Fall 12-10-2013

A Curriculum Project on Connecting Patterns and Equations in Algebra 1 Aligned with the New York State Common Core Standards

Kristina R. Graziadei
The College at Brockport, kgraz1@u.brockport.edu

Follow this and additional works at: http://digitalcommons.brockport.edu/ehd_theses

Part of the Curriculum and Instruction Commons

To learn more about our programs visit: http://www.brockport.edu/ehd/

Repository Citation
http://digitalcommons.brockport.edu/ehd_theses/290

This Thesis is brought to you for free and open access by the Education and Human Development at Digital Commons @Brockport. It has been accepted for inclusion in Education and Human Development Master’s Theses by an authorized administrator of Digital Commons @Brockport. For more information, please contact kmyers@brockport.edu.
A Curriculum Project on Connecting Patterns and Equations in Algebra 1 Aligned with the New York State Common Core Standards

Kristina R. Graziadei

A thesis project submitted to the Department of Education and Human Development of the State University of New York College at Brockport In partial fulfillment of the requirements for the degree of Master of Education – Adolescent Inclusive Mathematics
Table of Contents

Chapter 1: Introduction ................................................................................. 3
  Problem Statement ................................................................................. 3
  Secondary Algebra ................................................................................ 5

Chapter 2: Literature Review .................................................................... 6
  Modeling ................................................................................................ 7
  Aligning Curriculum to Common Core State Standards ....................... 8

Chapter 3: Unit Plan .................................................................................. 10
  Table 1 (Unit Calendar) ........................................................................ 10
  Table 2 (Assessment Design) ................................................................. 12
  Lesson Plans for Algebra 1 Unit 1 .......................................................... 12

Chapter 4: Validity of Curriculum Project .................................................. 38

Chapter 5: Reflections and Conclusions .................................................... 39
  Findings from Questionnaire ................................................................. 40
  Conclusions .......................................................................................... 42

References ............................................................................................... 43

Appendix A: Teacher Questionnaire........................................................... 45

Appendix B: Unit Assessments ................................................................. 46
Chapter 1

Introduction

Change is constant in the world of education. Teacher requirements and education policies are constantly evolving at a statewide and nationwide level, and it is critical that teachers adapt to these changes for successful classroom learning experiences. Through implementing new technologies in the classroom teachers may elevate students understanding, provide deeper meaning in content, and make connections to real life applications. Recent studies show that implementing technology and modeling strategies in the classroom have the potential to evoke a higher level of understanding among all students (NYSED, 2012).

This curriculum project aligned the Algebra 1: Unit 1- Connecting Patterns and Equations to the New York State (NYS) Common Core State Standards (CCSS) while focusing on implementing modeling in the classroom (Neher & Plourde, 2012; Newton & Kasten, 2013). The goal of this project is to provide teachers with a clear, concise, cohesive unit plan in which they can use in their own classroom. This curriculum project was tailored for many students and was aligned with the NYS CCSS.

Problem Statement

In 1989, the National Council of Teachers of Mathematics (NCTM) compiled state standards in hopes to elevate the learning standards in the United States (US) (CCSSI, 2010). More specifically, in 1995 the State of New York created the New York State Learning Standards for Mathematics (NYSLSM), which added to the existing NCTM standards (NCTM, 1995; NYSED, 2010). The majority of the US, which included 52 states and territories, adopted these new learning states by 2006. Despite adopting the NCTM standards studies including the Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA) proved that the US fell behind other countries in student
performance and assessment (Tienken, 2010; Tienken 2011; Woolard, 2012). These shortcomings played a role in the US adopting a new set of national standards. The nations that the US performance was compared to had national standards, which is problematic because in the US education is a state’s right with the federal government having no authority over each state’s education program. Thus the National Governors Association Center for Best Practices is the designer of the Common Core State Standards Initiative (CCSSI, 2010) and the states that opted to adopt the Common Core State Standards (CCSS) received Race to the Top (RTT) funding from the federal government.

By 2009 the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA) announced that 49 states and territories joined the CCSS Initiative (Tienken, 2010). In 2012 teachers felt the pressures of adopting an entirely new curriculum, which shifted from meeting the requirements of the NCTM standards to the aligning the content in the curriculum to the Common Core State Standards (CCSS) (CCSSI, 2012). While both sets of standards cover many similar concepts of teaching mathematics curriculum, the paradigm shift sought to achieve the concept of national standards and international competitiveness (Woolard, 2012).

The purpose of the CCSS is to create a curriculum in the mathematics classrooms that enhances student performance and assessment in the US comparatively on an international competitive level (Newton & Kasten, 2013). Due to the new standards, teachers were faced with the task of accommodating their curriculum to align with the CCSS (CCSSI, 2012).
Secondary Algebra

Algebra 1 in the secondary mathematics curriculum serves as the foundation of knowledge for the remainder of the mathematics sequence throughout high school (Pearson, 2012). The first unit in the school year is “Connecting Patterns and Formulas.” In this unit, students are introduced to variables and expressions while exploring operations using real-numbers (Pearson, 2012; Newton & Kasten, 2013). This project provided an explicit Unit Plan for Algebra 1, Unit 1: Connecting Patterns and Equations that attempted to foster learning and deep understanding among most students, but also implemented modeling and real-life application problem solving that aligning with the NYS CCSS. In addition to focusing on modeling and real-life application problem solving, this unit plan focused on the implementation of technology in the classroom to enhance modeling, which may increase performance for all students.

This curriculum project sought to provide a unit plan for teaching Unit One in Algebra I – Connecting Patterns and Equations with the potential to meet the needs of most students. The unit plan implemented technology and other teaching strategies and activities that could evoke deeper understanding among the students and help students understand patterns in business, economics, environment and behaviors to predict successful decisions (Pearson, 2012). As a result, students may be better prepared for real-life situations and make connections to more abstract concepts related to mathematics (Neher & Plourde, 2012).
Chapter 2: 
Literature Review

Paradigm Shift to Common Core State Standards (CCSS)

The shift to the CCSS for Mathematics brings the standards to the limelight of educational research for mathematics in the US (Newton & Kasten, 2013). This paradigm shift and adoption of the CCSS resulted in strong feelings from teachers, students, mathematics educators and researchers throughout the US (Newton & Kasten, 2013).

