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The Effect of Essential Guiding Questions on Adolescent Learning

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The Effect of Essential/Guiding Questions on Adolescents' Learning

By

Zachary William Abbe

A thesis submitted to the Department of Education and Human Development of the State University of New York College at Brockport in partial fulfillment of the requirements for the degree of

Master’s of Science in Education

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The Effect of Essential/Guiding Questions on Adolescents' Learning

By

Zachary William Abbe

Approved by

[Signature]
Advisor

8/2/07
Date

[Signature]
Chair, Graduate Committee

8/31/07
Date
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Selection and Statement of the topic

The topic that I have chosen for research is the effect of essential/guiding questions on adolescents’ learning.

Discussion of Significance of the topic:

One of the first steps in creating a lesson plan is to come up with essential questions or topics that the teacher wishes for the students to learn or be able to answer. But, how often do teachers come right out and say to their students, these are the essential questions for the lesson/unit. I want you to be able to both answer and discuss these at the conclusion of the lesson/unit. It is these essential questions/topics that students need to understand to be able to complete any sort of problem or example that we put in front of them. There is no reason for education to be a game of chance, students picking topics or problems to study and hoping that they will help them prepare for the test. I am going to examine how student learning changes if students are given the essential questions or topics before the lesson begins. I want to see if that can help them pinpoint the most valuable information in a lesson and put it to good use. I often notice students latching on to information that is not essential, or even trying to learn how to duplicate an example instead of learning the concept involved. Asking the questions in advance of the lesson may change the way students focus their attention. Would you start driving before you knew where you were going? So then why should students enter a lesson without knowing what they should get out of it?

In order to assess whether or not the essential questions had a positive effect on the students I plan on both comparing test averages and taking a survey. To start off I
will compare test averages, prior to the implementation of essential question, of the two classes using a t test to determine if there was a difference between the two to begin with. Then I will compare the study groups test scores before the implementation of essential questions versus the test scores after and check to see if there was a significant improvement in scores or not. Finally I will compare the essential test scores of the study group to the other classes to see if there was a difference, post implementation, between the two or not. The survey I have constructed is an 8 question survey in which students rank their opinion of essential questions based on a strongly disagree, disagree, neutral, agree, or strongly disagree scale. A score of 8-19 will be categorized as a negative opinion, a score of 20-28 will be categorized as neutral, and a score of 29-40 will be categorized as a positive opinion. (See appendix pg 20 for blank survey: Scoring guide pg 21)

Literature Review:

The first research article pertaining to this topic is titled “The effects of curriculum maps and guiding questions on the test performance of adolescents with learning disabilities” by Keith Lenz, Gary Adams, Janis A. Bulgren, Norman Pouliot, and Michelle Laraux. This study evaluated the effects of two different types of explicit instruction, curriculum maps and guiding questions. These two types were compared to the use of simple reviews of repeated information: Thirty high school students with learning disabilities were involved in this study. They were randomly assigned to six groups, making five students in each group. The students participated in all three of the interventions across three lessons. A 45-item test, including information from all three
lessons, was given as both a pre and posttest. Results from the tests showed that the curriculum map helped the most, but more important to my research the guiding questions were significantly higher than review of repeated information. Lenz (2002) concludes that based on the results of the study teachers can make their instruction more explicit and powerful by incorporating simple routines including the use of curriculum maps to depict the importance and structure of the content and then using these maps to lead and review learning through guided and interactive questioning. The guiding question intervention in this study that was shown to be helpful included three phases. First, a guiding question was posed to the students and written down. Students were then asked to list what must be known to answer the question, and what supporting questions must be answered before the guiding question could be answered. Second, three times during the lesson students were asked if the must be known information has been established yet and whether students knew answers to the other questions. Third, at the end of the lesson an answer to the guiding question was constructed with the students and the instructor questioned students about the “must be known” information and the answers to the other questions. (Lenz, 2002, para. 6)

Similar results were found in a research article titled “Guiding Questions Enhance Student Learning From Educational Video” by Timothy J. Lawson, James H. Bodle, Melissa A. Houlette, and Richard R. Haubner. A group of 127 psychology students watched a video about social psychology. Students in some sections of the course watched the video with no special instructions, while others were given 8 guiding questions in writing while watching the movie. After the movie students were given a test on information both from their text and the video. Those with the guiding questions
scored significantly higher on the video-related questions. There was no significant difference between scores on the textbook related questions, showing that the ability levels of the students were similar. (Lawson, 2006, para. 31)

Some schools have even gone as far as to commit themselves as Essential Schools. The goal of these schools is to “redesign the entire curriculum around thorough coverage of fewer areas, rather than offering an array of courses aimed to attract students for whom vastly different expectations are held.” (Cushman, 1989, p.1) In this type of school, teachers act more as coaches in order to provoke students to learn how to learn. This way student work is both active and collaborative. The work challenges students, helps them make connections, and leads to more creative thinking. If students learn how to think they can gain a broad understanding instead of narrow or rote expertise. The starting point to creating this type of atmosphere is to design curriculum around questions and problems instead of answers. These “essential questions, should shape the way students learn to think critically for themselves.” (Cushman, 1989, p. 2) These questions are typically higher-level questions that can only be answered through information they find in books and information they acquire through their own experimentation. It is through the students work that they become experts on one aspect of a problem, and learn to collaborate with other students.

