Abstract

The numerical range of a matrix, also known as its field of values, is a subset of the complex plane, loosely defined as the set of outputs of a function that acts on the n-dimensional complex unit sphere. Numerical ranges are defined algebraically, but have rich geometric properties that have been the subject of intense mathematical research in the last fifteen years, although the origins of the field can be traced back to work done in the 1950s. It is well known that the numerical range of a matrix is a compact and convex region. Despite this generic description, depending on the matrix, the convex boundary of the numerical range can take on a variety of shapes, such as an ellipse, a polygon, or even the union of flat and curved portions, and the richness of these possibilities increases as the dimension of the matrix increases. These sets can be segregated into several classes according to properties of the singular points of certain algebraic curves associated with numerical range boundaries. The speaker will present a number of his new results on the stability of each class of numerical range under perturbations of the matrix, particularly in the case where the dimension of the matrix is four. The types of numerical ranges of matrices with algebraic number entries will also be addressed from the point of view of matrix perturbations. Finally, the presenter will share some new observations on the density distribution of output points near the numerical range boundary.