The reason for the shift in the curriculum is to align the curriculum across the US so that there are national standards. Students’ performance and achievement in education, specifically mathematics, is considerably lower than that of other countries. A major difference in other countries that participated in the Third International Mathematics and Science Study (TIMSS) is that countries that have higher achievement levels than the US have national standards (Woolard, 2012; CCSSI, 2012). The TIMSS was the second assessment in a series of studies conducted by the International Association for the Evaluation of Educational Achievement (IEA) that measured trends in student achievement, among 38 participating countries, in mathematics and science (Beaton et al., 1996; Tienken, 2010). Other countries may have higher achievement rates due to the fact that there are common standards set throughout their nation, which differs from the previous state or NCTM standards across the US. Prior to the adoption of the CCSS, each state in the United States had their own set of state standards to follow. Currently forty-five states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the CCSS (CCSSI, 2012). By implementing the CCSS the US hopes to elevate both teaching, student achievement and assessment in the US (CCSSI, 2012; Woolard, 2012).
The mission of CCSS Initiative (2012) is to provide clear and concise expectations for students, so that teachers and parents know their role in helping the students (CCSSI, 2012). In addition to providing common understanding of expectations across the nation, the designers of the standards claimed that they are meant to aid students in learning real-life applications that help students to use their knowledge and skills to become success after high school and into college and a career (CCSSI, 2012). One of the key points in the mathematics standards that is applicable to this curriculum project is that the high school standards invite students to apply mathematical thinking to real world situations, similarly to the NCTM Standards, while asking students to think with mathematical reason (NYSED, 2012).

**Modeling**

Modeling in the classroom is a key component in the CCSS. Many ideas in the mathematics classroom are better understood through modeling and real-life application problems (Pearson, 2012). Modeling can connect classroom mathematics to decision making, working and everyday life (NYSED, 2011). Students and teachers can utilize modeling in many different ways and at different ability levels due to the fact that modeling can be quite simple or very complex. In order to differentiate classroom instruction teachers can create more elaborate models for accelerated students while modifying the model in a more simplistic manner for students who might need to move at a slower pace (NYSED 2011; NYSED, 2012; NYSED, 2013). This creates higher expectations for all students that fulfill requirements of the CCSS, while allowing different opportunities for students to reach these goals in various ways (NYSED, 2012).

One of the main focuses for the CCSS is not to merely enhance student performance, but how teachers’ practices influence student achievement, assessment and performance in the
Curriculum Project on Connecting Patterns and Equations

mathematics classroom (NYSED, 2012). Teachers must focus and be held responsible for planning instruction that evokes growth and achievement for all students in the classroom (NYSED, 2012). Students should be encouraged by inquiry based learning to make deep and meaningful connections through modeling (NYSED, 2012).

Aligning Curriculum to Common Core State Standards

The CCSS have caused many pedagogical shifts to be a necessary part of aligning the mathematics curriculum across the country. There are six major instructional shifts to be carried out by teachers and students included in the CCSS. The first shift is focus, which refers to teachers narrowing the amount of concepts taught in the classroom at a deeper level of understanding (Engageny, 2012). The second shift is coherence, which focuses on principals and teachers strategically connecting learning across grade levels. Fluency states that students are expected to have speed and accuracy with basic calculations while teachers must structure class time and homework for students to memorize core functions through repetition. The next shift, deeper understanding, states that students can deeply understand and operate math concepts easily. Application is the fifth shift, which means that students are expected to use math and correctly choose the concept to apply in a math problem without being prompted to do so. Lastly, dual intensity is the final shift. This means that students should practice and understand concepts with intensity. (Engageny, 2012).

This curriculum project aligns the New York State Common Core and Learning Standards for Mathematics (NYSCCLSM), and provided opportunity for mathematical modeling. This curriculum project aligns with NYSCCLSM because the curriculum was implemented by two veteran teachers in New York State. Each of the states that voluntarily adopted the CCSS had to align their curriculum to the standards specific that state (CCSSI,
Therefore the NYSCCLSM is very similar to the CCSS with the minute differences being what New York State added to the CCSS.

The unit on Connecting Patterns and Equations for the Algebra 1 curriculum contains content that will be used throughout the algebra stand in the high school content, therefore could be considered as high importance for success in mathematics courses. It is important that students start off strong in the beginning of the school year, and since this is the first unit of the year, it is important that students understand the structure and importance of modeling and technology in the mathematics classroom (Newton & Kasten, 2013). Teachers should demonstrate updated, research-based knowledge of learning and language acquisition (NYSED, 2012; Neher & Plourde, 2012).

This unit may serve as a resource for teachers to adopt and modify to fit the needs of their students. The concepts in this unit are foundational for the students’ future years of mathematics. Mathematical modeling, sense-making, abstract thinking, and reasoning are the key mathematical practices that this unit focuses on in the classroom (Pearson, 2012). The NYS CCSS for Mathematics, the classroom textbook written by Charles et al (2012), Algebra 1: Common Core and the New York State Education Department shaped the unit design in this curriculum project that focuses on three specific standards; Standard II- Reason abstractly and quantitatively, Standard IV- Model with mathematics and Standards VI- Attend to precision (NYSED, 2012, p. 50).
Chapter 3
Curriculum Design

This curriculum design is a guide to Unit 1 in Algebra 1, which began during the first week of a new school year. The suggested timeline of lessons throughout this Unit (Table 1) was based on a 45 minute class period, and could be adapted for 80 minute block scheduling by combining two lesson into one. This would change this 14 day unit into 7-8 day unit. For the sake of making this a clear and concise design the curriculum accommodated a 45 minute class period. The pacing of the curriculum was designed in correlation with the pacing guide found in the Pearson Algebra 1: Common Core textbook by Prentice Hall Inc. (2012). The Pearson text was aligned with the NYS CCSS, and added a baseline for this curriculum design. The

Table 1: Unit Calendar

Unit 1 Connecting Patterns and Equations (including which NYS CCSS will be met)

<table>
<thead>
<tr>
<th>Day</th>
<th>Concept for Lesson</th>
<th>NYS Common Core State Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Getting to Know Each Other</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Review</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Variables and Expression</td>
<td>A.SSE.1, A.SSE.1.a</td>
</tr>
<tr>
<td>4</td>
<td>Evaluating Expression (Order of Operations)</td>
<td>A.SSE.1, A.SSE.1.a</td>
</tr>
<tr>
<td>5</td>
<td>Real Numbers (Number Line)</td>
<td>N.RN.3</td>
</tr>
<tr>
<td>6</td>
<td>Properties of Real Numbers</td>
<td>N.RN.3</td>
</tr>
<tr>
<td>7</td>
<td>Adding and Subtracting Real Numbers</td>
<td>N.RN.3</td>
</tr>
<tr>
<td>8</td>
<td>Multiplying and Dividing Real Numbers</td>
<td>N.RN.3</td>
</tr>
<tr>
<td>9</td>
<td>The Distributive Property</td>
<td>A.SSE.1.a</td>
</tr>
<tr>
<td>10</td>
<td>An Introduction to Equations</td>
<td>A.CED.1</td>
</tr>
<tr>
<td>11</td>
<td>More on Equations</td>
<td>A.CED.1</td>
</tr>
<tr>
<td>12</td>
<td>Connecting Patterns, Equations and Graphs</td>
<td>A.CED.2, A.REI.10</td>
</tr>
<tr>
<td>13</td>
<td>Unit Review</td>
<td>A.SSE.1, A.SSE.1.a, N.RN.3, A.CED.1, A.CED.2, A.REI.10</td>
</tr>
<tr>
<td>14</td>
<td>Unit Test</td>
<td>A.SSE.1, A.SSE.1.a, N.RN.3, A.CED.1, A.CED.2, A.REI.10</td>
</tr>
</tbody>
</table>
curriculum design combined with instruction, activities and assessment (All worksheets, quizzes, and tests can be found in Appendix B) that sought to elevate students learning and understanding of connecting patterns and equations in Algebra 1.