A case study called “Using Questioning to Guide Student Thinking,” written by Emily Van Zee analyzed the way an experienced, award winning, physics teacher uses questioning to guide student thinking during a particular discussion. The approach to teaching by this teacher, Jim Minstrell, seems to be very similar to the Essential Schools. The study focuses in on a particular type of question, known as a reflective toss. This is
how the teacher tries to give students responsibility for thinking about a particular topic. The sequence of a reflective toss involves a student statement, followed by a teacher question, and hopefully completed with additional student statements. Van Zee (1997) proposes that the use of this type of questioning could help teachers shift to a more reflective discourse. This can be done by asking questions that help students make their meanings clear, analyze the discussion in their own thinking, and consider different points of view in an impartial setting.

In a section of her book, The Schooling Practices That Matter Most, Kathleen Cotton discusses the topic of classroom questioning. Cotton (2000) discusses the purposes of teachers' classroom questions. These include: to develop interest and motivate students to become actively involved in lessons, to evaluate students' preparation and check on homework or seatwork completion, to develop critical thinking skills and inquiring attitudes, to review and summarize previous lessons, to nurture insights by exposing new relationships, to assess achievement of instructional goals and objectives, and to stimulate students to pursue knowledge on their own. Cotton also includes general finding of researchers based on questioning. Conclusions include: instruction that includes posing questions during lessons is more effective in producing achievement gains, students perform better on test items previously asked, and oral questions posed during class are more effective in fostering learning than written questions. Lastly Cotton discusses that research has shown (it was not mentioned which one of her sources this information was from) with older students increasing the use of higher cognitive questions are positively related with: on task behavior, length of student responses, the number of relevant contributions by students, the number of student to
student interactions, student use of complete sentences, speculative thinking on the part of students, and relevant questions posed by students. Also with older students questions should be asked both before and after material is read or studied.

David Jakes shares similar opinions in his article titled Basing Learning Experiences in Essential Questions. He defines essential questions as a question which requires the student to develop a plan or course of actions, or one which requires the student to make a decision. Jakes points out that the answer to the essential question will require that students craft a response that involves knowledge construction. In other words answers to essential questions are a direct measure of student understanding.

Conclusions of Literature Review

After looking at multiple sources it appears that essential questions are a very important part of both the planning and implementation of classroom instruction. There are many definitions for essential questions, but they all imply that they must question only the most vital topics. They must also be based on concepts instead of examples and students must use critical thought to answer them. These questions are most useful when they are higher level questions. Higher level questions are useful in helping students to think more deeply and critically, in problem solving, promoting discussions, and encouraging students to seek information on their own. These questions should be asked before and after the lesson so that students are constantly searching for answers. Most importantly essential questions put the responsibility on the students. When it is a student’s responsibility to learn the answer to a topic they become more invested and they gain a better understanding of the topic. This goes right along with the concept of
integrated mathematics, the style that I teach, and that is one of the reasons that I want to conduct research on the topic of essential questions. I have found a lot of information on the benefits of essential questions and how they can help classroom learning but more research is needed that shows an increase in knowledge or test scores. Like it or not our education standards are currently based on test scores so it would be beneficial for the use of essential questions to see data that they can help increase scores.

Development of Hypothesis and Outcome Measures

It is my hypothesis that when essential questions are implemented into the classroom both student performance and understanding will increase. I work in a school that has a student centered investigation style math curriculum. Throughout the year I was noticing that students sometimes struggled to see where we were going with certain topics and investigations. It wasn’t until the investigations/lessons were over that the students understood what they were meant to be learning, so unfortunately by this time they may have missed some key information along the way. I began trying to come up with an idea of how I could help the students understand what information they need to know or learn by the end of a lesson/investigation before it is over. It was at this time that I began to look at the upcoming exponential unit in my Algebra class. This unit is out of the updated version of Core-Plus Mathematics: Contemporary Mathematics in Context. (A copy of two investigations completed during study are in the appendix pgs 25-35) This was our first use of the second edition, and at the beginning of every investigation there is one or more focus question(s) for the investigation. It was then that I realized these questions are just what I needed to help guide student focus and hopefully
increase their understanding and performance. The questions are upper level, concept based, and aligned perfectly with the investigations.

I took all of the focus questions from the book in the units we would complete, and typed them out on sheets of paper and stapled them together so each student could have their own “essential question packet.” (Essential question packet examples appendix pgs 22-23) At the beginning of each investigation I would post the essential question(s) on poster board in the room. The class would then discuss what we already knew and what we would need to find out about the essential question(s). The discussion at the beginning of the lesson was usually very informal and notes were not typically taken. Periodically during the investigation we would stop and go back to the essential question(s) to make sure everyone was still focused on the right topic. At the conclusion of each investigation we would hold a class discussion on the essential question(s) and the students would come up with an agreed upon solution to the question(s). (See appendix for student solution pg 24) Everyone would then write the solution in their essential question packet and use it to help complete journals, homework, or study for any upcoming tests/quizzes.

