/13 Prove the following triangles are congruent

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
In the accompanying diagram, \(HK\) bisects \(IJ\) and \(\angle H \cong \angle K\). Prove Triangle HJI is congruent to Triangle KJL.

---

Proof

Prove that \(<BAC\) is congruent to \(<BDC\), when \(AB \cong DB\).

---

Proof

Prove \(<BAC\) is congruent to \(<DEC\) when \(C\) is the midpoint of \(BD\) and \(AE\).

---

Proof

Prove \(DE \cong BA\) if \(BE\) is bisecting \(AD\) and \(<B \cong <E\).

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
7/23 Proof

$\overline{AB} \cong \overline{CB}, \overline{BD}$ bisects $\angle ABC$, prove $\overline{AD} \cong \overline{CD}$

7/24 Proof

Prove triangle $ABC \sim$ triangle $EDC$ when $\overline{AB} \parallel \overline{ED}$ and $\angle A \cong \angle E$.

7/25 Proof

Prove triangle $ABC$ is similar to triangle $DEF$. If $AB = 7$, $BC = 8$, and $AC = 9$. Also, $ED = 42$, and $EF = 48$, and $FD = 54$

7/26 Proof

Triangle $ABC$ and triangle $DEF$ are similar. Find all side lengths if you have a scale factor of 2.5 (from small to large), and $AB = 4$, $EF = 12.5$, and $AC = 8$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
7/30 Proof
Are the following triangles similar? Show your work!

7/31 Proof
Prove that the points make a rectangle.
A (-4, 6)
B (1, 7)
C (3, -3)
D (-2, -4)

8/1 proof
Prove that A(-3,2), B(-2,6), C(2,7) and D(1,3) is a rhombus.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
8/2 Proof

Prove that quadrilateral A(1,2), B(2,5), C(5,7) and D(4,4) is a parallelogram.

8/8 Proof

Prove ABCD is a square, A(2,2), B(5,-2),C(9,1) and D(6,5).

8/7 Proof

**Given:** BC is an altitude to AD, \( \angle ABC \cong \angle DBC \)

**Prove** \( \triangle BCA \cong \triangle BCD \)
8/8 proof
Given diameters $\overline{AB}, \overline{CD}$
Prove: $\overline{AC} \cong \overline{BD}$

8/13 proof
In the diagram below, $\overline{PA}$ and $\overline{PB}$ are tangent to circle $O$, $\overline{OA}$ and $\overline{OB}$ are radii, and $\overline{OP}$ intersects the circle at $O$.
Prove: $\angle AOP \cong \angle BOP$

8/14 proof
Given: $JKLM$ is a parallelogram.
$\overline{JK} \cong \overline{LN}$
$\angle LMN \cong \angle LNM$
Prove: $JKLM$ is a rhombus.

8/15 proof
Given: Quadrilateral $ABCD$ has vertices $A(-5,6), B(6,6), C(8,-3),$ and $D(-3,-3)$.
Prove: Quadrilateral $ABCD$ is a parallelogram but is neither a rhombus nor a rectangle.
[The use of the grid below is optional.]

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Constructions

7/11
• Copy any given line segment onto another, longer line, ray, or segment. Leave all construction marks!

Given segment AB

7/12
Construct a perpendicular bisector of AB

7/13
Construct a ray that is an angle bisector of angle ABC

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Construct a perpendicular segment through point K

Construct a perpendicular segment through point K where K is not on the line segment

Construct an equilateral triangle from the given segment of 5 cm.

Construct an isosceles triangle such that the two congruent sides are 4 cm.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Construct the locus of point with a dotted line that are equidistant from line M.

Construct the locus of points with a dotted line that are equidistant from parallel lines M and N

Construct the locus of points that are 7 units away from point P

Construct the locus of points from point P and point R on a segment

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
7/30 Construction Compound Locus

Put an X to indicate the solution(s) to the following problem. How many locations can a person be standing 8 units from P and equidistant from lines M and N?

---

7/31 Construction

How many locations can Bill stand if he needs to be equidistant from Points A and B, which are 8 units apart from each other, and 5 units from Point P, when P is 4 units from A?

---

8/1 How many locations are 3 units from the x axis and 6 units from the origin. Use the grid paper to graph.

---

8/2 How many points are 4 units from the point (2, -3) and 5 units from the y axis?

---

8/6 Construction

How many points are equidistant from the x and y axis and also 4 units from the line \( x = 6 \)?

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
8/8 Construction

In the diagram below, point $M$ is located on $\overline{AB}$.

Sketch the locus of points that are 1 unit from $\overline{AB}$ and the locus of points 2 units from point $M$.

Label with an $\times$ all points that satisfy both conditions.

8/13 Construction

How many points are both 4 units from the origin and 2 units from the line $y=4$?

8/14 Construction

Triangle $ABC$ has coordinates $A(2, -2)$, $B(2, 1)$, and $C(4, -2)$. Triangle $A'B'C'$ is the image of $\triangle ABC$ under $T_{5,-2}$.

On the set of axes below, graph and label $\triangle ABC$ and its image, $\triangle A'B'C'$.

8/15 Construction

On the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the perpendicular bisector of $\overline{AC}$.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Vocabulary

- Point
- Line
- Plane
- Segment
- Midpoint
- Slope
- Ray
- Acute
- Obtuse
- Right
- Angle Bisector
- Segment Bisector
- Parallel
- Perpendicular
- Slope
- Y intercept
- Positive Slope
- Negative Slope
- Zero Slope
- Undefined Slope
- Coplanar
• Colinear
• Intersection
• Skew
• Volume
• Base
• Height
• Altitude
• Surface Area
• Lateral Area
• Face
• Edge
• Slant Height
• Diameter
• Radius
• Center
• Standard Form of an Equation
• Negate
• Inverse
• Converse
• Contrapositive
• Logically Equivalent
• Rotation
• Reflection
• Dilation
• Scale Factor
• Translation
• Isometry
• Direct Isometry
• Opposite Isometry
• Origin
• X axis
• Y axis
• Clockwise
• Counter Clockwise
• Transformation Composition
• Secant
• Tangent
• Chord
• Central Angle
• Inscribed Angle
• Arc
• Proportion
• Perimeter
• Hypotenuse
• Leg
Topic of the Day

Find the midpoint of $\overline{AB}$: $A(-4, 7)$ and $B(8, 9)$

Find the length of $\overline{AB}$

Calculate the slope of $\overline{AB}$

Find the midpoint of $\overline{CD}$: $C(9, -1)$ and $D(-8, -7)$

Find the length of $\overline{CD}$

Calculate the slope of $\overline{CD}$
Find the midpoint of $\overline{EF}$: E(2, 9) and F(2, -10)

Find the length of $\overline{EF}$

Calculate the slope of $\overline{EF}$

Draw your own line segment with the following properties:

The midpoint is at the origin.

Its length is exactly 8 units.

It has $m = 0$
Convert the following linear equations into slope-intercept form $y = mx + b$

$$3x + 4y = 12$$
$$-\frac{1}{2}y = -2x - 2$$
$$-2x + 6y - 12 = 0$$

Now state the slope $m$ and y-intercept $b$ of each of the equations you just converted.

Next, write the slope of a line which is parallel to each of the three lines above.

Finally, write the slope of a line which is perpendicular to each of the three lines. Show how you calculated this value.
Given the linear equation $-3x + 2y = 13$, write three new linear equations in slope-intercept form which are parallel to the original equation.

Now write three linear equations in slope-intercept form which are perpendicular to the original equation. Explain how you can tell these will be perpendicular.

Given the linear equation $-\frac{3}{2}x + 4y = 16$, circle the following equation or equations which are parallel and underline the equation or equations which are perpendicular to the original equation.

$-3x - 8y = 8$  $8y = 3x - 32$  $-6x + 16y = 64$

$3y = -8x$  $24x + 9y = -27$  $x - 8y = -24$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Solving Linear and Quadratics Algebraically and Graphically

Algebraically:

Solve the following system of equations algebraically:

\[ y = x^2 + 4x - 2 \]
\[ y = 2x + 1 \]

Graphically:

Which ordered pair is in the solution set of the system of equations \( y = -x + 1 \) and \( y = x^2 + 5x + 6 \)?

Calculator:

Solve the following systems on the grid.

\[ y = x^2 + 4x + 1 \] and \(-20x + 4y - 12 = 0\)

Calculator steps:

Which ordered pair is a solution to the system of equations \( y = x \) and \( y = x^2 - 2 \)?

[A] (2, 2)  [B] (0, 0)
[C] (-1, 1)  [D] (-2, -2)

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Planes, lines, and points

1. Lines $k_1$ and $k_2$ intersect at point $E$. Line $m$ is perpendicular to lines $k_1$ and $k_2$ at point $E$.

Which statement is always true?
1) Lines $k_1$ and $k_2$ are perpendicular.
2) Line $m$ is parallel to the plane determined by lines $k_1$ and $k_2$.
3) Line $m$ is perpendicular to the plane determined by lines $k_1$ and $k_2$.
4) Line $m$ is coplanar with lines $k_1$ and $k_2$.

2. Lines $j$ and $k$ intersect at point $P$. Line $m$ is drawn so that it is perpendicular to lines $j$ and $k$ at point $P$. Which statement is correct?
1) Lines $j$ and $k$ are in perpendicular planes.
2) Line $m$ is in the same plane as lines $j$ and $k$.
3) Line $m$ is parallel to the plane containing lines $j$ and $k$.
4) Line $m$ is perpendicular to the plane containing lines $j$ and $k$.

3. In plane $P$, lines $m$ and $n$ intersect at point $A$. If line $k$ is perpendicular to line $m$ and line $n$ at point $A$, then line $k$ is
1) contained in plane $P$ 3) perpendicular to plane $P$
2) parallel to plane $P$ 4) skew to plane $P$

Point $P$ is on line $m$. What is the total number of planes that are perpendicular to line $m$ and pass through point $P$?
1) 1 3) 0
2) 2 4) infinite

Through a given point, $P$, on a plane, how many lines can be drawn that are perpendicular to that plane?
1) 1
2) 2
3) more than 2
4) none

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Volume of Prisms

If the Base of the prism looks like:

- **12 cm**
- **8 cm**
- **10 in**
- **7 cm**
- **5 cm**
- **10 cm**
- **8 cm**
- **10 m**
- **8 m**
- **6 m**

**Area =**

**Area =**

**Area =**

**Area =**

Use the reference sheet for the formula for the following volumes. Make sure you use the correct Base!

1. Find the volume of the triangular prism.

2. Find the volume:

   - **8 yd**
   - **9 yd**
   - **8 yd**
   - **8 yd**

   - **[A] 24 m³**
   - **[B] 140 m³**
   - **[A] 576 cubic yards**
   - **[B] 416 cubic yards**
   - **[C] 38 m³**
   - **[D] 280 m³**
   - **[C] 100 cubic yards**
   - **[D] 136 cubic yards**

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Which expression represents the volume, in cubic centimeters, of the cylinder represented in the diagram below?

3.

![Cylinder Diagram](Image)

1) $162\pi$
2) $324\pi$
3) $972\pi$
4) $3,888\pi$

4. Spaceship Earth at Epcot Center in Florida is a 180 ft geosphere. Estimate its volume by assuming it is a sphere with diameter 180 ft.

5. Find the volume of the cone that has a diameter of 8 feet and a height of 15 feet. (Use $3.14$ for $\pi$.)

[A] 251.2 ft$^3$  
[B] 753.6 ft$^3$
[C] 1004.8 ft$^3$  
[D] 376.8 ft$^3$

6. A regular pyramid with a square base is shown in the diagram below.

![Pyramid Diagram](Image)

A side, $s$, of the base of the pyramid is 12 meters, and the height, $h$, is 42 meters. What is the volume of the pyramid in cubic meters?
Lateral Area Practice

Sketch a picture of each problem then solve. Round all answers to the nearest tenths place.

1) An ice cream cone has a diameter of 4 inches, and a slant height of 7 inches. Calculate the lateral area to determine how much material is needed to make the cone.

2) A softball is about three inches in diameter. How much material does it take to cover the softball?

3) Campbell's Soup cans require labels for consumers to identify the contents of the can. The largest can has a radius of 4 cm, and a height of 10 cm. How much material is needed to create the can's label?

4) An hour glass is in the shape of two cones fixed to each other at the vertex. The entire height of the hour glass is 12 inches tall. Each slant height is 8 inches. The diameter of the largest part of the cone is also 12 inches. How much glass is needed to create this ancient clock?

5) A right cylinder has an approximate lateral area of 320 square inches. The radius of the cylinder is / inches. How tall is the cylinder?