For the first two days of instruction the goal was to facilitate activities and exercises that helped teachers get to know their students. Once this was achieved the curriculum design continued into the 12 day unit plan. Educators altered the activities and exercises included in the first day of this curriculum design based on the needs and preferences in accordance to one’s teaching style. This first day was intended to get to know one’s students outside the classroom. Students should feel comfortable and safe in the classroom, so the educator provided some information about oneself as well. The activities chosen helped teachers get to know the students’ likes, dislikes, hobbies, extracurricular activities, learning styles and needs. Also, the activity gave the students who might not know each other a chance to meet each of their classmates. This curriculum design included an example of an activity and exercise that was intended to achieve this goal, while getting the school year off on a positive note.

The second day of instruction focused on getting to know each students’ academic strengths and weaknesses, while gauging how much knowledge each student had retained from the previous year. It was important that before the students begin the new unit, that the teacher review older material that should be background knowledge for the new concepts of Algebra I. The provided Day 2 – Algebra I Entry Assessment, sought to help the students recall concepts from pre-algebra courses and foundational concepts and skills that they will need to be successful in Algebra I.
Once the students completed the first two days of instruction, the 12 day unit on Connecting Patterns and Equations began. The following lessons were used as a guideline and include the NYS CCSS that each lesson fulfills. All assessments (Appendix B) - handouts, homework, quizzes, and the unit exam – make up the suggested assessment design (Table 2) for the unit.

**Lesson Plans for Algebra 1 Unit 1: Connecting Patterns and Equations**

**Day 1 – Tuesday, September 2, 2013**

**Lesson: Getting to Know Each Other**

**Objective:** The objective for the first day of school is to set the stage for a successful school year. The teacher must help the students feel welcome and safe in the school environment, and facilitate in activities and exercises that make the students feel comfortable with their classmates as well as their teacher. In addition, the activities and exercises will provide useful information
about each students’ personality, hobbies/interests, learning preferences, strengths and needs. The information collected will be a source for future instruction, and be an invaluable piece of knowledge to help each student learn and achieve in the classroom.

**Instruction:** As soon as the students are seating it is important that they understand who their teacher is so that they are comfortable and more willing to introduce themselves as well. First, introduce yourself for the class. After the teacher’s introduction, pass out the activity, “Facts About Me” (found in Appendix B), to help the students be creative while help the teacher and students get to know each other. Make sure to do an example of what it should look like by presenting the class with your completed copy.

Pass the activity out and have the students fill it out. Provide students with colored pencils, markers, etc. and urge them to be creative. Depending on school policy you will need permission to take pictures of your students. Either get permission from students’ parents, have them bring in a picture of themselves to add if they so choose, or have them draw a picture of themselves. Inform the students after 20 minutes, they will be presenting this to the class so we can all get to know each other. Students should see that they have similar qualities and interests with some of their classmates, and the teacher can learn how to tailor lessons around the class throughout the school year.

**Facts About __________ Your Teacher _______**

My Birthday is ____________________________

**Directions:** In each box, draw pictures and explain in one sentence a little about yourself for that category. Please be creative!! Be prepared to share with the class once everyone is done.

<table>
<thead>
<tr>
<th>Photo of Yourself</th>
<th>Favorite Hobbies</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Photo" /></td>
<td><img src="image2.png" alt="Hobbies" /></td>
</tr>
</tbody>
</table>

I love singing and playing games like Just Dance and Rock Band. I love all types of music and discovering new bands!
<table>
<thead>
<tr>
<th>Favorite Candy</th>
<th>Dream Job</th>
</tr>
</thead>
<tbody>
<tr>
<td>I could eat Sour Patch Kids for breakfast lunch and dinner. (My dentist wouldn’t like me very much though☺)</td>
<td>I’m very lucky because my dream job is to be a math teacher! (Or maybe a professional singer?)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fun Fact</th>
<th>Favorite Movie/Book</th>
</tr>
</thead>
<tbody>
<tr>
<td>I’m full blooded Italian, and have been to Italy!</td>
<td>My favorite movies and books are the Harry Potter Series and the Hunger Games Series!</td>
</tr>
</tbody>
</table>

Once all of the students have finished and presented their poster collect them so that you can add their picture and hang them around the classroom. These facts will help build a positive relationship between the teacher and the students, and give insight to each student as an individual.

**Closure/Assessment**: For homework hand out the Pre-Algebra (pre-assessment found in Appendix B) Packet for students to complete. This will be a way for students to show what they remember from the previous year, and what content needs to be refreshed before beginning Unit 1.

**Day 2 – Wednesday, September 3, 2013**

**Lesson**: Pre-Algebra Review
Objective: The goal for this lesson is for the students to show what knowledge they recall from middle school algebra content. This will show the students strengths and weaknesses in their background knowledge and skills that will be needed to be successful in Algebra I.

Instruction: As soon as the students sit down go around the room checking if each student completed each question on the Pre-Algebra Review Packet. Record in your own gradebook - , ✓ , + based on completion of their work. Students are expected to have attempted each question since it is multiple choice, so grades should reflect from the work that they did. (i.e. Students who did not do the packet or did not answer all questions should get - , students who did them all but did not show any work on the sign should get ✓ , and students who answered all questions and showed work in the margins should receive +.)

Once you have checked each students’ work you will instruct them to get into pairs. Depending on how many students in the class assign each pair of students 2-3 questions from the review packet. The students have 5 minutes to go over their own questions and make sure they can explain each of their problems and show work. After 5 minutes (give more time if needed, but make sure to leave 30 minutes to have enough time for each problem) have the students who were assigned Question #1 go up to the board (Smart Board if available) and show their work for the problem. If help is needed the problem should be done as a class. Continue this process for all 15 problems and help the students when needed.

Make sure students keep their review sheets in a folder or binder, and no homework is assigned for this lesson. Answer any questions or concerns any of the students have. Students will start the Unit on Day 3.

Closure/Assessment: Throughout this lesson students will be observed on their individual work and group work informally. The teacher should see which students are the leaders of the class and who may need a little encouragement. Make sure that students are not copying each other’s work, but that they are working as a team to figure out the correct answers. Students should listen to each problem as it is presented and record it on their review packet.

Answers for Pre-Algebra Review Packet:

Day 3 – Thursday, September 4, 2013

Lesson: Variables and Expressions

NYS Common Core State Standard(s):
Algebra – Seeing Structure in Expressions - Interpret the Structure of Expressions
A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.
A.SSE.1.a – Interpret parts of an expression, such as terms, factors and coefficients.

**Objectives:** By the end of the lesson students should be able to
- Understand that algebra uses symbols to represent quantities that are unknown or that vary
- Represent mathematical phrases and real-world relationships by symbols and operations
- Define variable, quantity, algebraic expression and numerical expression

**Instruction:** As soon as students walk into class have the warm-up on the front board (Smart Board is available). After 5 minutes into class go over them as a class.