To measure the usefulness of the essential questions I surveyed the students to obtain their opinions of the usefulness of the essential questions. I also compared test scores before versus after the implementation of the essential questions and between both my study group and my control class to see if the implementation of the essential questions could improve test scores. (Sample exams taken during study in appendix pgs 37-42) I also plan on sharing observations that I have made during the use of essential questions in my classroom
Methods and Procedures

I have decided to use both quantitative and qualitative measures to help measure the effectiveness of the essential questions. I chose quantitative so that I could have numerical data to back up any results that come out of the study. My quantitative measures include my survey and the comparison of test scores before and after implementation of the essential questions. I also decided to have my own personal observations as a qualitative measure because there are times when test scores don’t necessarily tell all. There are a lot of other things that happen in a classroom that let you know what a student has learned. Often these things are hard to measure and can only be observed.

Analysis of Data and Interpretation of Results

The implementation of the essential questions began with the beginning of the fourth marking period in the school year. I have broken down the student average test scores first off by my control group versus my study group and then further broken down by before essential question implementation and post implementation. (Table of values appendix pg 25) Once the average test scores were calculated I used them to calculate t scores and test for significant differences between the data. My first test was between my control group and my case study before implementation to see if there was a difference in the performance level of the two classes to begin with. The calculated t score was .189 which is less than the critical value at the .05 level (2.048) so this concludes that there is no significant difference between the two classes before implementation. My next test was between the control group and my case study post implementation to see if my
hypothesis was correct and that the case study would perform at a higher level after the implementation. The calculated t score was .519 which is less than the critical value at the .05 level (1.701) so this concludes that there still was no significant difference in the performance of the two classes post implementation. From there I wanted to see if there would be a difference between each class pre and post implementation. First my control group had a decline in average scores but their calculated t score was 1.75 which was less than the critical value at the .05 level (2.120) so their decline was not seen as a significant difference. My case study group also had a decline in test averages with a calculated t score of 3.24 which was more than the critical value at the .05 level (2.179) so their decline was enough to be classified as a significant difference. So based on t scores of test averages there was not a significant difference between classes before or after the implementation but there was a difference between the case study before and after the implementation of the essential questions. It however was the opposite of my hypothesis that the essential questions could help the performance of students. Based on this information alone it would appear that the essential questions hurt the student’s performance instead of helping.

The student surveys however did show that the student’s held a positive opinion of the essential questions. This in turn shows that the students felt that the implementation of essential questions was helpful in their learning of mathematics.
This table shows the breakdown of the surveyed students. (See appendix for survey questions pg 20) The survey was out of 40 points and a score of 8-19 is a poor opinion of the essential questions, a score of 20-28 is a neutral opinion, and a score of 29-40 is a positive opinion. There were 8 students with a positive opinion, 5 with a neutral opinion, and 0 with a negative opinion. The average score of the survey is a 29.07 so it is not overwhelmingly positive but it is in the range none the less.

<table>
<thead>
<tr>
<th>Question</th>
<th>Avg Score</th>
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<tbody>
<tr>
<td>1</td>
<td>3.23</td>
</tr>
<tr>
<td>2</td>
<td>3.62</td>
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<tr>
<td>3</td>
<td>3.69</td>
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<td>3.62</td>
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<tr>
<td>8</td>
<td>3.92</td>
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</tbody>
</table>
opinion of the student. The scale was between 1 and 5. (The scale can be seen in the appendix pg 21) A score of 1 or 2 would be classified as a negative feeling toward the essential question characteristic. A score of 3 would be classified as a neutral feeling toward the essential question characteristic. A score of 4 or 5 would be classified as a positive feeling toward the essential question characteristic. So by looking at all of the questions separately we can see that the students were between a neutral and positive opinion about all aspects of the essential questions. This falls right in line with the students overall opinion from the first table.

Their lowest opinions by characteristics were in questions 1 and 6. Question 1 dealt with the fact that the essential questions were presented to the students before the start of the investigation, prior to any work on the topic. Overall the students did not have a negative opinion of receiving the questions in advance but some may not see any sort of benefit to this strategy. Question 6 dealt with the idea that instead of me helping students come up with an adequate solution to the essential questions, they were required to come up with the solutions as a class. The survey question asked if answering the questions through discussion made students take ownership over the material instead of being spoon fed by the teacher. I am not surprised that this is one of the lower opinions because it has been my experience that students are very uncomfortable when teachers refuse to act as a crutch for them, especially given that the students involved were freshman.

The characteristics with the highest opinions were from questions 5 and 8. Question 5 asked whether the essential questions helped students recognize what the key features of a lesson were. Given that this is one of the reasons I chose to implement the
essential questions, I am glad to see that the students were more able to recognize the key features of the investigations. Question 8 asked the students if they think the essential questions should be used in the future. Even though the students may have been pulled out of their comfort zone at times by the use of the essential questions they backed up their overall positive opinion of the essential questions by recommending that they be used again in my future classes.