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
6) Dave wants to wrap his rubber band ball to give to his twin brother Dale. Good news for Dale, Dave has been making this ball since 1994. The ball has a diameter of 16 inches. How much wrapping paper would it take to gift wrap the ball?

7) A cone has a lateral area of 134.5 square centimeters. Having a radius of 6 cm, what is the slant height of the cone? What is the height of the cone?

8) A sphere has a volume of 904 cubic inches. What is the surface area?

9) **BONUS** If you had a ball with a diameter of 8 inches, and a can with a height and diameter of 8 inches, which would use the most material to completely cover the shape? How much more material would be needed? (Remember to cover the two bases of the cylinder)

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Circle Graphs Practice

1) Graph the circle represented by the equation \( x^2 + y^2 = 25 \)
   
   What is the center? (__, __)
   
   What is the radius? ______

2) Graph the circle represented by the equation
   \((x - 3)^2 + (y + 4)^2 = 9\)

   What is the center? (__, __)
   
   What is the radius? ______

3) Write the equation of the circle shown in the following graph:

   What is the center? (__, __)
   
   What is the radius? ______

   Write the equation in standard form:

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Circle Graphs Practice

4) Write the equation of the circle shown in the following graph. Use standard form:

![Graph of a circle](image)

4B) What is the area of the circle shown in problem 4? Give your answer in terms of $\pi$ and round to the nearest tenth of a square unit.

5) (6, -4) and (-2, -4) are two of the farthest points Fifi, your prize poodle, can walk when tied to her chain.

Indicate where Fifi’s chain is staked to the ground with a small $\times$ on the graph.

Draw the locus of points where Fifi can walk when her chain is stretched to its limit.

Then write the equation of this relationship in standard form.

5B) What is the total area Fifi can reach when tied up to her chain? Give your answer in terms of $\pi$ and round to the nearest tenth of a square unit.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Logic Practice Problems

1) Write the inverse for “if I study, then I will pass.”

2) Write the converse for “if the dog barks, then he is hungry.”

3) Write the contrapositive for “if you sit on the stairs, then you will block a person’s path.”

4) Write the inverse for “if you do not breathe, then you will die.”

5) Write the converse for “if you don’t play the lottery, then you can save money.”

6) Write the contrapositive for “if you watch TV, then you pay a cable bill.”

7) Write the inverse for “if the day is Wednesday, then you will work.”

8) Write the converse for “if today is not Wednesday, then you do not work.”

9) Write the contrapositive for “if you do not water a plant, then the plant will die.”

10) Write the inverse for “if all the leaves are brown, then the sky is grey.”

11) Write the converse for “if you blink during the flash, then your picture will not be flattering.”

12) Write the contrapositive for “if the Bills do not win the Super Bowl, then they did not practice.”

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
13) Write the **inverse** for “if the Bills win the Super Bowl, then they must have paid someone off.”

14) Write the **converse** for “if you do not stop at the red light, then you could get a ticket.”

15) Write the **contrapositive** for “if your feet smell, then you must wash them.”

16) If the inverse is “if you eat candy, then you will get a cavity” then the original statement must be

17) If the converse is “if you walked then dog, then she pooped” then the original statement must be

18) If the contrapositive is “if you saw the movie, then you must like him” then the original is

19) If the inverse is “if you do not eat fruit, then you do not like smoothies” then the original is

20) If the converse is “if you saw Hannah Montana, then you are lucky” then the original is

21) If the contrapositive is “if you do not ride a bike, then you do not need a helmet” then the original statement must be

22) If the inverse is “if the Muppets are not real, then they should not be able to speak” then the original statement must be

23) If the converse is “if you eat spicy food, then you do not live in Toronto” then the original is

24) If the contrapositive is “if you are not home already, then you do not live here” then the original statement must be
Transformation Rules Sheet

Line Reflections:
\[ r_{x-axis} (x, y) = (x, -y) \]
\[ r_{y-axis} (x, y) = (-x, y) \]
\[ r_{y=x} (x, y) = (y, x) \]
\[ r_{y=-x} (x, y) = (-y, -x) \]

Point Reflection:
\[ R_{180^\circ} (x, y) = (-x, -y) \]

Rotations:
\[ R_{90^\circ} (x, y) = (-y, x) \]
\[ R_{180^\circ} (x, y) = (-x, -y) \]
\[ R_{270^\circ} (x, y) = (y, -x) \]
\[ R_{-90^\circ} (x, y) = (y, -x) \]

Translation:
\[ T_{a,b} (x, y) = (x + a, y + b) \]

Dilation:
\[ D_k (x, y) = (kx, ky) \]
**Isometry** - length is preserved - the figures are congruent.

**Direct Isometry** - orientation is preserved - the order of the lettering in the figure and the image are the same, either both clockwise or both counterclockwise.

**Opposite Isometry** - orientation is not preserved - the order of the lettering is reversed, either clockwise becomes counterclockwise or counterclockwise becomes clockwise.

<table>
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<tr>
<th>Line Reflection</th>
<th>Point Reflection</th>
<th>Translations</th>
<th>Rotations</th>
<th>Dilations</th>
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<td>Direct isometry</td>
<td>Direct isometry</td>
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<td>3. parallelism</td>
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<td>3. parallelism</td>
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<td>4. collinearity</td>
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<tr>
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<td>5. midpoint</td>
<td>5. midpoint</td>
<td>5. midpoint</td>
<td>Lengths not same.</td>
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</tbody>
</table>

**Reverse Orientation** (letter order changed) | Same Orientation (letter order the same) | Same Orientation (letter order the same) | Same Orientation (letter order the same) | Same Orientation (letter order the same) |

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Glide Reflection

**Opposite isometry**

**Properties preserved:**
1. distance
2. angle measure
3. parallelism
4. colinearity
5. midpoint

**Reverse Orientation**
(letter order changed)
Transformations practice

1. What is the image of point (8, -4) under the rotation $R_{90^\circ}$ about the origin?
   1) (8, 4)
   2) (4, 8)
   3) (−4, 8)
   4) (−8, 4)

2. In which quadrant would the image of point (5, −3)
   fall after a dilation using a factor of −3?
   1) I
   2) II
   3) III
   4) IV

3. In the diagram at the left, line segment $TM$ is the line of reflection for the figure.
   Use this diagram to answer the following questions
   a. What is the reflection image of segment $AB$?
   b. What is the reflection image of triangle $FUN$?
   c. What is the reflection image of point $U$?
   d. How does the length of segment $EC$ compare to the length of segment $BC$?

4. This graph illustrates a translation of $T_{(6, −2)}$
   0 True
   0 False
5. Under a dilation of scale factor 3 with the center at the origin, what will be the coordinates of the image of point $B$?

![Diagram showing points A(1,1), B(2,3), C(4,1) and a triangle ABC]

6. The image of point $(3, -5)$ under the translation that shifts $(x, y)$ to $(x - 1, y - 3)$ is
   1) $(-4, 8)$
   2) $(-3, 15)$
   3) $(2, 8)$
   4) $(2, -8)$

7. In the diagram below, which transformation was used to map $\triangle ABC$ to $\triangle A'B'C'$?

   ![Diagram showing $\triangle ABC$ and its image $\triangle A'B'C'$]

   1) dilation
   2) rotation
   3) reflection
   4) glide reflection
8. Under a dilation, triangle $A(0,0), B(0,4), C(6,0)$ becomes triangle $A'(0,0), B'(0,10), C'(15,0)$. What is the scale factor for this dilation?

Choose one:
- 2
- 2.5
- 4

9. Ms. Brewer’s art class is drawing reflected images. She wants her students to draw images reflected in a line. Which diagram represents a correctly drawn image?

10. Is this a reflection?

11. Which transformation is not an isometry?
1) $r_{y-x}$
2) $R_{0,90^\circ}$
3) $T_{3,5}$
4) $D_2$

12. Polygon $A'B'C'D'$ is a 180 degree counterclockwise rotation of polygon $ABCD$.

- True
- False
13. This drawing has been rotated $180^\circ$.

- yes
- no

14. What are the coordinates of point $(-1,4)$ under dilation $D_{-2}$?
1) $(-2,8)$
2) $(2,-8)$
3) $(-8,2)$
4) $(8,-2)$

15. Point $A$ is rotated $180^\circ$ in a counterclockwise direction about the origin. If the coordinates of $A$ are $(-1,3)$, what are the coordinates of $A'$, its image?

16. Which transformation represents a dilation?
1) $(8,4) \rightarrow (11,7)$
2) $(8,4) \rightarrow (-8,4)$
3) $(8,4) \rightarrow (-4,-8)$
4) $(8,4) \rightarrow (4,2)$

If $x = -2$ and $y = -1$, which point on the accompanying set of axes represents the translation $(x,y) \rightarrow (x+2,y+3)$?

1) $O$
2) $R$
3) $S$
4) $T$
1. The endpoints of $AB$ are $A(3, 2)$ and $B(7, 1)$. If $AB''$ is the result of the transformation of $AB$ under $D_2 \circ T_{-43}$, what are the coordinates of $A''$ and $B''$?
   1) $A''(-2, 10)$ and $B''(6, 8)$
   2) $A''(-1, 5)$ and $B''(3, 4)$
   3) $A''(2, 7)$ and $B''(10, 5)$
   4) $A''(14, -2)$ and $B''(22, -4)$

2. If the coordinates of point $A$ are $(-2, 3)$, what is the image of $A$ under $r_{y-\text{axis}} \circ D_3$?
   1) $(-6, -9)$
   2) $(9, -6)$
   3) $(5, 6)$
   4) $(6, 9)$

3. The coordinates of $\triangle JRB$ are $J(1, -2)$, $R(-3, 6)$, and $B(4, 5)$. What are the coordinates of the vertices of its image after the transformation $T_{2, -1} \circ r_{y-\text{axis}}$?
   [A] $(3, 1), (-1, -7), (6, -6)$
   [B] $(-1, 2), (3, 6), (-4, 5)$
   [C] $(1, -3), (5, 5), (-2, 4)$
   [D] $(3, -3), (-1, 5), (6, 4)$

4. What are the coordinates of point $A'$, the image of point $A(-4, 1)$ after the composite transformation $R_{90^\circ} \circ r_{x-y}$, where the origin is the center of rotation?
   [A] $(-1, -4)$  [B] $(-4, -1)$
   [C] $(4, 1)$  [D] $(1, 4)$
6. What is the image of point (1,1) under $r_{x=\text{max}} \circ R_{0,90^\circ}$?

[A] (1,1)  [B] (-1,1)  
[C] (1,-1)  [D] (-1,-1)

7. The coordinates of the vertices of parallelogram $ABCD$ are $A(-2,2)$, $B(3,5)$, $C(4,2)$, and $D(-1,-1)$. State the coordinates of the vertices of parallelogram $A''B''C''D''$ that result from the transformation $r_{y=\text{max}} \circ T_{2,-1}$. [The use of the set of axes below is optional.]

8. On the accompanying grid, graph and label $\triangle ABC$ with vertices $A(3,1)$, $B(0,4)$, and $C(-5,3)$. On the same grid, graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the transformation $r_{x=\text{max}} \circ r_{y=x}$.

9. What are the coordinates of point $A'$, the image of point $A(-4,1)$ after the composite transformation $R_{90^\circ} \circ r_{y=\text{max}}$ where the origin is the center of rotation?

[A] (-1,-4)  [B] (-4,-1)  
[C] (4,1)  [D] (1,4)
**Circle Segment Rules:**

Rule #1: Intersecting Chords Rule: (part)(part)=(part)(part)  
The "P-P" rule
1. In the accompanying diagram chord $\overline{CX}=10$, $\overline{AX}=6$, and $\overline{BX}=3$.  
   Find $\overline{BC}=$

Rule #2: Secant-Secant Rule: (whole)(outside part)=(whole)(outside part)  
The "Wo-Wo" rule
2. In the accompanying diagram chord $\overline{BX}=3$, $\overline{AX}=15$, and $\overline{CX}=9$.  
   Find $\overline{DX}=$

   Find $CD=$

Rule #3: Secant-Tangent Rule: (whole secant)(outside part)=(tangent part)$^2$  
The "Wo-tt" rule
3A. In the accompanying diagram chord $\overline{AX}=4$, $\overline{DX}=2$.  
   Find $\overline{CX}=$

   Find $\overline{CD}=$

3B. In the accompanying diagram chord $\overline{CD}=21$, $\overline{DX}=4$.  
   Find $\overline{AX}=$
Tangents, Secants in Circles Practice

1. Kimi wants to determine the radius of a circular pool without getting wet. She is located at point $K$, which is 4 feet from the pool and 12 feet from the point of tangency, as shown in the accompanying diagram.