**Warm Up:** Define each of the following terms.

1. Factor-
2. Greatest Common Factor -
3. Multiple -
4. Least Common Multiple -

**Answers** –
1. Factor - a number or quantity that when multiplied with another produces a given number or expression.
2. Greatest Common Factor - the largest number that divides a set of numbers without a remainder.
3. Multiple – a number that can be divided by another number without having a remainder.
4. Least Common Multiple – the smallest number (not zero) that is a multiple of two numbers given.

Once the warm up is complete hand out the in class worksheet (found in Appendix B) “Variables and Expressions” (The homework is on the attached in this packet as well.) The packet will be projected on the Smart Board (adapt this if technology is not available). Begin the lesson going over the key vocabulary for the lesson. Choose four different students to read each vocabulary word.

**Key Vocabulary:**
- **Quantity** – anything that can be measured or counted
- **Variable** – a symbol, for example a letter, which represents the value of a quantity
- **Algebraic Expression** – a mathematical phrase that includes one or more variables
- **Numerical Expression** – a mathematical phrase involving numbers and operation symbols, but no variables

Next go over the first example with the class. Explain the model is a visual representation of Example 1 on the handout. Answer any questions or concerns before asking the students to do Example 2 individually.
Example 1: What is an algebraic expression for 23 more than a number \( n \)?

Model →

\[
\begin{array}{c}
\text{n} \\
\hline
23
\end{array}
\]

\[ \text{n} + 23 \]

\( n + 23 \) is the algebraic expression

Now have the students try examples 2-5. Once students have finished go over the answers.

Example 2: What is an algebraic expression for 18 less than a number \( n \)?
The algebraic expression is \( n - 18 \)

Example 3: What is an algebraic expression for 8 times a number \( n \)?
The algebraic expression is \( 8n \)

Example 4: What is an algebraic expression for the quotient of a number \( n \) and 5?
The algebraic expression is \( \frac{n}{2} \)

Example 5: What is an algebraic expression for 3 more than twice a number \( x \)?
The algebraic expression is \( 3 + 2x \)

Now model how to write a word phrase that represents an algebraic expression by doing example 6 as a class, then have the students do example 7 by themselves. Go over the acceptable answers with the class.

Example 6: What word phrase represents the algebraic expression \( 3x \)?

\[ \rightarrow 3 \text{ times } \text{a number } x \rightarrow \text{the product of } 3 \text{ times a number } x \]

(Either of these answers are acceptable)

Example 7: What word phrase represents the algebraic expression \( 5x + 8 \)?

Lastly, example 8 is a real world application problem. Students should understand that algebraic expressions can be used to explain patterns among a group of numbers.

Example 8: Give any regular polygon, you can draw a segment from any one vertex to the other vertices. Each of these segments cut the regular polygon into non-overlapping triangles. Using the table below, give the rule and algebraic expression that describes the pattern.

<table>
<thead>
<tr>
<th>Number of Sides of the Regular Polygon</th>
<th>Number of Triangles</th>
</tr>
</thead>
</table>

In words: subtract 2 from the number of sides, \( n \), in a regular polygon

Algebraic Expression: \( n - 2 \)
<table>
<thead>
<tr>
<th>4</th>
<th>4 - 2 = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5 - 2 = 3</td>
</tr>
<tr>
<td>6</td>
<td>6 - 2 = 4</td>
</tr>
<tr>
<td>n</td>
<td>?</td>
</tr>
</tbody>
</table>

**Closure/Assessment:** For homework students have problems that follow the example problems in the “Variables and Expression” packet (Found in Appendix B).

**Day 4 – Friday, September 5, 2013**

**Lesson:** Evaluating Expressions (Order of Operations)

**NYS Common Core State Standard(s):**

Algebra – Seeing Structure in Expressions - Interpret the Structure of Expressions
A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.
A.SSE.1.a – Interpret parts of an expression, such as terms, factors and coefficients.

**Objective:** By the end of the lesson students should be able to –
- Define the key vocabulary words power, exponent, base, simplify and evaluate.
- Understand that power can be used to shorten the representation of repeated multiplication.
- Simplify expressions that include exponents.
- Evaluate expressions using the Order of Operations.

**Instruction:** Put the warm-up on the Smart Board (or white board). While students are completing the warm-up go around and check the students homework marking down - , ✔ or +.

**Warm-Up:**

1. Explain the difference between numerical and algebraic expressions.
   *Answer: An algebraic expression contains a variable while numerical expressions do not. Each expression contain numbers, operation signs, and represent a value.*
2. What is the algebraic expression for 9 more than three times a number \(x\)?
   *Answer: \(9 + 3x\)*
3. What is the word phrase for \(5t - 11\)?
   *Answer: 11 less than 5 times a number \(t\)*

Go over the warm-up questions, then display homework answer up on the Smart Board (or white board). Answer any questions the students might have then handout the packet for “Evaluating Expressions (Order of Operations)” (Found in Appendix B). Display the packet on the Smart Board (or other technology available).

*Homework Answer –
1. a) numerical b) algebraic*
2. \( \frac{n}{20} \)
3. 3 times a number \( y \) more than 14
4. 49 + 0.75n
5. The quotient of 8 and \( h \)
6. \( p - 2 \)

**Key Vocabulary:**

**Power** – a number that represents repeated multiplication containing an *exponent* and a *base* number. (*Example* - \( 9^4 \) where 9 is the base and 4 is the exponent, also represented by \( 9 \times 9 \times 9 \times 9 \))

**Simplify** – replacing a numerical or algebraic expression with its single or lowest numerical value.

**Evaluate** – replacing each variable in an algebraic expression with a number, then simplifying the expression using the Order of Operations.

Model how to simplify powers with the class by doing examples 1-3 as a class. Students will need their calculators.

Simplify each of the following expressions. Show the repeated multiplication for each example.

**Example 1:** \( 5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15625 \)

**Example 2:** \( (1.4)^3 = 1.4 \times 1.4 \times 1.4 = 2.744 \)

**Example 3:** 
\[
\left(\frac{4}{9}\right)^4 = \left(\frac{4}{9}\right) \times \left(\frac{4}{9}\right) \times \left(\frac{4}{9}\right) \frac{256}{6561}
\]

Go over the Order of Operations. PEMDAS. Ask the students to tell you what PEMDAS stands for, and for them to share the acronym that they learned to remember the Order of Operations. The most common acronym is Please Excuse My Dear Aunt Sally. The letters actually stand for Parentheses, Exponents, Multiplication and Division (from left to write), Addition and Subtraction (from left to write). Go over this with the students and answer any questions that may arise. Move on to examples 4 and 5 as a class, then have them individually do examples 6 and 7. Go over as a class once students have completed them.