My observations of the implementation are for the most part in favor of the essential questions. During the use of the essential questions I was most excited to see an improvement in student’s abilities to vocalize mathematical concepts. So if for nothing else this was one area where there was significant improvement. I believe the reason behind this is that I refused to given students the answers to any of the essential questions and I would not let them move on until everyone, including myself, thought that they as a class had come up with an adequate answer. This put the pressure on the students to voice their opinions so that the class could come up with the best answer possible. Cushman (1989) says that “essential questions should shape the way students learn to think critically for themselves.” The essential questions “can only be answered through information they find in books and information they acquire through their own experimentation.” I also noticed students using their essential question packets to help them though homework assignments, journal assignments, and to study for exams/quizzes. So it gave students another tool to refer to when needed.

One consistent theme with the essential question implementation is that both the students and I held a positive opinion of their usefulness in learning mathematics. The test scores on the other hand disagreed with our opinions. There are some things in my
research that may help to explain this. First, is that I had originally hoped for 42 students to take part in my research but I only received 30 parental consent letters so 12 of my students were not used in my data. This may possibly have skewed both my test scores and my survey results. In addition to this the most difficult topics we did all year took place in the fourth quarter. The fourth quarter was the time when I was conducting my research, so this may also have influenced my students lower test score averages post implementation. Also there were fewer test scores affecting the averages post implementation as opposed to before implementation. This left the post implementation scores more susceptible to outliers than the scores from before implementation. These of course could be completely irrelevant and the test scores may be accurate, and the essential questions may hurt students more than they helped. More research would be needed to be sure.

Discussion, Summary, and Reflection

During this study I learned that just because "I" think a strategy is helping the students, it does not necessarily translate directly into improved test scores. I do not however think that the essential questions are a dead issue, and test scores are not always the best judge of how useful a strategy is especially with a small sample size like I had. I plan on using essential questions this year in my Algebra classroom because I really enjoyed the way it forced the students to discuss concepts instead of completing memorized exercises. Not to mention I did receive positive feedback from students both on their survey's and in casual conversation in and out of class. In fact, question 8 in my survey had the highest score of any, and that was the question that asked the students if
the essential questions should be used in the future or not. I am interested to see if working with essential questions for a longer period of time will increase their effectiveness. The students this year may not have had enough time to adjust to the different style and a whole year implementation may yield much different results.

I do plan on making a few changes to my implementation of the essential questions. First, I will make the discussion of the essential questions before the lesson begins much more formal. This year the discussions were informal and no notes were taken. In the future I will set up the essential question packets in the form of a KWL. This way, students will be able to see the learning that takes place during the lesson and hopefully they will better understand how the lesson is centered around the essential question. This strategy is similar to that of the first phase of the guiding questions in the research article "The effects of curriculum maps and guiding questions on the test performance of adolescents with learning disabilities" by Keith Lenz, Gary Adams, Janis A. Bulgren, Norman Pouliot, and Michelle Laraux. In this first phase students were asked to list what must be known to answer the question, and what supporting questions must be answered before the guiding question could be answered. The results of this study showed that the use of these guiding questions held a significantly higher test average than just basic review so the process must be useful. I would also like to conduct some interviews with students at different points in the year to get student opinions on effectiveness and on any modifications that should be made. The anonymous surveys were useful but I feel that I could get some more meaningful information from an open conversation with a student. Instead of them being locked into the questions on the survey they may come up with something I had not thought of.
Over the next year I plan on collecting data for my own personal use and hopefully I will be able make a definite decision on whether I believe the essential questions are a useful part of my classroom or not.
References


Jakes, D. Basing Learning Experiences in Essential Questions. DC: Author


Wang, C. (2003). Questioning Techniques for Active Learning. Center for Development of Teaching and Learning, 1

Appendix
Essential Questions Survey
Please be honest and remember NO NAMES
SD = strongly disagree  D = disagree  N = neutral  A = agree  SA = strongly agree

1. Receiving the essential questions in advance helped me focus my learning.  SD  D  N  A  SA
2. The essential questions were a waste of time.  SD  D  N  A  SA
3. Looking back at my essential question sheets helped me prepare for my tests.  SD  D  N  A  SA
4. The essential questions just felt like extra work.  SD  D  N  A  SA
5. The essential questions helped me recognize what the key features of a lesson were.  SD  D  N  A  SA
6. Answering the questions through discussions, instead of Mr. Abbe giving us the answers, made me take ownership of the concepts.  SD  D  N  A  SA
7. The essential questions helped me summarize my learning in each investigation.  SD  D  N  A  SA
8. I think the essential questions should be used in future classrooms.  SD  D  N  A  SA
Essential Questions Survey
Please be honest and remember NO NAMES
SD= strongly disagree  D= disagree  N= neutral  A=agree  SA= strongly agree

1. Receiving the essential questions in advance helped me focus my learning.

2. The essential questions were a waste of time.

3. Looking back at my essential question sheets helped me prepare for my tests.

4. The essential questions just felt like extra work.

5. The essential questions helped me recognize what the key features of a lesson were.

6. Answering the questions through discussions, instead of Mr. Abbe giving us the answers, made me take ownership of the concepts.

7. The essential questions helped me summarize my learning in each investigation.

8. I think the essential questions should be used in future classrooms.

Box opinion

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<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>SD</td>
<td>D</td>
<td>N</td>
<td>A</td>
<td>SA</td>
</tr>
</tbody>
</table>

8 - 19  20 - 28  29 - 40

Positive  Neutral  Poor opinion
Algebra 1 Period 1
Essential Questions
Unit: Exponential Functions
Lesson 1 Exponential Growth

**Invest 1**

What are the basic patterns of exponential growth in variations of the Pay-It-Forward process?