   What is the radius of the pool?
   1) 16 ft
   2) 20 ft
   3) 32 ft
   4) $4\sqrt{10}$ ft

2. $\overline{AB}$ is tangent to $\odot O$ at $A$ (not drawn to scale). Find the length of the radius $r$, to the nearest tenth.

3. In the accompanying diagram, $\overline{PAB}$ and $\overline{PCD}$ are secants drawn to circle $O$, $PA = 8$, $PB = 20$, and $PD = 16$.

   What is $PC$?
   1) 6.4
   2) 10
   3) 12
   4) 40

4. In the diagram below of circle $O$, secant $\overline{AB}$ intersects circle $O$ at $D$, secant $\overline{AOC}$ intersects circle $O$ at $E$, $AE = 4$, $AB = 12$, and $DB = 6$.

   What is the length of $\overline{OC}$?
   1) 4.5
   2) 7
   3) 9
   4) 14
5. Find the value of $x$.

6. In the accompanying diagram, $BA$ is tangent to circle $O$ at $A$. Radii $OA$ and $OC$ are drawn, and $OC$ is extended to intersect $BA$ at $B$. If $BA = 15$ and $OB = 17$, find the measure of a radius of circle $O$.

7. Solve for $x$.

8. $AB$, $CB$ tangents.
Find $x$.

9. $AB$, $CB$ tangents.
Find $x$.

10. Given: circle with two secants.
Solve for $x$. 
Central Angles:

- $360^\circ$
- $180^\circ$
- $90^\circ$
- Solve for $x$

Inscribed angles:

- Through the center
- Center inside angle
- Center outside angle

Parallel Chords:

- Through center
- Connecting to center
Vertical Angels Inside the circle:

Through the center

Not through the center

Tangents through the circle:
Bingo Worksheet
Working with Arcs and Angles in Circles
In each diagram, find the value of $x$. 

1. $275^\circ$
2. $17^\circ$
3. $x$
4. $115^\circ$
5. 
6. $100^\circ$
7. $105^\circ$
8. $x$
9. $85^\circ$
10. $105^\circ$
11. $55^\circ$
12. $x$
13. $x$
14. $25^\circ$
15. 
16. $x$
17. $110^\circ$
18. $x$
19. $x$
20. 
21. $270^\circ$
22. $x$
23. $x$
24. $200^\circ$
25. 

[Images of various circle diagrams with angles indicated]
G.G.15: Similarity 1: Investigate, justify, and apply theorems about similar triangles

1. The accompanying diagram shows two similar triangles.

Which proportion could be used to solve for \( x \)?
1) \( \frac{x}{24} = \frac{9}{15} \)
2) \( \frac{24}{9} = \frac{15}{x} \)
3) \( \frac{32}{x} = \frac{12}{15} \)
4) \( \frac{32}{12} = \frac{13}{x} \)

2. A triangle has sides whose lengths are 5, 12, and 13. A similar triangle could have sides with lengths of
1) 3, 4, and 5
2) 6, 8, and 10
3) 7, 24, and 25
4) 10, 24, and 26

3. In the accompanying diagram, \( \triangle QRS \) is similar to \( \triangle LMN \), \( RQ = 30 \), \( QS = 21 \), \( SR = 27 \), and \( LN = 7 \). What is the length of \( ML \)?

4. The Rivera family bought a new tent for camping. Their old tent had equal sides of 10 feet and a floor width of 15 feet, as shown in the accompanying diagram.

If the new tent is similar in shape to the old tent and has equal sides of 16 feet, how wide is the floor of the new tent?
5 The accompanying diagram shows a section of the city of Tacoma. High Road, State Street, and Main Street are parallel and 5 miles apart. Ridge Road is perpendicular to the three parallel streets. The distance between the intersection of Ridge Road and State Street and where the railroad tracks cross State Street is 12 miles. What is the distance between the intersection of Ridge Road and Main Street and where the railroad tracks cross Main Street?

6 In the accompanying diagram, triangle A is similar to triangle B. Find the value of n.

7 In the diagram below, \( \triangle ABC \sim \triangle EFG \), \( \angle C = 4x + 30 \), and \( \angle G = 5x + 10 \). Determine the value of x.
G.G.45: Similarity 2: Investigate, justify, and apply theorems about similar triangles

1. The perimeter of $\triangle A'B'C'$, the image of $\triangle ABC$, is twice as large as the perimeter of $\triangle ABC$. What type of transformation has taken place? 
   1) dilation 
   2) translation 
   3) rotation 
   4) reflection

2. Two triangles are similar. The lengths of the sides of the smaller triangle are 3, 5, and 6, and the length of the longest side of the larger triangle is 18. What is the perimeter of the larger triangle? 
   1) 14 
   2) 18 
   3) 24 
   4) 42

3. Delroy's sailboat has two sails that are similar triangles. The larger sail has sides of 10 feet, 24 feet, and 26 feet. If the shortest side of the smaller sail measures 6 feet, what is the perimeter of the smaller sail? 
   1) 15 ft 
   2) 36 ft 
   3) 60 ft 
   4) 100 ft

4. The base of an isosceles triangle is 5 and its perimeter is 11. The base of a similar isosceles triangle is 10. What is the perimeter of the larger triangle? 
   1) 15 
   2) 21 
   3) 22 
   4) 110

5. On a scale drawing of a new school playground, a triangular area has sides with lengths of 8 centimeters, 15 centimeters, and 17 centimeters. If the triangular area located on the playground has a perimeter of 120 meters, what is the length of its longest side? 
   1) 24 m 
   2) 40 m 
   3) 45 m 
   4) 51 m
6 The ratio of the corresponding sides of two similar squares is 1 to 3. What is the ratio of the area of the smaller square to the area of the larger square?
   1) $1: \sqrt{3}$
   2) $1:3$
   3) $1:6$
   4) $1:9$

7 Two triangles are similar, and the ratio of each pair of corresponding sides is 2:1. Which statement regarding the two triangles is not true?
   1) Their areas have a ratio of 4:1.
   2) Their altitudes have a ratio of 2:1.
   3) Their perimeters have a ratio of 2:1.
   4) Their corresponding angles have a ratio of 2:1.

8 Given $\triangle ABC \sim \triangle DEF$ such that $\frac{AB}{DE} = \frac{3}{2}$. Which statement is not true?
   1) $\frac{BC}{EF} = \frac{3}{2}$
   2) $\frac{m\angle A}{m\angle D} = \frac{3}{2}$
   3) $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF} = \frac{9}{4}$
   4) $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{3}{2}$

9 Which is not a property of all similar triangles?
   1) The corresponding angles are congruent.
   2) The corresponding sides are congruent.
   3) The perimeters are in the same ratio as the corresponding sides.
   4) The altitudes are in the same ratio as the corresponding sides.
Mean Proportional in a Right Triangle

The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

\[ \Delta ACB \sim \Delta ADC \sim \Delta CDB \]

Why do we care that these three triangles are all similar to one another?

Because now we can set up proportions to solve for the measures of missing sides among these triangles.

**Rule #1:** The altitude to the hypotenuse of a right triangle is the mean proportional between the segments into which it divides the hypotenuse.

\[ \frac{AD}{CD} = \frac{CD}{DB} \]

**Altitude Rule:**

\[ \frac{\text{part of hyp}}{\text{altitude}} = \frac{\text{altitude}}{\text{other part hyp}} \]

**Practice #1:**

Find \( x \):

[Diagram of a triangle with altitude and segments labeled]
Mean Proportional in a Right Triangle

Rule #2 Each leg of a right triangle is the mean proportional between the entire hypotenuse and part of the leg on the hypotenuse.

\[
\frac{AB}{CB} = \frac{CB}{DB}
\]

or

\[
\frac{AB}{CA} = \frac{CA}{AD}
\]

Practice #2A:

Find \(x\):

Practice #2B:

Find \(x\) to nearest tenth:
Mean Proportional of a Triangle Practice

1. The accompanying diagram shows a 24-foot ladder leaning against a building. A steel brace extends from the ladder to the point where the building meets the ground. The brace forms a right angle with the ladder.

   \[ \begin{align*}
   [A] & \quad 10^2 + x^2 = 24^2 \\
   [B] & \quad \frac{10}{x} = \frac{x}{14} \\
   [C] & \quad \frac{10}{x} = \frac{x}{24} \\
   [D] & \quad 10^2 + x^2 = 14^2
   \end{align*} \]

If the steel brace is connected to the ladder at a point that is 10 feet from the foot of the ladder, which equation can be used to find the length, \( x \), of the steel brace?

2. The accompanying diagram shows part of the architectural plans for a structural support of a building. \( PLAN \) is a rectangle and \( AS \parallel LN \).

   \[ \begin{align*}
   [A] & \quad \frac{AS}{SN} = \frac{AS}{LS} \\
   [B] & \quad \frac{AN}{LN} = \frac{AS}{LS} \\
   [C] & \quad \frac{AS}{LS} = \frac{LS}{SN} \\
   [D] & \quad \frac{LS}{SN} = \frac{AS}{SN}
   \end{align*} \]

Which equation can be used to find the length of \( AS \)?
Mean Proportional of a Triangle Practice

3. In the diagram below of right triangle \( ACB \), altitude \( CD \) intersects \( AB \) at \( D \). If \( AD = 3 \) and \( DB = 4 \), find the length of \( CD \) in simplest radical form.

![Diagram of triangle ACB with altitude CD](image)

4. In the diagram below, the length of the legs \( AC \) and \( BC \) of right triangle \( ABC \) are 6 cm and 8 cm, respectively. Altitude \( CD \) is drawn to the hypotenuse of \( \triangle ABC \).

![Diagram of right triangle ABC with altitude CD](image)

What is the length of \( AD \) to the nearest tenth of a centimeter?

[A] 4.0  [B] 6.0  [C] 6.4  [D] 3.6
5. In the diagram below of right triangle $ACB$, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

![Diagram of right triangle ACB with altitude CD drawn to hypotenuse AB.]

If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?


6. Four streets in a town are illustrated in the accompanying diagram. If the distance on Poplar Street from $F$ to $P$ is 12 miles and the distance on Maple Street from $E$ to $M$ is 10 miles, find the distance on Maple Street, in miles, from $M$ to $P$.

![Diagram of a triangle with streets labeled Elm, Fern, Poplar, and Maple.]

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Triangle Special Points

Centroid: The point where all three of a triangle’s medians intersect. Assuming that the figure has equal density, this is where you could put a stick under the triangle and have it balance.

Orthocenter: The point where all three of a triangle’s altitudes intersect.

Incenter: The point where all three of a triangle’s angle bisectors intersect. This point is also the center of the biggest circle which fits entirely inside a given triangle.
Triangle Special Points

Circumcenter: The point where all three of a triangle’s perpendicular bisectors intersect. It is the center of the only circle which contains all three vertices of the triangle.

Key Rule: The centroid of any triangle divides each median into a 2:1 ratio.

In the diagram below of \( \triangle TEM \), medians \( \overline{TB}, \overline{EC}, \) and \( \overline{MA} \) intersect at \( D \), and \( TB = 9 \). Find the length of \( \overline{T} \)
Special Triangle Points Practice

1 In which triangle do the three altitudes intersect outside the triangle?
   1) a right triangle
   2) an acute triangle
   3) an obtuse triangle
   4) an equilateral triangle

2 The diagram below shows the construction of the center of the circle circumscribed about $\triangle ABC$.

![Diagram of constructions](image)

This construction represents how to find the intersection of
   1) the angle bisectors of $\triangle ABC$
   2) the medians to the sides of $\triangle ABC$
   3) the altitudes to the sides of $\triangle ABC$
   4) the perpendicular bisectors of the sides of $\triangle ABC$
3. In the diagram below of \( \triangle ABC \), \( \overline{CD} \) is the bisector of \( \angle BCA \), \( \overline{AE} \) is the bisector of \( \angle CAB \), and \( \overline{BG} \) is drawn.

Which statement must be true?
1) \( DG = EG \)
2) \( AG = BG \)
3) \( \angle AEB = \angle AEC \)
4) \( \angle DBG = \angle EBG \)

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4. Find \( DE \).

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5. Given \( AC = 42, \ CB = 46, \ AB = 48 \).
   \( D, E, F \) are midpoints.
   Find the perimeter of triangle \( DEF \).

Choose:
- 34
- 48
- 68
- 136
Test 2 Review

1. In the accompanying diagram chord $\overline{AX} = 4$, $\overline{DX} = 2$.

   Find $\overline{CX} = \underline{\quad}$.

   Find $\overline{CD} = \underline{\quad}$.