Simplify each of the following expressions. (Remember PEMDAS)

**Example 4:** \( (6-5)^2 \times 8 = \)
\[
Answer: (6-5)^2 \times 8 = (1)^2 \times 8 = 1 \times 8 = 8
\]

**Example 5:** \( 3 \times 7 - 3^2 = \)
\[
Answer: 3 \times 7 - 3^2 = 3 \times 7 - 9 = 21 - 9 = 12
\]

**Example 6:** \( 12 - 25 ÷ 5 = \)
\[
Answer: 12 - 25 ÷ 5 = 12 - 5 = 7
\]

**Example 7:** \( \frac{4 + 3^4}{7 - 2} = \)
\[
Answer: \frac{4 + 3^4}{7 - 2} = \frac{4 + 81}{7 - 2} = \frac{85}{5} = 17
\]

Next, we will evaluate algebraic expression. Have one student read out loud the definition of evaluate (located on the top of the handout). Model example 8 with the class then let them do example 9 by themselves. Go over with the students when they are finished.
Evaluate each of the following expressions for \(x = 4\) and \(y = 2\).

Example 8: \(x^2 + x - 12 \div y^2\)

\[
\text{Answer: } 4^2 + 4 - 12 \div 2^2 = 16 + 4 - 12 \div 4 = 16 + 4 - 3 = 17
\]

Example 9: \(89 - 2x^2 + 18y\)

\[
\text{Answer: } 89 - 2(4)^2 + 18(2) = 89 - 2(16) + 18(2) = 89 - 32 + 36 = 93
\]

**Closure/Assessment:** For homework students will evaluate the real-world expression word problems located under “Homework” at the end of the “Evaluating Expressions (Order of Operations)” handout (Found in Appendix B).

**Day 5 – Monday, September 6, 2013**

**Lesson:** Real Numbers (Number Line)

**NYS Common Core State Standard(s):**

- Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
- N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

**Objective:** By the end of this lesson should be able to-
- Define and list examples of a square root, radicand, radical, perfect square, set, element of a set, subset, rational numbers, natural numbers, whole numbers, integers, irrational numbers, real numbers, and an inequality.
- Classify numbers by their characteristics.
- Understand that square roots of other nonnegative numbers can be approximated.
- Identify patterns between square roots being the inverse operation of squaring.

**Instruction:** Put the warm-up on the Smart Board (if available). While students are completing the warm-up go around and check the students homework marking down -, ✔ or +. Ask 3 different students to go up to the board and write their solution on the board. Go over as a class. Pass out handout, “Real Numbers (The Number Line)” (Found in Appendix B).

**Warm Up:**

Simplify the following expressions and show your step by step solution.

1. \(10^7\) (Answer: \(10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10,000,000\))

2. \((5+2)^2 - 3\) (Answer: \((7)^2 - 3 = 49 - 3 = 46\))

Evaluate the following expression for \(x = 1\) and \(y = 2\).

3. \(2x + y^2 - xy\) (Answer: \(2(1) + (2)^2 - (1)(2) = 2 + 4 - 2 = 4\))

For this lesson the students will be filling out a chart of key terms, writing their definitions and writing examples of each term in the third column. Students should be learning how to classify numbers throughout this lesson.
**Key Terms:**

- **Square Root** – a number that produces a specific quantity when multiplied by itself. A square root is the inverse operation of squaring a number.
  - Ex. \(8^2 = 64\), where 8 is a square root of 64. Written \(\sqrt{64} = 8\)
  - Ex. \(\frac{16}{25} = \frac{4}{5}\), so \(\frac{4}{5}\) is a square root of \(\frac{16}{25}\)

- **Radical** – a radical symbol (or square root sign) is \(\sqrt{\text{a}}\) and indicates a nonnegative square root.
*Have the students write a square root with a nonnegative number under it in the example box.*

- **Radicand** – The expression under the radical symbol.

radical symbol \(\sqrt{a}\) radicand (where \(a\) is a nonnegative number)
*For the terms Radical and Radicand make sure the students draw the picture representations. Also make sure to explain to the students that you can find the exact square roots of some nonnegative numbers, and you can approximate the square roots of other nonnegative numbers. This is a good introduction into defining a perfect square.*

- **Integer** – A number that is not a fraction.
  - Ex. \{…-2, -1, 0, 1, 2…\}

- **Perfect Square** – The square root of an integer.
  - Ex. 36 is a perfect square because \(6^2 = 36\).
  - Ask the students to give another example.

- **Set** – a well-defined collection of objects.
  - Ex. \{1, 3, 5\}
  - Ex. \{shoes, coat, boots, pants, hat\}

- **Element of a Set** – each of the object(s) in a set.
  - Ex. \{2, 4, 5\}, where 2, 4, 5 are the elements of the set of numbers
  - Ex. \{orange, yellow, red\}, where orange, yellow and red are the elements of the set of colors.

- **Rational Number** – any number that can be written as \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b \neq 0\). *(Remind students that if the denominator of a fraction is 0 the fraction is undefined.)* If in decimal form the decimal either terminates (ends after a few decimal places such as 3.245) or is a repeated decimal (ex- 2.74333333…).  
  - Infinite amount of examples, ask students to shout out a few.

- **Natural Number** – any integer greater that zero.
  - \{1, 2, 3, …\}

- **Whole Number** – any integer great than or equal to zero.
  - \{0, 1, 2, …\}

- **Irrational Number** – any number that cannot be represented as the quotient of two integers \(\frac{a}{b}\). If in decimal form the numbers do not end or repeat.
  - \(\pi = 3.14159265359…\)
  - \(\sqrt{11}\)

- **Real Number** – the set of rational and irrational numbers.
  - Infinite amount of examples, ask students to shout out any real number.

- **Inequality** – a mathematical sentence that compares the values of two expressions using some symbol of inequality.
Lastly, remind students what a number line is and briefly go over the following example. Instruct students to classify the set of numbers, put the numbers in order from least to greatest, and plot on the numbers line.

Set #1 \{9, 1, 5, 0\} – Whole Numbers; 0, 1, 5, 9

\[
\begin{array}{cccc}
0 & 1 & 5 & 9 \\
\end{array}
\]

Set #2 \{\sqrt{2}, 0.4, -1.4, \frac{2}{3}\} – Irrational Numbers; -1.4, \frac{2}{3}, 0.4, \sqrt{2}

\[
\begin{array}{ccc}
-1.4 & \frac{2}{3} & 0.4 & \sqrt{2} \\
\end{array}
\]

Assessment/Closure: Ask the students if they have any questions on classifying real numbers, and ensure they are comfortable putting numbers on a number line. For homework, hand out the “Real Numbers and the Number Line” (Found in Appendix B) for the students to complete at home. This is assignment will be handed in during the next class.

Day 6 – Tuesday, September 7, 2013

Lesson: Properties of Real Numbers

NYS Common Core State Standard(s):
Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers

N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

Objective: By the end of this lesson students will be able to:
- Define equivalent expressions, deductive reasoning and counterexample.
- Understand the relationships and properties of real numbers (Commutative Properties of Addition and Multiplication, Associative Properties of Addition and Multiplication, and Identity Properties of Addition and Multiplication)
- Illustrate properties of real numbers using numerical expression.