How can these patterns be expressed with symbolic rules?
Invest 1

What patterns of change appear in tables and graphs of (time, height) values for flying pumpkins and other projectiles?

What functions model those patterns of change?
Invest 1

What mathematical patterns in tables, graphs, and symbolic rules are typical of exponential decay relations?

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td>a \cdot b</td>
</tr>
<tr>
<td>2</td>
<td>a \cdot b \cdot b</td>
</tr>
</tbody>
</table>

\( b < 1 \) so this represents decreasing at a decreasing rate.

Graph:

Also dec at a dec rate

Rule: \( y = a(b^x) \)

\( b < 1 \) so dec at dec rate
<table>
<thead>
<tr>
<th>B4 Implementation Case Study</th>
<th>After Implementation Case Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>67.8</td>
<td>72</td>
</tr>
<tr>
<td>80.67</td>
<td>47</td>
</tr>
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<td>91.2</td>
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</tr>
<tr>
<td>9.539465355</td>
<td>15.81169206</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>
Think About This Situation

Continue Trevor's kind of Pay It Forward thinking.

a) How many people would receive a Pay It Forward good deed at each of the next several stages of the process?

b) What is your best guess about the number of people who would receive Pay It Forward good deeds at the tenth stage of the process?

c) Which of the graphs above do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increases as the process continues over time?

In this lesson, you will discover answers to questions like these and find strategies for analyzing patterns of change called exponential growth. You will also discover some basic properties of exponents that allow you to write exponential expressions in useful equivalent forms.

Investigation 1 Counting in Tree Graphs

The number of good deeds in the Pay It Forward pattern can be represented by a tree graph that starts like this:

```
  *---*---*---*
  /   /   /   /
 /   /   /   /   /   /   /   /
```

The vertices represent the people who receive and do good deeds. Each edge represents a good deed done by one person for another. As you work on the problems of this investigation, look for answers to these questions:

- What are the basic patterns of exponential growth in variations of the Pay It Forward process?
- How can these patterns be expressed with symbolic rules?
At the start of the Pay It Forward process, only one person does good deeds—for three new people. In the next stage, the three new people each do good things for three more new people. In the next stage, nine people each do good things for three more new people, and so on, with no person receiving more than one good deed.

a. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the (stage, number of good deeds) data.

<table>
<thead>
<tr>
<th>Stage of Process</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Good Deeds</td>
<td>9</td>
<td>9</td>
<td>27</td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

b. How does the number of good deeds at each stage grow as the tree progresses? How is that pattern of change shown in the plot of the data?

c. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done?

Consider now how the number of good deeds would grow if each person touched by the Pay It Forward process were to do good deeds for only two other new people, instead of three.

a. Make a tree graph for several stages of this Pay It Forward process.

b. Make a table showing the number of good deeds done at each of the first 10 stages of the process and plot those sample (stage, number of good deeds) values.

c. How does the number of good deeds increase as the Pay It Forward process progresses in stages? How is that pattern of change shown in the plot of the data?

d. How many stages of this process will be needed before a total of 25,000 good deeds will have been done?

In the two versions of Pay It Forward that you have studied, you can use the number of good deeds at one stage to calculate the number at the next stage.

a. Use the words NOW and NEXT to write rules that express the two patterns.

b. How do the numbers and calculations indicated in the rules express the patterns of change in tables of (stage, number of good deeds) data?

c. Write a rule relating NOW and NEXT that could be used to model a Pay It Forward process in which each person does good deeds for four other new people. What pattern of change would you expect to see in a table of (stage, number of good deeds) data for this Pay It Forward process?

What are the main steps (not keystrokes) required to use a calculator to produce tables of values like those you made in Problems 1 and 2?
It is also convenient to have rules that will give the number of
good deeds $N$ at any stage $x$ of the Pay It Forward process,
without finding all the numbers along the way to stage $x$. When
students in one class were given the task of finding such a rule
for the process in which each person does three good deeds for
others, they came up with four different ideas:

\[ N = 3x \]
\[ N = x + 3 \]
\[ N = 3^x \]
\[ N = 3x + 1 \]

a. Are any of these rules for predicting the number of good deeds $N$
correct? How do you know?
b. How can you be sure that the numbers and calculations expressed
in the correct “$N = ...$” rule will produce the same results as the
NOW-NEXT rule you developed in Problem 2?
c. Write an “$N = ...$” rule that would show the number of good deeds
at stage number $x$ if each person in the process does good deeds
for two others.
d. Write an “$N = ...$” rule that gives the number of good deeds at stage
$x$ if each person in the process does good deeds for four others.