2. In the accompanying diagram of right triangle $ABC$, altitude $\overline{AD}$ divides hypotenuse $\overline{BC}$ into segments with lengths of 4 and 5. Find the length of leg $\overline{AB}$.

   \[ [1] \ 4.5 \quad [2] \ 2\sqrt{5} \quad [3] \ 6 \quad [4] \ 7.5 \]

3. The number of points equidistant from two parallel lines and also equidistant from two points on one of the given lines is exactly.

   \[ [1] \ 1 \quad [2] \ 2 \quad [3] \ 3 \quad [4] \ 4 \]

4. Which statement is logically equivalent to “If Andrea gets a job, she buys a new car”?

   [1] Andrea gets a job and she buys a new car.
   [2] If Andrea does not buy a new car, she does not get a job.
   [3] If Andrea does not get a job, she does not buy a new car.
   [4] If Andrea buys a new car, she gets a job.

5. In the accompanying diagram of circle $O$, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$ and $m\angle{ACD} : m\angle{CBD} : m\angle{BDE} : m\angle{DA} = 4 : 2 : 6 : 8$.
   What is the $m\angle{DEB}$?

   \[ [1] \ 36^\circ \quad [2] \ 90^\circ \quad [3] \ 100^\circ \quad [4] \ 126^\circ \]

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
6. In the accompanying diagram, $ABC$ is an equilateral triangle with a perimeter of 30. What is the length of altitude $h$ of this triangle?

- [1] $5\sqrt{2}$
- [2] $5\sqrt{3}$
- [3] $10\sqrt{2}$
- [4] $10\sqrt{3}$

7. A triangle has vertices $A(3,2)$, $B(4,1)$ and $C(4,3)$. What are the coordinates of point $B$ under the glide reflection $T_{(0,1)} \circ r_{x=0}$?

- [1] $(2,-1)$
- [2] $(4,0)$
- [3] $(1,2)$
- [4] $(-4,2)$

In circle $O$, the secant from $S$ to $D$ intersects the circle at $C$ and the secant from $S$ to $B$ intersects the circle at $A$.

If $SC = 6$, $CD = 8$ and $SA = 4$, find $AB$.

- [1] $16/3$
- [2] $17$
- [3] $21$
- [4] $28/3$

9. What is the equation of a line parallel to the line whose equation is $3y + 5x = 6$ and whose $y$-intercept is 4?

- [1] $y = 5x + 4$
- [2] $y = \frac{5}{3}x + 2$
- [3] $y = 5x - 4$
- [4] $y = -\frac{5}{3}x + 4$

Which point satisfies the system of equations, $y = 2x + 2$ and $y = -x^2 + 2$?

- [1] $(0,2)$
- [2] $(-1,0)$
- [3] $(1,2)$
- [4] $(2,-4)$
Angle Notes

Interior:  

Exterior:  

Sum of interior angles of triangle =  

Isosceles Triangle  

Sum of interior angles of quadrilateral =  

Parallelogram  

Sum of interior angles of any polygon  

Pentagon  hexagon  octagon  n-gon
** Regular polygon: 

** To find the measure of an interior angle of a regular polygon use the formula: **

Ex: Find the measure of an interior angle of a regular hexagon.

Exterior Angles: 

Solve for x.

\[
\begin{array}{c}
\text{75°} \\
\text{55°} \\
\text{x°} \\
\end{array}
\quad
\begin{array}{c}
\text{x°} \\
\text{60°} \\
\text{140°} \\
\end{array}
\]

The sum of the exterior angles =

Use the formula \[\text{Exterior Angle} = \frac{360°}{n}\] to calculate each exterior angle of a regular polygon.

Find the number of degrees in each exterior angle of a regular pentagon.

If each exterior angle of a regular polygon contains 40°, how many sides does the polygon have?
1. In the diagram of $\triangle ABC$ below, $\overline{AB} \cong \overline{AC}$. The measure of $\angle B$ is $40^\circ$.

![Diagram of $\triangle ABC$ with $\overline{AB} \cong \overline{AC}$ and $\angle B = 40^\circ$.]

What is the measure of $\angle A$?
1) $40^\circ$
2) $50^\circ$
3) $70^\circ$
4) $100^\circ$

2. In the accompanying diagram, isosceles $\triangle ABC \cong \triangle DEF$, $\angle C = 5x$, and $m\angle D = 2x + 18$. Find $m\angle B$ and $m\angle BAG$.

![Diagram of isosceles triangles $\triangle ABC$ and $\triangle DEF$ with $\angle C = 5x$ and $\angle D = 2x + 18$.]

3. In the accompanying diagram of $\triangle ABC$, $\triangle ABC$ is an equilateral triangle and $AD = AB$. What is the value of $x$, in degrees?

![Diagram of equilateral triangle $\triangle ABC$ with $AD = AB$.]

4. In the accompanying diagram of $\triangle ABC$, $\overline{AB}$ is extended to $D$, exterior angle $CBD$ measures $145^\circ$, and $m\angle C = 75$.

What is $m\angle C\angle D$?
1) $35$
2) $70$
3) $110$
4) $220$

5. In the diagram below, $\triangle ABC$ is shown with $\overline{AC}$ extended through point $D$.

![Diagram of $\triangle ABC$ with $\overline{AC}$ extended through point $D$.]

If $m\angle BCD = 6x + 2$, $m\angle BAC = 3x + 15$, and $m\angle ABC = 2x - 1$, what is the value of $x$?

- $m\angle BCD = 6x + 2$
- $m\angle BAC = 3x + 15$
- $m\angle ABC = 2x - 1$

If the sum of the angles in a triangle is $180^\circ$, then $m\angle BCD + m\angle BAC + m\angle ABC = 180^\circ$.

Solving for $x$: $6x + 2 + 3x + 15 + 2x - 1 = 180$

$11x + 16 = 180$

$11x = 164$

$x = 15^\circ$
6. The pentagon in the diagram below is formed by five rays.

What is the degree measure of angle $x$?
1) 72
2) 96
3) 108
4) 112

9. The measures of five of the interior angles of a hexagon are $150^\circ$, $100^\circ$, $80^\circ$, $165^\circ$, and $150^\circ$. What is the measure of the sixth interior angle?

[A] $180^\circ$  [B] $105^\circ$  [C] $80^\circ$  [D] $75^\circ$

10. In the accompanying diagram, $AB \parallel CD$. From point $E$ on $AB$, transversals $EF$ and $EG$ are drawn, intersecting $CD$ at $H$ and $I$, respectively.

If $m\angle CHF = 20$ and $m\angle DIG = 60$, what is $m\angle HEI$?

11. Find the measure of one of the interior angles of a regular polygon with twelve sides.

[A] $150^\circ$  [B] $30^\circ$  [C] $165^\circ$  [D] $15^\circ$
Triangle Inequality Notes

The sum of any two sides of a triangle is bigger than the third side!

Why? Because if two sides are equal to the length of the third side, then the two short sides are stuck on top of the third side!

And if the two shorter sides are less than than the length of the third side, then the two short sides can’t meet to form a triangle!

So what’s the fastest way to check three side lengths to see if they can form a triangle, without having to complete a construction?

It’s easy!

1. Select the two smallest sides.
2. Add them together.
3. Compare them to the longest side.
   a. IF the sum is greater than the third side, then YES!, you have the three side lengths of a triangle.
   b. IF the sum is equal to the third side, then NO!, the three sides cannot form a triangle.
   c. IF the sum is less than the third side, then NO!, the three sides cannot form a triangle.
Triangle Inequality Notes

Let's try three of these problems together:

1) Which set of numbers represents the lengths of the sides of a triangle?
   1) \{5, 18, 13\}
   2) \{6, 17, 22\}
   3) \{16, 24, 7\}
   4) \{26, 8, 15\}

2) The plot of land illustrated in the accompanying diagram has a perimeter of 34 yards. Find the length, in yards, of each side of the figure. Could these measures actually represent the measures of the sides of a triangle? Explain your answer.

![Diagram of a triangle with sides labeled x + 3, 3x - 1, and 4x.]

3) A plot of land is in the shape of rhombus \(ABCD\) as shown below.

![Diagram of a rhombus ABCD with diagonals AC and BD.]

Which can not be the length of diagonal \(AC\)?
   1) 24 m
   2) 18 m
Triangle Inequality Practice

1) If the lengths of two sides of a triangle are 4 and 10, what could be the length of the third side?
   1) 6
   2) 8
   3) 14
   4) 16

2) Sara is building a triangular pen for her pet rabbit. If two of the sides measure 8 feet and 15 feet, the length of the third side could be
   1) 13 ft
   2) 7 ft
   3) 3 ft
   4) 23 ft

3) Which set can not represent the lengths of the sides of a triangle?
   1) {4, 5, 6}
   2) {5, 5, 11}
   3) {7, 7, 12}
   4) {8, 8, 8}

4) Phil is cutting a triangular piece of tile. If the triangle is scalene, which set of numbers could represent the lengths of the sides?
   1) {2, 4, 7}
   2) {4, 5, 6}
   3) {3, 5, 8}
   4) {5, 5, 8}

5) In the diagram below of ΔABC, D is a point on AB, AC = 7, AD = 6, and BC = 18.

   ![Diagram of triangle ABC with point D on AB]

   The length of DB could be
   1) 5
   2) 12
   3) 19
   4) 25
Triangles: Sides and Angles

Have you ever played Pac-Man? The wider his mouth opens, the bigger his mouth appears.

You can get credit for this totally obvious relationship on the Geometry Regents exam!

The greater an angle, the longer the opposite side of the triangle!

Let’s practice:

Find the largest side of the triangle. (not drawn to scale)
Triangle Sides and Angles Practice

1) On the banks of a river, surveyors marked locations A, B, and C. The measure of \( \angle ACB \) is 70° and the measure of \( \angle ABC \) is 65°.

Which expression shows the relationship between the lengths of the sides of this triangle?
1) \( AB < BC < AC \)
2) \( BC < AB < AC \)
3) \( BC < AC < AB \)
4) \( AC < AB < BC \)

2) In \( \triangle ABC \), \( m \angle A = 95 \), \( m \angle B = 50 \), and \( m \angle C = 35 \). Which expression correctly relates the lengths of the sides of this triangle?
1) \( AB < BC < CA \)
2) \( AB < AC < BC \)
3) \( AC < BC < AB \)
4) \( BC < AC < AB \)

3) In \( \triangle PQR \), \( PQ = 8 \), \( QR = 12 \), and \( RP = 13 \). Which statement about the angles of \( \triangle PQR \) must be true?
1) \( m \angle Q > m \angle P > m \angle R \)
2) \( m \angle Q > m \angle R > m \angle P \)
3) \( m \angle R > m \angle P > m \angle Q \)
4) \( m \angle P > m \angle R > m \angle Q \)

4) In the diagram below of \( \triangle ABC \) with side \( AC \) extended through D, \( m \angle A = 37 \) and \( m \angle BCD = 117 \). Which side of \( \triangle ABC \) is the longest side? Justify your answer.
Mixed Review Problems

1. In two similar triangles, the ratio of the lengths of a pair of corresponding sides is 7:8. If the perimeter of the larger triangle is 32, find the perimeter of the smaller triangle.
   \[ \begin{array}{cccc}
   \end{array} \]

2. Which of the following statements is NOT true about the centroid of a triangle?
   \[ \begin{array}{ll}
   [1] & \text{The centroid of a triangle is point where the medians of the triangle are concurrent.} \\
   [2] & \text{The centroid of a triangle may be located inside the triangle, on a side of the triangle, or outside of the triangle.} \\
   [3] & \text{The centroid of a triangle divides the medians of the triangle into a ratio of 2:1.} \\
   [4] & \text{The centroid of a triangle is the center of gravity of a triangular shape of uniform thickness and density.} \\
   \end{array} \]

3. Point C (3,4) is the midpoint of \( \overline{AB} \). If the coordinates of \( A \) are (7,6), the coordinates of \( B \) are:
   \[ \begin{array}{ll}
   [1] & (-1,2) \\
   [2] & (2,1) \\
   [3] & (5,5) \\
   [4] & (11,8) \\
   \end{array} \]

4. In \( \triangle ABC \), \( \overline{BC} \) is extended to \( E \), and \( D \) is a point on \( \overline{BC} \).
   \[ \begin{array}{ll}
   [1] & m<ADE > m<ABC \\
   [2] & m<ADE = m<ABC \\
   [3] & m<ACB = m<ABC \\
   [4] & m<ABC > m<ADE \\
   \end{array} \]

5. If the lengths of two sides of a triangle measure 7 and 15, then the length of the third side could measure:
   \[ \begin{array}{cccc}
   \end{array} \]

6. In the accompanying diagram of right triangle \( ABC \), altitude \( AD \) divides hypotenuse \( BC \) into segments with lengths of 4 and 5. Find the length of leg \( AB \).
   \[ \begin{array}{ll}
   [1] & 4.5 \\
   [2] & 2\sqrt{5} \\
   [3] & 6 \\
   [4] & 7.5 \\
   \end{array} \]
7. In the accompanying diagram, \( \overline{AB} \parallel \overline{CD} \), \( \angle FHE \), \( m\angle AEP = 40 \), and \( m\angle FHG = 60 \). Find \( m\angle HGD \).