Instruction: Begin with collecting the homework, “Real Numbers and the Number Line,” from each of the students and have the students do the warm-up that is up on the Smart Board (or other available technology). Go over the warm up as soon as the students are finished.

Warm Up: Complete the following questions.
1. Simplify $\sqrt{\frac{36}{81}}$. (Answer $= \frac{6}{9} = \frac{2}{3}$)

2. Classify the number 14 as a set of the real number system. *Hint: It fits into more than one category* (Answer: Integer, Natural, Whole, Rational)

3. Write an inequality to compare the numbers 7 and $\sqrt{50}$.
   (Answer: $7 > \sqrt{50}$)

Hand out the “Properties of Real Number’s” packet (Found in Appendix B). Let’s talk about Properties of Real Numbers. If two algebraic expressions have the same value for all values of the variables then we can call these equivalent expressions. The following properties illustrate expressions that are equivalent for all real numbers. *Note- we will be using variables $a, b, and c$ to represent any real number*

**Commutative Property of Addition** – Changing the order of the addends does not change the sum. $a + b = b + a$ (ex. $1 + 2 = 2 + 1$)

**Commutative Property of Multiplication** – Changing the order of the factors does not change the product. $a * b = b * a$ (ex. $4 * 6 = 6 * 4$)

**Associative Property of Addition** – Changing the grouping of the addends does not change the sum. $(a + b) + c = a + (b + c)$ (ex. $(3 + 7) + 4 = 3 + (7 + 4)$)

**Associative Property of Multiplication** – Changing the grouping of the factors does not change the product. $(a * b) * c = a * (b * c)$ (ex. $(4 * 9) * 2 = 4 * (9 * 2)$)

**Identity Property of Addition** – The sum of any real number and 0 is the original number. $a + 0 = a$ (ex. $89 + 0 = 89$)

**Identity Property of Multiplication** - The product of any real number and 1 is the original number. $a * 1 = a$ (ex. $\sqrt{50} * 1 = \sqrt{50}$)

**Zero Property of Multiplication** – The product of $a$ and 0 is 0. $a * 0 = 0$ (ex. $19 * 0 = 0$)

**Multiplicative Identity** – The product of $a$ and -1 is $-a$. $a * -1 = -a$ (8 * -1 = -8)

Instruct students to get into groups and do the “Let’s Practice” section. Model how to the first one and then give them 10 minutes to do the rest. Go over as a class once each group is finished.


Next, students will use deductive reasoning. Explain that deductive reasoning is the process in which we will logically make conclusions from facts we are given. Also explain to students what a counterexample is. A counterexample is an example that proves that a statement is false.

Using deductive reasoning guide the students through examples 1-3. Let them do number 3 individually and then go over as a class.

Example 1: For all real numbers $a$ and $b$, $a + b = a * b$. (Answer: false, $4 + 6 \neq 4 * 6$ is a counterexample)
Example 2: For all real numbers \( x, y, \text{ and } z \), \( x \cdot (y \cdot z) = (x \cdot y) \cdot z \). (Answer: true, by the Associative Property of Multiplication)

Example 3: For all real numbers \( a \), \( a \cdot 0 = a \). (Answer: false, \( 7 \cdot 0 \neq 7 \) is a counterexample)

**Assessment/Closure:** Students will have a quiz on the material from days 2-6 next class so for homework they should go over there in-class packets and homework from each day. Instruct students to have any questions on the material ready to ask during the first 5 minutes of day 7. Also, if possible, make yourself available during your free period(s) and/or after school.

**Day 7 – Wednesday, September 8, 2013**

**Lesson:** Adding and Subtracting Real Numbers (Quiz of 1.1-1.4)

**NYS Common Core State Standard(s):**
- Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
  - N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

**Objective:** By the end of the lesson the students will be able to-
- Model a real-world situations by adding and subtracting real numbers.
- Identify and utilize rule for adding numbers with the same sign or use rules for subtracting numbers with the same sign.

**Instruction:** For the first 5 minutes of class students will have time to ask questions from day 2 through day 6 of instruction. Instruct students to clear their desks, take out a pencil, and they can use a calculator. Pass out the quiz (Found in Appendix B). Students should be able to finish the quiz in 15-20 minutes.

When the students are finished with the quiz go over some rules for adding and subtracting real numbers. Display the rules on the Smart Board (or any available board).

**Rules for Adding and Subtracting Real Numbers**

1. **If the signs are the same keep them the same and add.**
   Ex. \( 2 + 29 = 31 \); \( -9 + -8 = -17 \)

2. **If the signs are different take the sign of the larger real number and subtract.**
   Ex. \( -2 + 30 = 28 \); \( 40 + (-79) = -39 \)

   You can also use a number line to help the students better understand. Do the next questions (3 and 4) that show adding and subtracting with a number line. Instruct the students to explain what direction the numbers are moving. For instance, for questions 3 we can say that we are starting at number 9 and moving 3 units to the right ending up at number 12. Similarly, for questions 4 we say that we are starting at -8 and moving to the left for units ending up at -12.
For further practice we will be logging onto the computer and projecting in front of the class (using Smart Board or projector) using the website below:

http://www.ictgames.com/mummyNumberLine/mummyNumberLine.html

Read the directions for the game (found on the bottom of the web page) and do a few problems with the class.

Assessment/Closure: Before moving into the lesson be sure that you have collected each of the quizzes. Instruct the students to practice more on this website outside of school. The homework problems are found on the handout “Adding and Subtracting Real Numbers” (Found in Appendix B). Also pass back the homework “Real Numbers and the Number Line.” This may help them do their homework.

Day 8 – Thursday, September 9, 2013

Lesson: Multiplying and Dividing Real Numbers

NYS Common Core State Standard(s):
Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

Objective: By the end of the lesson the students will-
- Recognize patterns in multiplying positive and negative integers.
- Define multiplicative inverse and reciprocal.
- Derive the rules for multiplying real numbers by using inductive reasoning.
- Evaluate products and quotients of real numbers.

Instruction: Begin instruction by choosing 9 students to put up the homework questions on the board. (One student for each problem) Only spend about one or two minutes on each. If the students don’t understand the answer makes sure to show them on the number line. Answers for homework: 1. 7 2. 5 3. -21 4. 0.9 5. 48 6. \(\frac{5}{8}\) 7. 4.6 8. -11 9. $48.54$

After going over the homework, pass back the quizzes and go over the answers. The answers for the quiz are 1. \(\frac{7}{4}\) 2. 5n-2 3. 150 more than the product of 12 and the number of students n; 12n + 150 (students could have used any variable not just n) 4. \(\frac{24}{7}\) 5. 12 6. 256 7. Irrational numbers; rational numbers and integers; rational numbers; -4, \(\frac{4}{3}\), \(\sqrt{105}\), 8. Associative Property of Addition 9. 10; the natural numbers are whole numbers starting with 1, therefore -10-0 do not count in the set. 10. (y * x) * z; z * (x * y). If the students have any questions go over them together on the board.
Next we will move onto the handout, “Multiplying and Dividing Real Numbers” (Found in Appendix B). Instruct students to take out their calculators. First we will try to notice some patterns of multiplying real numbers. Look at the first table with the students and go over. Students should recognize patterns and answer the following questions:

1. What is the sign of the product of a positive number and a negative number?
2. What is the sign of the product of two negative numbers? (When they answer this questions explain the correlation between the English language and a double negative. Ex. I didn’t see nothing. Which really means, I saw something.)