### Summarize the Mathematics

Look back at the patterns of change in the number of good deeds in the different
Pay It Forward schemes—three per person and two per person.

a. Compare the processes by noting similarities and differences in:
   i. Patterns of change in the tables of (stage, number of good deeds) data;
   ii. Patterns in the graphs of (stage, number of good deeds) data;
   iii. The rules relating $N$ and $\text{NEXT}$ numbers of good deeds; and
   iv. The rules expressing number of good deeds $N$ as a function of stage number $x$.

b. Compare patterns of change in numbers of good deeds at each stage of the Pay It
   Forward process to those of linear functions that you have studied in earlier work.
   i. How are the $\text{NOW-NEXT}$ rules similar, and how are they different?
   ii. How are the “$y = ...$” rules similar, and how are they different?
   iii. How are the patterns of change in tables and graphs of linear functions similar to
      those of the Pay It Forward examples, and how are they different?

Be prepared to share your ideas with the rest of the class.
**Check Your Understanding**

The patterns in spread of good deeds by the Pay It Forward process can be observed in other quite different situations. For example, when bacteria infect some part of your body, they often grow and split into pairs of genetically equivalent cells over and over again.

a. Suppose a single bacterium lands in a cut on your hand. It begins spreading an infection by growing and splitting into two bacteria every 20 minutes.

i. Complete a table showing the number of bacteria after each 20-minute period in the first three hours. (Assume none of the bacteria are killed by white blood cells.)

<table>
<thead>
<tr>
<th>Number of 20-Min Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria Count</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

ii. Plot the (number of time periods, bacteria count) values.

iii. Describe the pattern of growth of bacteria causing the infection.

b. Use NOW and NEXT to write a rule relating the number of bacteria at one time to the number 20 minutes later. Then use the rule to find the number of bacteria after fifteen 20-minute periods.

c. Write a rule showing how the number of bacteria \( N \) can be calculated from the number of stages \( x \) in the growth and division process.

d. How are the table, graph, and symbolic rules describing bacteria growth similar to and different from the Pay It Forward examples? How are they similar to, and different from, typical patterns of linear functions?
The current distance record for Punkin' Chunkin' is over 4,000 feet. Such a flight would take the pumpkin very high in the air, as well.

4 Which of these graphs is most likely to fit the pattern relating pumpkin height to time in flight? Explain your choice.

5 What pattern would you expect to find in data tables relating pumpkin height to elapsed time?

In work on investigations of this lesson, you will explore several strategies for recognizing, modeling, and analyzing patterns like those involved in the motion of a flying pumpkin.
Investigation 12  Pumpkins in Flight

It turns out that the height of a flying pumpkin can be modeled well by a quadratic function of elapsed time. You can develop rules for such functions by reasoning from basic principles of science. Then you can use a variety of strategies to answer questions about the relationships. As you work on the problems of this investigation, look for answers to these questions:

- What patterns of change appear in tables and graphs of (time, height) values for flying pumpkins and other projectiles?
- What functions model these patterns of change?

Punkin' Droppin' At Old Dominion University in Norfolk, Virginia, physics students have their own flying pumpkin contest. Each year they see who can drop pumpkins on a target from 10 stories up in a tall building, while listening to music by the group Smashing Pumpkins.

![Image of a pumpkin being dropped from a height]

By timing the flight of the falling pumpkins, the students can test scientific discoveries made by Galileo Galilei, nearly 400 years ago. Galileo used clever experiments to discover that gravity exerts force on any free-falling object so that if the distance fallen, will be related to time squared by the function

$$d = 16t^2$$

(time in seconds and distance in feet).

For example, suppose that the students dropped a pumpkin from a point that is 100 feet above the ground. At a time 0.7 seconds after being dropped, the pumpkin will have fallen $100 \times 0.7^2 = 49$ feet, leaving it 100 - 49 = 51 feet above the ground.

This model ignores the resistive effects of the air as the pumpkin falls. But, for fairly compact and heavy objects, the function $d = 16t^2$ describes motion of falling bodies quite well.

Create a table like the one below to show estimates for the pumpkin's distance fallen and height above ground in feet at various times between 0 and 3 seconds.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance Fallen (ft)</th>
<th>Height Above Ground (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>100 - 4 = 96</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>22.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td></td>
</tr>
</tbody>
</table>

31
Use data relating height and time to answer the following questions about flight of a pumpkin dropped from a position 100 feet above the ground.

a. What function rule shows how the pumpkin’s height \( h \) is related to time \( t \)?
b. What equation can be solved to find the time when the pumpkin is 10 feet from the ground? What is your best estimate for the solution of that equation?
c. What equation can be solved to find the time when the pumpkin hits the ground? What is your best estimate for the solution of that equation?
d. How would your answers to parts a, b, and c change if the pumpkin were to be dropped from a spot 75 feet above the ground?

High Punkin’ Chunkin’ Compressed-air cannons, medieval catapults, and whirling slings are used for the punkin’ chunkin’ competitions.

Imagine pointing a punkin’ chunkin’ cannon straight upward. The pumpkin height at any time \( t \) will depend on its speed and height when it leaves the cannon.

Suppose a pumpkin is fired straight upward from the barrel of a compressed air cannon at a point 20 feet above the ground, at a speed of 90 feet per second (about 60 miles per hour).

a. If there were no gravitational force pulling the pumpkin back toward the ground, how would the pumpkin’s height above the ground change as time passes?
b. What function rule would relate height above the ground \( h \) in feet to time in the air \( t \) in seconds?
c. How would you change the function rule in Part b if the pumpkin chunklet used a stronger cannon that fired the pumpkin straight up into the air with a velocity of 120 feet per second?

d. How would you change the function rule in Part b if the end of the cannon barrel was only 15 feet above the ground, instead of 20 feet?