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & 95^\circ & \text{[3]} & 120^\circ & \hline \text{[2]} & 100^\circ & \text{[4]} & 140^\circ & \hline \end{array} \]

8. If the measures of three angles of a triangle are represented by \((y + 30)^\circ\), \((4y + 30)^\circ\), and \((10y - 30)^\circ\), then the triangle must be:

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & \text{obtuse} & \text{[2]} & \text{isosceles} & \text{[3]} & \text{scalene} & \text{[4]} & \text{right} & \hline \end{array} \]

9. The number of points equidistant from two parallel lines and also equidistant from two points on one of the given lines is exactly:

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & 1 & \text{[2]} & 2 & \text{[3]} & 3 & \text{[4]} & 4 & \hline \end{array} \]

10. The lengths of the bases of an isosceles trapezoid are 6 centimeters and 12 centimeters. If the length of each leg is 5 centimeters, what is the area of the trapezoid?

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & 18 \text{ cm}^2 & \text{[2]} & 36 \text{ cm}^2 & \text{[3]} & 45 \text{ cm}^2 & \text{[4]} & 90 \text{ cm}^2 \hline \end{array} \]

11. In the accompanying diagram, \( \overline{PQ} \) and \( \overline{PS} \) are tangents drawn to circle \( O \), and chord \( \overline{OS} \) is drawn. If \( \angle P = 40 \), what is \( \angle PQS \)?

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & 140^\circ & \text{[3]} & 70^\circ & \hline \text{[2]} & 80^\circ & \text{[4]} & 60^\circ & \hline \end{array} \]

12. Which statement is logically equivalent to “If Andrea gets a job, she buys a new car”?

\[ \begin{array}{c|c|c|c|c} \hline \text{[1]} & \text{Andrea gets a job and she buys a new car.} & \hline \text{[2]} & \text{If Andrea does not buy a new car, she does not get a job.} & \hline \text{[3]} & \text{If Andrea does not get a job, she does not buy a new car.} & \hline \text{[4]} & \text{If Andrea buys a new car, she gets a job.} & \hline \end{array} \]
13. In the accompanying diagram, $AB$ and $CD$ intersect at $E$, $E$ is the midpoint of $AB$, and $\angle A \equiv \angle B$. Which statement can be used to prove that $\triangle ADE \cong \triangle BCD$?

[1] ASA (Angle-Side-Angle)  
[2] HL (Hypotenuse-Leg)  
[3] SSS (Side-Side-Side)  
[4] SAS (Side-Angle-Side)

14. For this rectangular solid, which plane(s) contain $D$ and are parallel to plane $FEH$?

[1] planes $DAB$ and $HAD$  
[2] only plane $DAB$  
[3] planes $DCB$ and $FCB$  
[4] only plane $HAD$

15. In the accompanying diagram of circle $O$, chords $AB$ and $CD$ intersect at $E$ and $m\angle AC : m\angle CB : m\angle BD : m\angle DA = 4 : 2 : 6 : 8$.

What is the $m\angle DEB$?

[1] $36^\circ$  
[2] $90^\circ$  
[3] $100^\circ$  
[4] $126^\circ$

16. The sum of the measures of the interior angles of a regular pentagon is:

[1] $180^\circ$  
[2] $360^\circ$  
[3] $540^\circ$  
[4] $720^\circ$

17. Which statement is false about the line whose equation is $y = -2x - 5$?

[2] It is parallel to the line whose equation is $y = 2x + 5$.  
[3] Its y-intercept is -5.  
[4] It is perpendicular to the line whose equation is $y = \frac{1}{2}x - 5$. 

18. In the accompanying diagram, \( ABC \) is an equilateral triangle with a perimeter of 30. What is the length of altitude \( h \) of this triangle?

- [1] \( 5\sqrt{2} \)
- [2] \( 5\sqrt{3} \)
- [3] \( 10\sqrt{2} \)
- [4] \( 10\sqrt{3} \)

![Equilateral Triangle Diagram]

19. Which of the following points is a solution to the system \( y = 4 - x \) and \( y = -x^2 + 2x + 4 \)?

- [1] \((1,3)\)
- [2] \((4,1)\)
- [3] \((-1,-3)\)
- [4] \((3,1)\)

20. Which equation represents a circle with center \((1,-3)\) and radius 4?

- [1] \((x - 1)^2 + (y + 3)^2 = 16\)
- [2] \((x - 1)^2 + (y + 3)^2 = 4\)
- [3] \((x + 1)^2 + (y - 3)^2 = 16\)
- [4] \((x + 1)^2 + (y - 3)^2 = 4\)

21. The diagram at the right shows the construction of a perpendicular, \( PX \), to a line \( l \) from point \( P \). The arc drawn from point \( P \) intersects line \( l \) at \( A \) and \( B \), and the arcs drawn from points \( A \) and \( B \) intersect \( PX \) at \( C \). Which of the statements is not always true about this construction?

- [1] \( PA = PB \)
- [2] \( AX = BX \)
- [3] \( PX = CX \)
- [4] \( AC = BC \)

![Construction Diagram]

22. The distance between points \((4a,3b)\) and \((3a,2b)\) is

- [1] \( a^2 + b^2 \)
- [2] \( \sqrt{a^2 + b^2} \)
- [3] \( a + b \)
- [4] \( \sqrt{a + b} \)

23. A translation maps \((x, y)\) to \((x - 5, y + 3)\). In which quadrant does the point \((-3,-2)\) lie under the same translation?

- [1] I
- [2] II
- [3] III
- [4] IV
1) [2pts]

What is the length of the line segment that joins the points whose coordinates are (4,7) and (-3,5)?

1) \( \sqrt{5} \)
2) \( \sqrt{53} \)
3) \( \sqrt{193} \)
4) \( 3\sqrt{6} \)

3) [2pts]

Find the midpoint of (3, -2) and (-11, 12).

[A] (-8, 10)  [B] (7, 7)
[C] (-4, 5)  [D] (14, 14)

5) [1pt]

What is the y-intercept of the graph of the line whose equation is \( y = -\frac{2}{5}x + 4 \)?

[A] 0  [B] \(-\frac{2}{5}\)  [C] 4  [D] \(-\frac{5}{2}\)

2) [2pts]

Which expression describes the slope of a line that is parallel to the x-axis?

[A] \( \frac{8-3}{2-1} \)  [B] \( \frac{-2-(-2)}{1-(-2)} \)
[C] \( \frac{1-(-2)}{0-2} \)  [D] \( \frac{4-2}{3-5} \)

4) [2pts]

Find the slope of the line passing through the points (8, -5) and (4,-2)

6) [1pt]

An angle that is 50 degrees is classified as

[A] obtuse    [B] acute
[C] Straight    [D] right
Name:___________________________________________________ Quiz 2 (10pts)

1) [2pts]

What is the slope of a line that is perpendicular to the line whose equation is $3x + 4y = 12$?

1) $\frac{3}{4}$

2) $\frac{3}{4}$

3) $\frac{4}{5}$

4) $\frac{4}{3}$

2) [2pts]

Which equation represents a line parallel to the line whose equation is $2y - 5x = 10$?

1) $5y - 2x = 25$

2) $5y + 2x = 10$

3) $4y - 10x = 12$

4) $2y + 10x = 8$

3) [2pts]

The coordinates of point $R$ are $(-3,2)$ and the coordinates of point $T$ are $(4,1)$. What is the length of $\overline{RT}$?

[A] $4\sqrt{3}$  [B] $\sqrt{10}$  
[C] $2\sqrt{2}$  [D] $5\sqrt{2}$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4) [1pt]

Given: \( y = \frac{1}{4} x - 3 \)
\( y = x^2 + 8x + 12 \)

In which quadrant will the graphs of the given equations intersect?
1) I
2) II
3) III
4) IV

5) [1pt]

Which construction is represented by the diagram below?

[A] copying \( AB \)  [B] bisecting \( AB \)  [C] rotating \( AB \)  [D] drawing a line parallel to \( AB \)
6) [2pt]

Which graph could be used to find the solution to the following system of equations?

\[ y = -x + 2 \]

\[ y = x^2 \]
1) [2pts]

In the diagram of \( \triangle ABC \) and \( \triangle DEF \) below, 
\( AB \cong DE, \angle A \cong \angle D, \) and \( \angle B \cong \angle E. \)

\[\begin{align*}
\text{A} & \text{F} \\
\text{C} & \text{D} \\
\text{B} & \text{E}
\end{align*}\]

Which method can be used to prove 
\( \triangle ABC \cong \triangle DEF? \)
1) SSS
2) SAS
3) ASA
4) HL

2) [2pts]

Which condition does not prove that two triangles
are congruent?
1) SSS \( \cong \) SSS
2) SSA \( \cong \) SSA
3) SAS \( \cong \) SAS
4) ASA \( \cong \) ASA

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
3) [2pts]

In three-dimensional space, two planes are parallel and a third plane intersects both of the parallel planes. The intersection of the planes is a

[A] plane  [B] point
[C] pair of parallel lines
[D] pair of intersecting lines

4) [2pts]

Determine if the two lines $3x + 2y = 6$ and $y = -\frac{3}{2}x - 2$ are parallel, perpendicular, or neither.

5) [2pts] Graph each equation. State the point(s) of intersection. Round all answers to the nearest tenths place if necessary.

$Y = x^2 - 4x - 5$ and $y = x + 3$
Name: ____________________________  Quiz 4 (10pts)

1) [2pts]

A transversal intersects two lines. Which condition would always make the two lines parallel?
1) Vertical angles are congruent.
2) Alternate interior angles are congruent.
3) Corresponding angles are supplementary.
4) Same-side interior angles are complementary.

2) [2pts]

What is the length, to the nearest tenth, of the line segment joining the points \((-4, 2)\) and \((146, 52)\)?
1) 141.4
2) 150.5
3) 151.9
4) 158.1

3) [2pts]

In the diagram of \(\triangle ABC\) and \(\triangle DEF\) below, \(\overline{AB} \cong \overline{DE}, \angle A \cong \angle D,\) and \(\angle B \cong \angle E.\)

Which method can be used to prove \(\triangle ABC \cong \triangle DEF\)?
1) SSS
2) SAS
3) ASA
4) HL
4) [2pts]

A regular pyramid with a square base is shown in the diagram below.

A side, $s$, of the base of the pyramid is 12 meters, and the height, $h$, is 42 meters. What is the volume of the pyramid in cubic meters?

5) [2pts] Graph each equation. State the point(s) of intersection. Round all answers to the nearest tenths place if necessary.

\[ Y = x^2 - 6x + 10 \quad \text{and} \quad y = x + 4 \]
Quiz 5 [10pts]

1) [2pts]

*Find the volume of the cone that has a diameter of 8 feet and a height of 15 feet. (Use 3.14 for π.)*

[A] 251.2 ft³  
[B] 753.6 ft³  
[C] 1004.8 ft³  
[D] 376.8 ft³

2) [2pts]

*In the diagram below, line k is perpendicular to plane P at point T.*

![Diagram showing line k perpendicular to plane P at point T](image)

Which statement is true?
1) Any point in plane P also will be on line k.  
2) Only one line in plane P will intersect line k.  
3) All planes that intersect plane P will pass through T.  
4) Any plane containing line k is perpendicular to plane P.

3) [2pts]

Line k is drawn so that it is perpendicular to two distinct planes, P and R. What must be true about planes P and R?
1) Planes P and R are skew.  
2) Planes P and R are parallel.  
3) Planes P and R are perpendicular.  
4) Plane P intersects plane R but is not perpendicular to plane R.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4) Calculate the surface area of a bowling ball if the diameter is 13 inches.
Include a sketch and your work (round to the nearest tenths place).

5) A can has a radius of 5 inches and a lateral surface area of 115.6 square inches. Sketch, show work, and calculate the height of the can (Round to the nearest tenths place)
1) [2pts] In the accompanying diagram, $HK$ bisects $IL$ and $\angle H = \angle K$.