Multiplication Rules:

1. The product of two positive numbers is positive.
2. The product of a positive number and a negative number is negative.
3. The product of two negative numbers is positive.

Inverse Property of Multiplication: For every nonzero real number $a$, there is a number $\frac{1}{a}$ that $a \cdot \frac{1}{a} = 1$, where $\frac{1}{a}$ is the multiplicative inverse.

Ex. The multiplicative inverse of -4 is $\frac{1}{4}$ because $-4 \left( -\frac{1}{4} \right) = 1$

The multiplicative inverse is an example of a reciprocal. A reciprocal is a nonzero real number of the form $\frac{a}{b}$ is $\frac{b}{a}$.

Division Rules:

1. The quotient of two real numbers with different signs is negative.
   Ex. $-20/5 = -4$
2. The quotient of two real numbers with the same sign is positive.
   Ex. $-20/-5 = 4; 20/5 = 4$
3. The quotient of 0 and any nonzero real number is 0.
   Ex. $0/20 = 0$
4. The quotient of any real number and 0 is undefined.
   Ex. $20/0 = $ undefined

Now go over how to divide fraction. Students should remember to multiply by the RECIPROCAL, multiply fractions across (numerators * numerators/denominators * denominators), then SIMPLIFY! Go over the two example problems, let them do the second one individual and go over to make sure they did it correctly.

\[
1. \quad \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}
\]

\[
2. \quad \frac{-9}{10} \div \frac{4}{5} = \frac{-9}{8}
\]

Assessment/Closure: Answer any questions students might have from the lesson, and instruct that for homework the students must complete “Let’s Practice” on the class handout, “Multiplying and Dividing Real Numbers” (Found in Appendix B).

Day 9 – Friday, September 10, 2013
Lesson: The Distributive Property

NYS Common Core State Standard(s):
Algebra – Seeing Structure in Expressions - Interpret the Structure of Expressions
A.SSE.1.a – Interpret parts of an expression, such as terms, factors and coefficients.

Objective: By the end of the lesson students will be able to-
- Define the distributive property, constant, coefficient and like terms.
- Model the distributive property.
- Understand that the distributive property can be used to simplify the product of a number and a sum or difference.
- Recognize that an algebraic expression can be simplified by combining the parts of the expression that are alike.

Instruction: Begin the class by going around checking for homework completion while students complete the warm up, which is projected on the board (Smart Board if available). After going over the warm up answers, go over the homework answer, and answer any questions students might have. Answer for homework: 1. -160 2. 3/2 3. -4/81 4. 16 5. -0.9 6. 3 7. -6, 8. -5/9 9. 13 10. -4.

Warm Up:
Evaluate the following expressions:
1. -4 + 9 (answer: 5)  2. 4 - (-6) (answer: 10)  3. -4 * 9 (answer: -36)
4. \[ \frac{1}{2} \div \frac{3}{4} \] (answer: \( \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \))

Pass out the handout for today’s lesson, “The Distributive Property” (Found in Appendix B). Let’s first give the students the definition of the property and then provide an area model.

The **distributive property** can be used to simplify the product of a number and a sum or difference. This is a property of real numbers that helps you simply expression. Here is an area model that shows \( 8(x + 5) = 8(x) + 8(5) \).

\[ \begin{align*}
8 & \quad 8 \\
\times + 5 & \quad x \quad 5 \\
\end{align*} \]

\( 8 \) \( (x + 5) = 8x + 40 \)

Next explain the different Algebraic and Numerical Examples of the Distributive Property.

- \( a (b + c) = ab + ac \) \( \Rightarrow \) \( 7(3 + 2) = 7(3) + 7(2) \)
- \( (b + c) a = ba + bc \) \( \Rightarrow \) \( (3 + 2)x = 3(x) + 2(x) \)
- \( a (b - c) = ab - ac \) \( \Rightarrow \) \( 20(4 - 5) = 20(4) - 20(5) \)
- \( (b - c) a = ba - ca \) \( \Rightarrow \) \( (4 - 5)x = 4(x) - 5(x) \)
Next, model simplifying expressions using the distributive property, and then how to rewrite and simplify expressions with fractions.

Example 1: \(3(x + 8) = 3(x) + 3(8) = 3x + 24\)  
Example 2: \((5b-4)(-1) = 5b(-1) – 4(-1) = -5b + 4\)

Example 3: \(\frac{4x-16}{3} = \frac{1}{3}(4x - 16) = \frac{1}{3}(4x) - \frac{1}{3}(16) = \frac{4x}{3} - \frac{16}{3}\)

When simplifying expressions it is important to know when to combine parts of the expression that are alike, called “Like Terms”. In algebraic expressions a **term** is a number, variable, or product of a number and one or more variables. There are a few types of terms, such as a **constant** (a term that has no variable) and a **coefficient** (a numerical factor in front of a term). For example if we are given the following algebraic expression, \(4x + (-5xy) – 12\), we can classify \(4x\), \(-5xy\), and \(12\) as terms. More specifically, \(4\) and \(-5\) are coefficients, and \(12\) is a constant.

Next do the practice problems by combining like terms to simplify. Try drawing different shapes around each of different types of like terms. (For example, circle any constant terms and put a box around each of the terms that are connected to an \(x\))

1. \(8x + 3x – 2 = 11x – 2\)  
2. \(5x – 3 + 6x + 4 = 11x + 1\)

**Assessment/Closure:** Ask students if they have any questions about the Distributive Property of Combining Like Terms. For homework there is a “Homework” set of problems at the end of the in class worksheet, “The Distributive Property” (Found in Appendix B).

**Day 10 & 11 – Monday, September 11, 2013/Tuesday, September 12, 2013**

**Lesson:** An Introduction to Equations

**NYS Common Core State Standard(s):**
- Algebra– Creating Equations – Create Equations that Describe Numbers or Relationships
- A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**Objective:** By the end of the two day lesson students will be able to-
- Define the key words equations, open sentence, solution of an equation.
- Solve equations using tables and mental math.
- Recognize relationships between quantities and represent these relationships on a table, in an equations and graphically.

**Instruction:** Start off the lesson going over the answer to the questions from the homework. Go around the room, choosing students to give their answer. If any students have different answers or trouble, put the question up on the board and go through the step by step process.