4 Now think about how the flight of a launched pumpkin results from the combination of three factors:
- initial height of the pumpkin's release,
- initial upward velocity produced by the pumpkin-launching device, and
- gravity pulling the pumpkin down toward the ground.

a. Suppose a compressed air cannon fires a pumpkin straight up into the air from a height of 20 feet and provides an initial upward velocity of 90 feet per second. What function rule would combine these conditions and the effect of gravity to give a relation between the pumpkin's height $h$ in feet and its flight time $t$ in seconds?

b. How would you change your function rule in Part a if the pumpkin is launched at a height of 15 feet with an initial upward velocity of 120 feet per second?

5 By now you may have recognized that the height of a pumpkin shot straight up into the air at any time in its flight will be given by a function that can be expressed with rule in the general form

$$h = h_0 + vt - 16t^2.$$  

In those functions, $h$ is measured in feet and $t$ in seconds.

a. What does the value of $h_0$ represent? What units are used to measure $h_0$?

b. What does the value of $v$ represent? What units are used to measure $v$?

When a pumpkin is not launched straight up into the air, we can break its velocity into a vertical component and a horizontal component. The vertical component, the upward velocity, can be used to find a function that predicts change over time in the pumpkin’s height. The horizontal component can be used to find a function that predicts change over time in the horizontal distance traveled.
The pumpkin's height in feet $t$ seconds after it is launched will still be given by $h = -16t^2 + v_0t + h_0$. It is fairly easy to measure the initial height ($h_0$) from which the pumpkin is launched, but it is not so easy to measure the initial upward velocity ($v_0$).

a. Suppose that a pumpkin leaves a cannon at a point 24 feet above the ground when $t = 0$. What does that fact tell about the rule giving height $h$ as a function of time in flight $t$?

b. Suppose you were able to use a stop watch to discover that the pumpkin shot described in Part a returned to the ground after 6 seconds. Use that information to find the value of $v_0$.

7) Suppose that you were able to use a ranging tool that records the height of a flying pumpkin every half second from the time it left a cannon. A sample of the data for one pumpkin launch appears in the following table.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in feet)</td>
<td>18</td>
<td>40</td>
<td>80</td>
<td>70</td>
<td>70</td>
<td>60</td>
<td>80</td>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>

a. Plot the data on a graph and experiment with several values of $v_0$ and $h_0$ in search of a function that models the data pattern well.

b. Use a calculator or computer tool that offers quadratic curve-fitting to find a quadratic model for the sample data pattern. Compare that automatic curve fit to what you found with your own experimentation.

c. Use the rule that you found in Part b to write and solve equations and inequalities matching these questions about the pumpkin shot.

i. When was the pumpkin 60 feet above the ground?

ii. For which time(s) was the pumpkin at least 60 feet above the ground?

d. Use the rule you found in Part b to answer the following questions.

i. What is your best estimate for the maximum height of the pumpkin?

ii. How do you know if you have a good estimate? When does the pumpkin reach that height?
Summarize the Mathematics

In this investigation, you used several strategies to find rules for quadratic functions that relate the position of flying objects to time in flight. You used those function rules and resulting tables and graphs to answer questions about the problem situations.

1. How can the height from which an object is dropped or launched be seen:
   i. In a table of (time, height) values?
   ii. On a graph of height over time?
   iii. In a rule of the form \( h = h_0 + vt - 16t^2 \) giving height as a function of time?

2. How could you determine the initial upward velocity of a flying object from a rule in the form \( h = h_0 + vt - 16t^2 \) giving height as a function of time?

3. What strategies can you use to answer questions about the height of a flying object over time?

Be prepared to share your ideas and strategies with others in your class.

Check Your Understanding

In Game 3 of the 1970 NBA championship series, the L.A. Lakers were down by two points with three seconds left in the game. The ball was inbounded to Jerry West, whose image is silhouetted in today's NBA logo. He launched and made a miraculous shot from beyond midcourt, a distance of 60 feet, to send the game into overtime (there was no 3 point line at that time).

Through careful analysis of the game tape, one could determine the height at which Jerry West released the ball, as well as the amount of time that elapsed between the time the ball left his hands and the time the ball reached the basket.

This information could then be used to write a rule for the ball's height \( h \) in feet as a function of time in flight \( t \) in seconds.

a. Suppose the basketball left West's hands at a point 8 feet above the ground. What does that information tell about the rule giving \( h \) as a function of \( t \)?

b. Suppose also that the basketball reached the basket (at a height of 10 feet) 2.5 seconds after it left West's hands. Use this information to determine the initial upward velocity of the basketball.

c. Write a rule giving \( h \) as a function of \( t \).
d. Use the function you developed in Part c to write and solve equations and inequalities to answer these questions about the basketball shot.

i. At what other time(s) was the ball at the height of the rim (10 feet)?

ii. For how long was the ball higher than 20 feet above the floor?

iii. If the ball had missed the rim and backboard, when would it have hit the floor?