What is the most direct method of proof that could be used to prove $\triangle HIJ \cong \triangle KLP$?
1) $HI \cong HL$.
2) $SAS \cong SAS$
3) $AAS \cong AAS$
4) $ASA \cong ASA$

2) [2pts]

Point $P$ is on line $m$. What is the total number of planes that are perpendicular to line $m$ and pass through point $P$?
1) 1
2) 2
3) 0
4) infinite

3) [2pts]

A right circular cylinder has an altitude of 11 feet and a radius of 5 feet. What is the lateral area, in square feet, of the cylinder, to the nearest tenth?
1) 172.7
2) 172.8
3) 345.4
4) 345.6

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4) Construct the perpendicular bisector of the following line segment. Leave all construction marks:

5) [2pts] Draw the graph of the relation represented by the equation \((x - 2)^2 + (y - 4)^2 = 16\)
1) [2pts] What is the slope of a line perpendicular to the line whose equation is $5x + 3y = 8$?
   
   1) $\frac{5}{3}$
   
   2) $\frac{3}{5}$
   
   3) $\frac{-3}{5}$
   
   4) $\frac{-5}{3}$

2) [2pts]

A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box?

1) 21.3 in
2) 16 in
3) 8 in
4) 4 in

3) [2pts] The lateral area of a cone is $20\pi$ in.$^2$. If the radius is 10 in., find the slant height.

[A] 0.5 in.  
[B] 2.0$\pi$ in.  
[C] 2.0 in.  
[D] 0.5$\pi$ in.
4) [2 pts] \[ \overline{AC} \cong \overline{DC} \text{ and } \overline{BC} \cong \overline{CE}. \text{ Prove } \triangle ABC \cong \triangle DEC. \]

5) [2pts] Construct a segment perpendicular to \( \overline{XY} \) through point \( Z \).
Answer all 15 questions in this part. Each correct answer will receive 2 credits. Partial credit may be given if work is shown.

A right circular cylinder has an altitude of 11 feet and a radius of 5 feet. What is the lateral area, in square feet, of the cylinder, to the nearest tenth?

(1) 172.7, (2) 172.8, (3) 345.4, (4) 345.6

A transversal intersects two lines. Which condition would always make the two lines parallel?

(1) Vertical angles are congruent.
(2) Alternate interior angles are congruent.
(3) Corresponding angles are supplementary.
(4) Same-side interior angles are complementary.

What is the converse of the statement “If Bob does his homework, then George gets candy”?

(1) If George gets candy, then Bob does his homework.
(2) Bob does his homework if and only if George gets candy.
(3) If George does not get candy, then Bob does not do his homework.
(4) If Bob does not do his homework, then George does not get candy.

Given:

\[ y = \frac{1}{4}x - 3 \]

\[ y = x^2 + 8x + 12 \]

In which quadrant will the graphs of the given equations intersect?

(1) I, (2) II, (3) III, (4) IV
[5] Which diagram shows the construction of an equilateral triangle?

(1)  

(3)  

[6] What is an equation for the circle shown in the graph below?

[7] In plane $P$, lines $m$ and $n$ intersect at point $A$. If line $k$ is perpendicular to line $m$ and line $n$ at point $A$, then line $k$ is

(1) contained in plane $P$  
(2) parallel to plane $P$  
(3) perpendicular to plane $P$  
(4) skew to plane $P$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
What is the length, to the nearest tenth, of the line segment joining the points \((-4,2)\) and \((146,52)\)?

(1) 141.4  
(2) 150.5  
(3) 151.9  
(4) 158.1

What is the slope of a line perpendicular to the line whose equation is \(y = 3x + 4\)?

(1) \(\frac{1}{3}\)  
(2) \(-\frac{1}{3}\)  
(3) 3  
(4) -3

Two lines are represented by the equations \(-\frac{1}{2}y = 6x + 10\) and \(y = mx\). For which value of \(m\) will the lines be parallel?

(1) -12  
(2) -3  
(3) 3  
(4) 12

Based on the construction below, which statement must be true?

![Diagram](image)

Which equation represents the circle whose center is \((-2,3)\) and whose radius is 5?

(1) \((x - 2)^2 + (y + 3)^2 = 5\)  
(2) \((x + 2)^2 + (y - 3)^2 = 5\)  
(3) \((x + 2)^2 + (y - 3)^2 = 25\)  
(4) \((x - 2)^2 + (y + 3)^2 = 25\)

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
[13] Lines $j$ and $k$ intersect at point $P$. Line $m$ is drawn so that it is perpendicular to lines $j$ and $k$ at point $P$. Which statement is correct?

1. Lines $j$ and $k$ are in perpendicular planes.
2. Line $m$ is in the same plane as lines $j$ and $k$.
3. Line $m$ is parallel to the plane containing lines $j$ and $k$.
4. Line $m$ is perpendicular to the plane containing lines $j$ and $k$.

[14] What is the distance between the points $(-3,2)$ and $(1,0)$?

1. $2\sqrt{2}$
2. $2\sqrt{3}$
3. $5\sqrt{2}$
4. $2\sqrt{5}$

[15] Which expression represents the volume, in cubic centimeters, of the cylinder represented in the diagram below?

![Diagram of a cylinder with a radius of 12 cm and a height of 27 cm]

1. $162\pi$
2. $324\pi$
3. $972\pi$
4. $3,888\pi$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Part II

Answer all questions in this part. Each correct response with appropriate work will receive 2 credits. A correct solution with no work will receive a maximum of 1 credit.

16) Graph the relation represented by the equation $(x - 2)^2 + (y + 4)^2 = 25$

17) Construct the angle bisector of the given angle. Leave all construction marks:
18) Find the volume of the triangular prism.

19) Calculate the lateral area of the following figure. Round your answer to the nearest tenth.
Part III (4 points each)

20) Given the conditional statement, “If I wear sandals, then it isn’t snowing outside,” please write the following statements:

Converse:

Inverse:

Contrapositive:

Which of the three new statements is logically equivalent to the original statement?

21) Solve the following system of equations. Round each value to the nearest tenth:

\[
\begin{align*}
y &= x^2 + x - 1 \\
y &= -2x + 1
\end{align*}
\]

22) Given: \(X\) is the midpoint of \(\overline{AD}\).

\(X\) is the midpoint of \(\overline{BC}\).

Prove: \(\overline{BD} \cong \overline{AC}\)

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
1) [1 pt] In the accompanying diagram, $\triangle ABC$ is similar to
but not congruent to $\triangle A'B'C'$.

Which transformation is represented by $\triangle A'B'C'$
1) rotation
2) translation
3) reflection
4) dilation

2) [2 pts] What are the coordinates of $M'$, the image of
$M(2,4)$, after a counterclockwise rotation of 90°
about the origin?
1) \((-2,4)\)
2) \((-2,-4)\)
3) \((-4,2)\)
4) \((-4,-2)\)

3) [1 pt] Which statement is logically equivalent to “If I eat,
then I live”?
1) If I live, then I eat.
2) If I eat, then I do not live.
3) I live if and only if I eat.
4) If I do not live, then I do not eat.
4) [2 pts] What is an equation of the line that passes through the point \((-2, 5)\) and is perpendicular to the line whose equation is \(y = \frac{1}{2}x + 5\)?

1) \(y = 2x + 1\)
2) \(y = -2x + 1\)
3) \(y = 2x + 9\)
4) \(y = -2x - 9\)

5) [2 pts] The inside of an ice cream cone has radius 5 cm and height 6 cm. Assume that you receive exactly a half scoop of ice cream in the shape of a hemisphere, and that the ice cream shop is cheap and leaves the inside of the cone empty. Solve the total volume of ice cream. Round your answer to the nearest tenth.

6) [2 pts] Using a compass and straightedge, and \(AB\) below, construct an equilateral triangle with all sides congruent to \(AB\). [Leave all construction marks.]

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
1) [2 pt] If the letter P is rotated 180 degrees, which is the resulting figure?
   1) p
   2) d
   3) a
   4) b

2) [2 pts] What is the negation of the statement “The Sun is shining”?
   1) It is cloudy.
   2) It is daytime.
   3) It is not raining.
   4) The Sun is not shining.

3) [2 pt] The center of a circular sunflower with a diameter of 4 centimeters is (−2, 1). Which equation represents the sunflower?
   1) \((x - 2)^2 + (y + 1)^2 = 2\)
   2) \((x + 2)^2 + (y - 1)^2 = 4\)
   3) \((x - 2)^2 + (y - 1)^2 = 4\)
   4) \((x + 2)^2 + (y - 1)^2 = 2\)
4) [2 pts]

What is the locus of all points in the plane 4 cm from a given line?

[A] a line perpendicular to the first  [B] two parallel lines 4 cm from from either side of the first
[C] a parallel line 4 cm from the first  [D] a circle whose center is on the line

5) [2 pts]

Given point \(A(-2, 3)\). State the coordinates of the image of \(A\) under the composition \(T_{(-3, -4)} \circ R_{x = \pm 3}\).

[The use of the accompanying grid is optional.]
1) [2 pt] What is the equation of a line that passes through the point \((-3, -11)\) and is parallel to the line whose equation is \(2x - y = 4\)?
   1) \(y = 2x + 5\)
   2) \(y = 2x - 5\)
   3) \(y = \frac{1}{2}x + \frac{25}{2}\)
   4) \(y = -\frac{1}{2}x - \frac{25}{2}\)

2) [2 pts] In the diagram below of circle \(O\), secant \(AB\) intersects circle \(O\) at \(D\), secant \(AOC\) intersects circle \(O\) at \(E\). \(AE = 4\), \(AB = 12\), and \(DB = 6\).

   (Not drawn to scale)

What is the length of \(OC\)?
   1) 4.5
   2) 7
   3) 9
   4) 14
3) [2 pt] In the diagram below of \( \triangle PRT \), \( Q \) is a point on \( PR \), \( S \) is a point on \( TR \), \( QS \) is drawn, and \( \angle RPT \equiv \angle RSQ \).

Which reason justifies the conclusion that \( \triangle PRT \sim \triangle SRQ \)?
1) AA  
2) ASA  
3) SAS  
4) SSS

4) [2 pts] Which statements could be used to prove that \( \triangle ABC \) and \( \triangle A'B'C' \) are congruent?
1) \( AB \equiv A'B', BC \equiv B'C', \text{ and } \angle A \equiv \angle A' \)  
2) \( AB \equiv A'B', \angle A \equiv \angle A', \text{ and } \angle C \equiv \angle C' \)  
3) \( \angle A \equiv \angle A', \angle B \equiv \angle B', \text{ and } \angle C \equiv \angle C' \)  
4) \( \angle A \equiv \angle A', AC \equiv A'C', \text{ and } BC \equiv B'C' \)
1) [2 pts] In the diagram of circle $O$ below, chord $CD$ is parallel to diameter $AOB$ and $m\angle AC = 30$.

![Diagram of a circle with chords and angles labeled]

What is $m\angle CD$?
1) 150
2) 120
3) 100
4) 60

2) [2 pts] Which transformation produces a figure that is always the mirror image of the original figure?
1) line reflection
2) dilation
3) translation
4) rotation

3) [2 pts] In the accompanying diagram, chord $CD$ is parallel to diameters $AO$. If $m\angle ACO = 25$, what is $m\angle COE$?

![Diagram with angles labeled]

1) 25
2) 65
3) 130
4) 155

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4) [4 pts]
Complete the partial proof below for the accompanying diagram by providing reasons for steps 3, 6, 8, and 9.

Given: $AFCD$, $AB \perp BC$, $DE \perp EF$, $BC \parallel FE$,
$AB \cong DE$
Prove: $AC \cong FD$

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
<tr>
<td>1 $AFCD$</td>
<td>1 Given</td>
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<tr>
<td>2 $AB \perp BC$, $DE \perp EF$</td>
<td>2 Given</td>
</tr>
<tr>
<td>3 $\angle B$ and $\angle E$ are right angles</td>
<td>3</td>
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<tr>
<td>4 $\angle B \cong \angle E$</td>
<td>4 All right angles are congruent.</td>
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<tr>
<td>5 $BC \parallel FE$</td>
<td>5 Given</td>
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<tr>
<td>6 $\angle BCA \cong \angle EFD$</td>
<td>6</td>
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<td>7 $AB \cong DE$</td>
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<td>8 $\triangle ABC \cong \triangle DEF$</td>
<td>8</td>
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<tr>
<td>9 $AC \cong FD$</td>
<td>9</td>
</tr>
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Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
1. [1pt] The image of function $f(x)$ is found by mapping each point on the function $(x,y)$ to the point $(y,x)$. This image is a reflection of $f(x)$ in
   (1) the x-axis
   (2) the y-axis
   (3) the line whose equation is $y = x$
   (4) the line whose equation is $y = -x$

2. [1pt] In the accompanying diagram of triangles $BAT$ and $FLU$, $\angle B \cong \angle F$ and $\overline{BA} \cong \overline{FL}$.

Which statement is needed to prove $\triangle BAT \cong \triangle FLU$?
   (1) $\angle A \cong \angle L$
   (2) $\overline{AT} \cong \overline{LU}$
   (3) $\angle A \cong \angle U$
   (4) $\overline{BA} \parallel \overline{FL}$

3. [1pt] Which type of transformation is $(x,y) \rightarrow (x + 2, y - 2)$?
   (1) dilation
   (2) reflection
   (3) rotation
   (4) translation

4. [1pt] What is the length of the altitude of an equilateral triangle whose side has a length of 8?
   (1) $32$
   (2) $4\sqrt{2}$
   (3) $4\sqrt{3}$
   (4) $4$
In a coordinate plane, how many points are both 5 units from the origin and 2 units from the x-axis?

5. [1pt]
(1) 1  (3) 3
(2) 2  (4) 4

6. [1pt] What is the contrapositive of the statement, “If I am tall, then I will bump my head”?
(1) If I bump my head, then I am tall.
(2) If I do not bump my head, then I am tall.
(3) If I am tall, then I will not bump my head.
(4) If I do not bump my head, then I am not tall.

7. [2pt] show all work
In the diagram below, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle $O$ from an external point $A$, $AB = 8$, and $AC = 4$.

![Diagram]

What is the length of $\overline{CD}$?

8. [2pt] show all work
On a scale drawing of a new school playground, a triangular area has sides with lengths of 8 centimeters, 15 centimeters, and 17 centimeters. If the triangular area located on the playground has a perimeter of 120 meters, what is the length of its longest side?
1. [1pt] Point $P'$ is the image of point $P(-3,4)$ after a translation defined by $T_{(7,-1)}$. Which other transformation on $P$ would also produce $P'$?

(1) $r_{y-x}$  
(2) $r_{y-axis}$  
(3) $R_{90^\circ}$  
(4) $R_{90^\circ}$

2. [1pt] Which equation, or set of equations, represent(s) the locus of points equidistant from the two lines $x = 6$ and $x = -2$?

[1] $x = 2$  
[2] $x = 4$  
[3] $y = 4$  
[4] $x = 10$ and $x = -6$

3. [1pt] A circle has the equation $(x - 2)^2 + (y + 4)^2 = 16$. Under a translation $T_{(3,5)}$ the center of the circle will be located at which point?

[1] $(5,1)$  
[2] $(1,9)$  
[3] $(-6,20)$  
[4] $(7,-1)$

4. [1pt] In circle $O$, the secant from $S$ to $D$ intersects the circle at $C$ and the secant from $S$ to $B$ intersects the circle at $A$.

If $SC = 6$, $CD = 8$ and $SA = 4$, find $AB$.

[1] $16/3$  
[2] $17$  
[3] $21$  
[4] $28/3$

5. [1pt] Quadrilateral $PQRS$ is plotted in the coordinate plane as shown. Find the length of diagonal $PR$.

[1] $13$  
[2] $6\sqrt{2}$  
[3] $\sqrt{73}$  
[4] $\sqrt{122}$

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
6. show all work. [2pts]

Given circle $O$ with tangent $\overline{CB}$. Find the value of $x$.

7. [3pts] show all work and sketch a picture

At a certain time of the day, the shadow of a boy 5 feet tall is 8 foot long. The shadow of a tree at this same time is 28 feet long. How tall is the tree to the nearest foot?
1. [1 pt] In the diagram of \( \triangle ABC \) and \( \triangle DEF \) below, \( AB \equiv DE \), \( \angle A \equiv \angle D \), and \( \angle B \equiv \angle E \).

Which method can be used to prove \( \triangle ABC \cong \triangle DEF \)?

(1) SSS \hspace{1cm} (3) ASA
(2) SAS \hspace{1cm} (4) HL

2. [1 pt] Point \( A \) is located at \((4, -7)\). The point is reflected in the x-axis. Its image is located at

(1) \((-4, 7)\) \hspace{1cm} (3) \((4, 7)\)
(2) \((-4, -7)\) \hspace{1cm} (4) \((7, -4)\)

3. [1 pt] In the diagram of circle \( O \) below, chords \( AB \) and \( CD \) are parallel, and \( BD \) is a diameter of the circle.

If \( mAD = 60 \), what is \( m\angle CDB \)?

(1) 20 \hspace{1cm} (3) 60
(2) 30 \hspace{1cm} (4) 120

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4. [1pt] In the diagram below, the length of the legs $AC$ and $BC$ of right triangle $ABC$ are 6 cm and 8 cm, respectively. Altitude $CD$ is drawn to the hypotenuse of $\triangle ABC$.

![Diagram of right triangle with altitude](image)

What is the length of $AD$ to the nearest tenth of a centimeter?

(1) 3.6  (3) 6.4
(2) 6.0  (4) 4.0

5. [2pts] Show all work
In the diagram of $\triangle ABC$ below, $AB = 10$, $BC = 14$, and $AC = 16$. Find the perimeter of the triangle formed by connecting the midpoints of the sides of $\triangle ABC$.

![Diagram of triangle with midpoints](image)

6. [4pts]
The coordinates of the vertices of parallelogram $ABCD$ are $A(-2, 2)$, $B(3, 5)$, $C(4, 2)$, and $D(-1, -1)$. State the coordinates of the vertices of parallelogram $A'B'C'D'$ that result from the transformation $r_{y-axis} \circ T_{x=-y}$ [The use of the set of axes below is optional.]

![Grid and points](image)
1. What is an equation of the line that passes through the point \((-2, 5)\) and is perpendicular to the line whose equation is \(y = \frac{1}{2}x + 5\)?
   1) \(y = 2x + 1\)
   2) \(y = -2x + 1\)
   3) \(y = 2x + 9\)
   4) \(y = -2x - 9\)

2. In the diagram below of circle \(O\), secant \(AB\) intersects circle \(O\) at \(D\), secant \(AOC\) intersects circle \(O\) at \(E\). \(AE = 4\), \(AB = 12\), and \(DB = 6\).

   ![Diagram of a circle with secants and points labeled A, B, D, E, and O.]

   What is the length of \(OC\)?
   1) 4.5
   2) 7
   3) 9
   4) 14
3. Solve for $x$.
1) $40^\circ$
2) $80^\circ$
3) $100^\circ$
4) $120^\circ$

4. The accompanying diagram shows two similar triangles.

Which proportion could be used to solve for $x$?
1) $\frac{x}{24} = \frac{9}{15}$
2) $\frac{24}{9} = \frac{15}{x}$
3) $\frac{32}{x} = \frac{12}{15}$
4) $\frac{32}{12} = \frac{15}{x}$

5. A circle has the equation $(x - 2)^2 + (y + 4)^2 = 16$. Under a translation $T_{(3,5)}$ the center of the circle will be located at which point?

6. In the diagram below of right triangle $ACB$,
   altitude $CD$ is drawn to hypotenuse $AB$.

If $AB = 36$ and $AC = 12$, what is the length of $AD$?

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
7. In the accompanying diagram of right triangle $ABC$, altitude $AD$ divides hypotenuse $BC$ into segments with lengths of 4 and 5. Find the length of leg $AB$.

8. Which point satisfies the system of equations, $y = 2x + 2$ and $y = -x^2 + 2$?

   [1] (0,2)  [2] (-1,0)  [3] (1,2)  [4] (2,-4)

9. Which condition does not prove that two triangles are congruent?
   1) $SSS \cong SSS$
   2) $SSA \cong SSA$
   3) $SAS \cong SAS$
   4) $ASA \cong ASA$

10. Find the midpoint of (3,-2) and (-11, 12).

   1) (-8,10)
   2) (-4,5)
   3) (7,7)
   4) (14,14)
11. What is the length of the line segment that joins the points whose coordinates are (4, 7) and (−3, 5)?
   1) \( \sqrt{5} \)
   2) \( \sqrt{53} \)
   3) \( \sqrt{193} \)
   4) \( 3\sqrt{6} \)

12. In which quadrant would the image of point (5, −3) fall after a dilation using a factor of −3?
   1) I
   2) II
   3) III
   4) IV

13. In the accompanying diagram chord \( CD = 21 \), \( DX = 4 \).

   ![Diagram](image)

   1) 5
   2) 10
   3) 15
   4) 20

14. A triangle has vertices \( A(3, 2), B(4, 1) \) and \( C(4, 3) \). What are the coordinates of point \( B \) under the glide reflection \( T_{(0, 1)} \circ R_{x=0} \).
   [1] (2, −4)
   [2] (4, 0)
   [3] (4, 2)
   [4] (−4, 2)

15. In which triangle do the three altitudes intersect outside the triangle?
   1) a right triangle
   2) an acute triangle
   3) an obtuse triangle
   4) an equilateral triangle

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Part II
Answer all questions in this part. Each correct response with appropriate work will receive 2 credits. A correct solution with no work will receive a maximum of 1 credit.

16) Find an equation of the line passing through the point (6, 5) and perpendicular to the line whose equation is $2y + 3x = 6$.

17) Using a compass and straightedge and construct an isosceles triangle with two of the sides measuring 5 cm, and the third side measuring 7 cm. [Leave all construction marks.]

18) The accompanying diagram shows two lengths of wire attached to a wheel, so that $AB$ and $AC$ are tangent to the wheel. If the major arc $BC$ has a measure of $220^\circ$, find the number of degrees in $\angle BAC$.

19) Find the value of $x$.

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
Part III (4 points each)

20) A city is planning to build a new park. The park must be 4 units away from Happy Road at y=3. The park also must be exactly 5 miles from the center of town, which is located at the origin on the coordinate graph. On the set of axes below, sketch the compound loci and label with an X all possible locations for the new park.

21) Farmington, New York, has plans for a new triangular park. If plotted on a coordinate grid, the vertices would be A(3, 3), B(5, -2), and C(-3, -1). However, a tract of land has become available that would enable the planners to increase the size of the park, which is based on the following transformation of the original triangular park, \( R_{270} \circ D_2 \). On the grid below, graph and label both the original park \( \Delta ABC \) and its image, the new park \( \Delta ABC \) following the transformation.
22) Given: $\triangle ABC$ and $\triangle EDC$, $C$ is the midpoint of $BD$ and $AE$
Prove: $AB \parallel DE$

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<th>Statement</th>
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Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
1. [1 pt]
In three-dimensional space, two planes are parallel and a third plane intersects both of the parallel planes. The intersection of the planes is a
1) plane  
2) point  
3) pair of parallel lines  
4) pair of intersecting lines

2. [1 pt] Which piece of paper can be folded into a pyramid?

3. [2 pts] Construct a perpendicular from $\overline{AB}$ that passes through $P$. 

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Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4. [2 pts] In the diagram below, town C lies on straight road $p$. Sketch the points that are 6 miles from town C. Then sketch the points that are 3 miles from road $p$. How many points satisfy both conditions?

5. [2 pts] The degree measures of the angles of $\triangle ABC$ are represented by $x$, $3x$, and $5x - 54$. Find the value of $x$.

6. [2 pts] In the diagram below of $\triangle ACT$, $D$ is the midpoint of $AC$, $O$ is the midpoint of $AT$, and $G$ is the midpoint of $CT$.

If $AC = 10$, $AT = 18$, and $CT = 22$, what is the perimeter of parallelogram $CDOG$?
1. [1pt] In the accompanying diagram of $\triangle ABC$, $\overline{AB}$ is extended to $D$, exterior angle $CBD$ measures $145^\circ$, and $m\angle C = 75$.

What is $m\angle CAB$?
1) 35
2) 70
3) 110
4) 220

2. [1pt] Find the measure of one of the interior angles of a regular polygon with twelve sides.