iv. What was the maximum height of the shot, and when did the ball reach that point?
1. Two exponential relationships are represented by graphs (1) and (2) and also by tables (A) and (B).

   a. Write the number of the graph beside its corresponding table. (1 point)

   ![Graphs](image)

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th></th>
<th>(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1 2 3 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>6 12 24 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>1 2 3 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>1.5 0.75 0.375 0.1875</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. The equations for the relationships in part a are both in the form $y = a(b)^x$. Explain the differences you would expect to see in the equations for each of the two relationships. Include information about the values of both $a$ and $b$. (3 points)
2. Coffee, tea, and some soft drinks contain the drug caffeine. One hour after ingestion, 75% of the original amount of caffeine remains. At the end of each hour after that, 75% of the amount at the beginning of the hour remains. Suppose a person consumes 40 milligrams of caffeine.

a. How much of that 40 milligrams will remain after 1 and after 2 hours? (2 points)

after 1 hour ____________ after 2 hours ____________

b. Write a NOW-NEXT equation that can be used to calculate the amount of caffeine that will remain after any number of hours. (2 points)

\[ \text{NEXT} = \text{PREVIOUS} \times 0.75 \]

c. Write a rule beginning \( y = \ldots \) that can be used to calculate the amount of caffeine that will remain \( x \) hours after the initial dose. (2 points)

\( y = 40 \times (0.75)^x \)

d. Find the amount of caffeine remaining after 6 hours. Show or explain your work. (2 points)

\[ 40 \times (0.75)^6 = 11.39 \text{ milligrams} \]

e. Write and solve an inequality that gives an estimate of how long it will take for less than 1 milligram of caffeine to remain? Show your work. Round your answer to the nearest tenth of an hour. (4 points)

\[ 40 \times (0.75)^t < 1 \]

\[ t > \log_{0.75} \frac{1}{40} \approx 9.7 \text{ hours} \]
3. Write each of the following expressions in simpler equivalent exponential form. (3 points each)

a. \( \frac{24x^2y^4}{4x^2y^2} \)

b. \( \left( \frac{20x^2y^4}{10x^3y} \right)^3 \)

4. Find values of \( x \) and \( y \) that will make the following equations true statements. (2 points each)

a. \( \left( \frac{3^x}{n} \right)^4 = \frac{3^x}{n^2} \)

b. \( \frac{4^x}{4^y} = 4^2 \)

5. Write each of the following expressions in equivalent exponential form without using negative exponents. (2 points each)

a. \( x^{-4} \)

b. \( \left( \frac{3}{7} \right)^{-2} \)
6. Write the expression in an equivalent form using a radical and then in simplest numerical form. (2 points)

\[
\left( \frac{4}{25} \right)^{\frac{1}{3}}
\]

7. Suppose that you performed the following experiment:

- Roll 50 dice and remove all dice that showing 5 dots on the top face.
- Roll the remaining dice and remove all dice that show 5 dots on the top face.
- Repeat the roll and remove process, recording the number of dice left on each roll.

a. Complete the table below showing your prediction of the number of dice remaining after each roll and remove stage of the experiment. (2 points)

<table>
<thead>
<tr>
<th>Roll Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Dice Left</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write NOW-NEXT and "y = ..." equations that model the relationship between roll number and dice left shown in your table. (4 points)

\[
\text{NEXT} = \text{expression}
\]

\[
y = \text{expression}
\]

c. Suppose that the experiment was changed so that 60 dice were rolled and all of the dice showing 2 dots and showing 3 dots were removed. What "y = ..." equation would be a good model the data in this new experiment? (2 points)
Quiz 2

Name ____________________________

Total: 27 points

Date ____________________________

Write the number of your answer on the blank in front of each question. Each question is worth two points.

_____ 1. The larger root of the equation \((x + 4)(x - 5) = 0\) is

(1) 4
(2) -4
(3) 5
(4) -5

_____ 2. Which expression is a factor of \(x^2 + 5x - 24\)?

(1) \((x + 8)\)
(2) \((x + 3)\)
(3) \((x - 8)\)
(4) \((x + 24)\)

_____ 3. The expression \((x - 4)^2\) is equivalent to

(1) \(x^2 - 16\)
(2) \(x^2 + 16\)
(3) \(x^2 - 8x + 16\)
(4) \(x^2 + 8x + 16\)

_____ 4. The solution set for the equation \(x^2 - 6x - 16 = 0\) is

(1) \{8, 2\}
(2) \{-8, 2\}
(3) \{-8, -2\}
(4) \{8, -2\}

_____ 5. Written in factored form the expression \(x^2 - 25\) is

(1) \(x(x - 25)\)
(2) \(x(x - 5)\)
(3) \((x + 5)(x - 5)\)
(4) \((x + 5)^2\)
6. Sketch a graph of the equation $y = (x + 1)(x + 3)$. Label the following features on your sketch: x-intercepts, y-intercept, line of symmetry, minimum or maximum point. (5 points)

7. Find the product of each expression. (2 points each)
   a. $(x - 3)(x + 7)$
   b. $3x(x + 5)$
   c. $(x - 4)(x - 2)$

8. Factor each expression. (2 points each)
   a. $x^2 + 9x + 18$
   b. $x^2 - 100$
   c. $x^2 + 2x$