[A] $150^\circ$  [B] $30^\circ$  [C] $165^\circ$  [D] $15^\circ$

3. [1pt] The number of points equidistant from two parallel lines and also equidistant from two points on one of the given lines is exactly:


4. [1pt] What is an equation of a circle with its center at $(-3, 5)$ and a radius of 4?
1) $(x - 3)^2 + (y + 5)^2 = 16$
2) $(x + 3)^2 + (y - 5)^2 = 16$
3) $(x - 3)^2 + (y + 5)^2 = 4$
4) $(x + 3)^2 + (y - 5)^2 = 4$

5. [1pt] A polygon is transformed according to the rule: $(x, y) \rightarrow (x + 2, y)$. Every point of the polygon moves two units in which direction?
1) up
2) down
3) left
4) right

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
6. [1pt] Given the system of equations:
\[ y = x^2 - 4x \]
\[ x = 4 \]
The number of points of intersection is
1) 1
2) 2
3) 3
4) 0

7. [1pt] Which transformation produces a figure similar but not congruent to the original figure?
1) \( T_{1,3} \)
2) \( D_{\frac{1}{2}} \)
3) \( R_{90^\circ} \)
4) \( r_{y=-x} \)

8. [1pt] If the endpoints of \( \overline{AB} \) are \( A(-4,5) \) and \( B(2,-5) \), what is the length of \( \overline{AB} \)?
1) \( 2\sqrt{34} \)
2) \( 2 \)
3) \( \sqrt{61} \)
4) \( 8 \)

9. [1pt] What is the image of point \( A(4,2) \) after the composition of transformations defined by \( R_{90^\circ} \circ r_{y=-x} \)?
1) \( (-4,2) \)
2) \( (4,-2) \)
3) \( (-4,-2) \)
4) \( (2,-4) \)

10. [1pt] In \( \triangle ABC \), \( m\angle A = x \), \( m\angle B = 2x + 2 \), and \( m\angle C = 3x + 4 \). What is the value of \( x \)?
1) 29
2) 31
3) 59
4) 61
1. [1pt] Which set can \textit{not} represent the lengths of the sides of a triangle?
   1) \{4, 5, 6\}
   2) \{5, 5, 11\}
   3) \{7, 7, 12\}
   4) \{8, 8, 8\}

2. [1pt] In \(\triangle PQR\), \(PQ = 8\), \(QR = 12\), and \(RP = 13\). Which statement about the angles of \(\triangle PQR\) must be true?
   1) \(m\angle Q > m\angle P > m\angle R\)
   2) \(m\angle Q > m\angle R > m\angle P\)
   3) \(m\angle R > m\angle P > m\angle Q\)
   4) \(m\angle P > m\angle R > m\angle Q\)

3. [1pt] What is the sum, in degrees, of the measures of the interior angles of a pentagon?
   1) 180
   2) 360
   3) 540
   4) 900

4. [1pt] In the accompanying diagram of \(\triangle ABC\), \(\overline{AB}\) is extended to \(D\), exterior angle \(CBD\) measures 145°, and \(m\angle C = 75\).

What is \(m\angle CAB\)?
   1) 35
   2) 70
   3) 110
   4) 220
5. [1pt] A circle has the equation \((x - 2)^2 + (y + 4)^2 = 16\). Under a translation \(T_{(3,5)}\), the center of the circle will be located at which point? 


6. [1pt] In the diagram below of right triangle \(ACB\), altitude \(CD\) is drawn to hypotenuse \(AB\).

![Diagram of right triangle ACB with altitude CD drawn to hypotenuse AB]

If \(AB = 36\) and \(AC = 12\), what is the length of \(AD\)?


7. [2pts] Show all work.

The endpoints of \(PQ\) are \(P(-3,1)\) and \(Q(4,25)\). Find the length of \(PQ\).

8. [4pts] \(V = bh\) where \(b\) is the area of the base. Show all work.

The volume of a cylinder is 12,566.4 cm\(^3\). The height of the cylinder is 8 cm. Find the radius of the cylinder to the nearest tenth of a centimeter.
1. [1 pt] Triangle $ABC$ has vertices $A(1,3)$, $B(0,1)$, and $C(4,0)$. Under a translation, $A'$, the image point of $A$, is located at (4,4). Under this same translation, point $C'$ is located at
  1) (7,1)
  2) (5,3)
  3) (3,2)
  4) (1, -1)

2. [1 pt] What is the slope of a line perpendicular to the line whose equation is $y = -\frac{2}{3}x - 5$?
  1) $-\frac{3}{2}$
  2) $\frac{3}{2}$
  3) $\frac{2}{3}$
  4) $\frac{1}{2}$

3. [1 pt] In the diagram below, circle $O$ has a radius of 5, and $CE = 2$. Diameter $AC$ is perpendicular to chord $BD$ at $E$.

   ![Diagram](https://example.com/diagram.png)

What is the length of $BD$?
  1) 12
  2) 10
  3) 8
  4) 4

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4. [1 pt] In \( \triangle ABC \), point \( D \) is on \( AB \), and point \( E \) is on \( BC \) such that \( DE \parallel AC \). If \( DB = 2 \), \( DA = 7 \), and \( DE = 3 \), what is the length of \( AC \)?

1) 8
2) 9
3) 10.5
4) 13.5

5. [1 pt] In the diagram of \( \triangle ABC \) below, \( AB \cong AC \). The measure of \( \angle B \) is 40°.

What is the measure of \( \angle A \)?

1) 40°
2) 50°
3) 70°
4) 100°
6. [2 pts] Write an equation of the line that passes through the point \((6, -5)\) and is parallel to the line whose equation is \(2x - 3y = 11\).

7. [3 pts] Using a compass and straightedge, and \(\overline{AB}\) below, construct an equilateral triangle with all sides congruent to \(AB\). [Leave all construction marks.]
1. [1pt] The lateral faces of a regular pyramid are composed of

(1) squares  (3) congruent right triangles
(2) rectangles  (4) congruent isosceles triangles

2. [1pt] After a composition of transformations, the coordinates $A(4,2)$, $B(4,6)$, and $C(2,6)$ become $A'(-2,-1)$, $B'(-2,-3)$, and $C'(-1,-3)$, as shown on the set of axes below.

Which composition of transformations was used?
(1) $R_{180} \circ D_2$  (3) $D_\frac{1}{2} \circ R_{180}$
(2) $R_{90} \circ D_2$  (4) $D_1 \circ R_{90}$

3. [1pt] In $\triangle ABC$, $m\angle A = 95$, $m\angle B = 50$, and $m\angle C = 35$. Which expression correctly relates the lengths of the sides of this triangle?
(1) $AB < BC < CA$  (3) $AC < BC < AB$
(2) $AB < AC < BC$  (4) $BC < AC < AB$

4. [1pt] In a coordinate plane, how many points are both 5 units from the origin and 2 units from the x-axis?
(1) 1  (3) 3
(2) 2  (4) 4

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
5. [1pt] Square LMNO is shown in the diagram below.

What are the coordinates of the midpoint of diagonal LN?

(1) \( \left( \frac{4}{2}, -\frac{1}{2} \right) \) 
(2) \( \left( -\frac{3}{2}, \frac{1}{2} \right) \)
(3) \( \left( -\frac{1}{2}, \frac{3}{2} \right) \) 
(4) \( \left( \frac{1}{2}, 4\frac{1}{2} \right) \)

6. [1pt] In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.

What is the volume of the cone to the nearest cubic inch?

(1) 201 
(2) 481 
(3) 603 
(4) 804

7. [1pt] A circle is represented by the equation \( x^2 + (y + 3)^2 = 13 \). What are the coordinates of the center of the circle and the length of the radius?

(1) (0,3) and 13 
(2) (0,3) and \( \sqrt{13} \) 
(3) (0, -3) and 13 
(4) (0, -3) and \( \sqrt{13} \)

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
8. [1pt] Which graph represents a circle with the equation 

\[(x - 5)^2 + (y + 1)^2 = 9?\]

![Graphs of circles](image)

(1) ![Graph](image)

(2) ![Graph](image)

(3) ![Graph](image)

(4) ![Graph](image)


In the diagram below, \(\triangle ABC \sim \triangle EFG\), \(m\angle C = 4x + 30\), and \(m\angle G = 5x + 10\). Determine the value of \(x\).
Name:                    Quiz 20

1. [1 pt]  What is the inverse of the statement “If two triangles are not similar, their corresponding angles are not congruent”?  
            1) If two triangles are similar, their corresponding angles are not congruent.  
            2) If corresponding angles of two triangles are not congruent, the triangles are not similar.  
            3) If two triangles are similar, their corresponding angles are congruent.  
            4) If corresponding angles of two triangles are congruent, the triangles are similar.

2. [1 pt]  Which transformation is not always an isometry?  
            1) rotation  
            2) dilation  
            3) reflection  
            4) translation

3. [1 pt]  In the diagram below, circle A and circle B are shown.

[Diagram of circles A and B]

What is the total number of lines of tangency that are common to circle A and circle B?  
            1) 1  
            2) 2  
            3) 3  
            4) 4

Questions and images obtained from http://jmap.org/jmap_resources_by_topic.htm#geo
4. [1 pt] What is the negation of the statement “Squares are parallelograms”?
   1) Parallelograms are squares.
   2) Parallelograms are not squares.
   3) It is not the case that squares are parallelograms.
   4) It is not the case that parallelograms are squares.

5. [1 pt] Based on the diagram below, which statement is true?

        a
      /   
   110° /     
      /       
    /         
   120° /           
         /             
       /               
     /                 
   60° /                   
      /                       
    /                           
   115° /                                 
         /                                    
      /                                      
    /                                        
  b

1) \( a \parallel b \)
2) \( a \parallel c \)
3) \( b \parallel c \)
4) \( d \parallel e \)
6. [1pt] In the diagram below of circle $O$, chords $\overline{AD}$ and $\overline{BC}$ intersect at $E$, $m\overarc{AC} = 87$, and $m\overarc{BD} = 35$.

What is the degree measure of $\angle CE A$?
1) 87
2) 61
3) 43.5
4) 26

7. [1pt] Which expression represents the volume, in cubic centimeters, of the cylinder represented in the diagram below?

1) $162\pi$
2) $324\pi$
3) $972\pi$
4) $3,888\pi$
8) [1 pt] If the endpoints of $\overline{AB}$ are $A(4, 5)$ and $B(2, 5)$, what is the length of $\overline{AB}$?

1) $2\sqrt{34}$
2) $2$
3) $\sqrt{61}$
4) $8$

9) [2 pts] List the five ways to prove two triangles congruent:

1. ________________________________
2. ________________________________
3. ________________________________
4. ________________________________
5. ________________________________
Supporting Data

Figure 3.2 displays the average scores of the students who used this curriculum in separate summer school programs within three years. Grades were calculated for the first three weeks (Session A) and the last three weeks (Session B) of each summer school session. The scores on the Geometry Regents Exams are also indicated in the graph.

Summer 2010 had 36 upstate New York students use this geometry curriculum.

Summer 2011 had 37 upstate New York students use this geometry curriculum.

Summer 2012 had 39 upstate New York students use this geometry curriculum.

Figure 3.2  Data collected from the author’s grade books for the 2010, 2011, and 2012 summer school sessions.
Chapter Four: Conclusion and Recommendations

The high school geometry curriculum proposed may provide students with the skills to build self-efficacy and confidence in mathematics through the grade tracking of open note quizzes. General education and inclusive education students can benefit from the cyclical nature of the content which can improve their overall retention skills. The usage of key concepts, vocabulary, necessary steps, and testing strategies support students as they retake the state exam. Dividing the material into sections creates obtainable goals during each class time. Distributing a bound packet to each student reduces the time needed to pass out individual pages and more time can be used for building student knowledge. Allowing students to keep the packet in the classroom reduces the possibility of lost materials. Students transition between warm up, to construction, to proof, to vocabulary, to the topic of the day, and then to the quiz frequently which can make the two hour class period feel a little shorter for the teacher and students. Specific geometric visual aids and computer software could be utilized to support the learning styles of many students. Allowing students to work on material during class instead of assigning homework increases productivity and allows the students to have a summer life without the pressure of completing items for school. The data displayed demonstrates an overall success of the students that have used the curriculum during three previous summer school programs.
Summer school programs exist because adequate funding and sufficient enrollment. The prospect of eliminating traditional summer school programs increases the need for credit recovery initiatives. Assuming schools have proper funding, reforms to summer school programs could be a part of future education. Programs designed through continued enrichment and skill building may replace the traditional high school credit recovery format.

Future research may be made regarding the new standards and the impact of teacher practices and student learning. New teaching practice could be tracked to determine the effectiveness of overall achievement beginning in early grade levels and ending at high school.
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Vita

Kyle Kucsmas teaches mathematics during the typical school year for a high school in a suburb of Rochester, New York. He is certified in both general and special education mathematics for grades 7-12. Kyle has taught Geometry, Integrated Algebra, Algebra 2-Trigonometry, Intermediate Algebra, Financial Math Applications, and Algebra-Geometry Connections. He has taught mathematics for summer school programs since 2007. Geometry has been a summer school concentration for him since 2009. Kyle has obtained undergraduate degrees Liberal Arts: Math and Science, and Adolescent Math Education for grades 7-12. He has earned a graduate degree in Education and Human Development and is currently pursuing an additional graduate degree in Educational Administration.