*Answer from homework:* 1. \(6a + 60\) 2. \(90 – 10t\) 3. \(4.5 – 12c\) 5. 13 6. \(2x/5 + 7/5\) 7. \(5-8t/5\) 8. \(20x\) 9. \(3n\) 10. \(-3x + y + 11\).
Hand out the classwork packet, “An Introduction to Equations” (Found in Appendix B). This packet will be used on both day 10 and 11, so make sure you let the students know that they must make sure to keep this with their math material and bring to class both days. First we will define an equation. An **equation** is a mathematical sentence that uses an equal sign (=). Equations are used to represent the relationship between two quantities that have the same value. An equation is only true if the quantities on each side of the equal sign have the same value. An equation can be classified as an **open sentence** if it contains one or more variables and may be true or false depending on the values of its variables.

Next you will classify equations as true, false, or open and explain to the class this reasoning.

**Example 1:** 24 + 18 = 20 + 22 (True; because each expression equals 42)
**Example 2:** 6 * 4 = 22 (False; 6 * 4 = 24 which is not equal to 22)
**Example 3:** 2x – 10 = 40 (Open; because there is a variable)

If an equation contains a variable then there is a **solution of an equation**, which is a value for that variable that makes the equation true. Next, model for the class how to find solutions of an equations.

**Example 1:** Is x = 6 a solution of the equation 32 = 3x + 12?
-We want to substitute 6 for x to get 32 = 3(6) + 12 = 18+12 = 30. Since 32 ≠ 30, we can say that x ≠ 6 and that x = 6 is not a solution of the equation 32 = 3x + 12.

Now let’s try writing an equation given a real-life situation.

**Example 2:** An art student wants to make a model of the Mayan Great Ball Court in Chichen Itza, Mexico. The length of the court is 2.4 times its width. The length of the student’s model is 54 in. What should the width of the model be?
-Let’s try 2.4 in, 22.5 in, and 11.25 in. First we must figure out what equation to use by reading through the word problem. 54 = 2.4w, where w is the width for the model. Now let’s plug each of the three terms into the equation and find which one is true.

Check (2.4 in): 54 = 2.4(2.4)     Check (22.5 in): 54 = 2.4(22.5)     Check (11.25 in): 54 = 2.4(11.25)
54≠5.76     54 = 54 ✔     54 ≠ 27
Therefore we have proven that w = 22.5 in to complete the art students model!

Let’s try finding solutions using mental math. For examples 3 and 4 let’s find the solutions of the equation using mental math. Make sure to always check your answer by plugging it back in for the equation.

**Example 3:** x + 8 = 12 → Think, what number plus 8 equals 12? Answer: 4 (check!)
**Example 4:** \( \frac{a}{8} = 9 \) → Think, what number divided by 8 equals 9? Answer: 72 (check!)
Let’s try using a table to find a solution of an equation. For the following examples use a table and mental math to starting values. Do example 5 with the class, let them do example 6 in groups. (Give them 5-8 minutes) Go over as a class.

### Example 5: What is the solution of $5n + 8 = 48$?

<table>
<thead>
<tr>
<th>( n )</th>
<th>(-9x-5)</th>
<th>(5n+8) Value of (-9x-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-9(-1)-5</td>
<td>33</td>
</tr>
<tr>
<td>-2</td>
<td>-9(-2)-5</td>
<td>38</td>
</tr>
<tr>
<td>-3</td>
<td>-9(-3)-5</td>
<td>43</td>
</tr>
<tr>
<td>-4</td>
<td>-9(-4)-5</td>
<td>48</td>
</tr>
</tbody>
</table>

Since we want to find a solution that makes the equation equal 28 we notice that 28 is between 22 and 31, therefore we can estimate that our solution is between -3 and -4!

### Example 6: What is the solution of $25 - 3p = 55$?

<table>
<thead>
<tr>
<th>( p )</th>
<th>(-9x-5)</th>
<th>(5n+8) Value of (-9x-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>25-3(-6)</td>
<td>43</td>
</tr>
<tr>
<td>-7</td>
<td>25-3(-7)</td>
<td>46</td>
</tr>
<tr>
<td>-8</td>
<td>25-3(-8)</td>
<td>49</td>
</tr>
<tr>
<td>-9</td>
<td>25-3(-9)</td>
<td>52</td>
</tr>
<tr>
<td>-10</td>
<td>25-3(-10)</td>
<td>55</td>
</tr>
</tbody>
</table>

From the table we can see that a solution for the equation $5n+8$ is 8. Also, notice the pattern as \( n \) increases by 1 that the value of the equation goes up by 5.

Let’s use a table to estimate a solution. If we identify a pattern in the values of an expression we can make an estimate. Model how to do example 7, then let them do example 8 in small groups. (Give them 5-8 minutes) Go over as a class.

### Example 7: What is an estimate of the solution of $-9x – 5 = 28$?

- First we must find the integer values of \( x \) between which the solution must lie
  - $-9(0) – 5 = -5$ and $-9(1) – 5 = -14$.
- So we can make the conclusion that as \( x \) become greater than 0 the value of the equation is going down. We want to find a solution that makes our equation 28 therefore we should start with -1 since it is lower than 0. Now let’s make a table.

<table>
<thead>
<tr>
<th>( n )</th>
<th>(5n+8) Value of (-9x-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>13</td>
</tr>
<tr>
<td>-3</td>
<td>22</td>
</tr>
<tr>
<td>-4</td>
<td>31</td>
</tr>
</tbody>
</table>

From the table we can see that a solution for the equation $5n+8$ is 8. Also, notice the pattern as \( n \) decreases by 3 that the value of the expression goes down by 3. So we see that -10 is a solution of the equation $25-3p$. We should recognize that the solution needs to be a negative number because 55 is greater than 25 so we need to multiply -3 by a negative to switch the equation to addition.

Example 7: What is an estimate of the solution of $-9x – 5 = 28$?
- First we must find the integer values of \( x \) between which the solution must lie
- $-9(0) – 5 = -5$ and $-9(1) – 5 = -14$.
- So we can make the conclusion that as \( x \) become greater than 0 the value of the equation is going down. We want to find a solution that makes our equation 28 therefore we should start with -1 since it is lower than 0. Now let’s make a table.
Example 8: What is an estimate of the solution of $3x + 3 = -22$? Use a table.

<table>
<thead>
<tr>
<th>n</th>
<th>$3x + 3$</th>
<th>Values of $3x + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>3(-6) + 3</td>
<td>-15</td>
</tr>
<tr>
<td>-7</td>
<td>3(-7) + 3</td>
<td>-18</td>
</tr>
<tr>
<td>-8</td>
<td>3(-8) + 3</td>
<td>-21</td>
</tr>
<tr>
<td>-9</td>
<td>3(-9) + 3</td>
<td>-24</td>
</tr>
</tbody>
</table>

Since -22 is between -21 and -24 we can estimate that the solution for the equation is between -8 and -9.

**Assessment/Closure:** At the end of each day be sure to ask if the students have any questions on what they’ve learned that day (after day 11 ask if they have questions on anything from either day 10/11). The homework is attached to the end of the “An Introduction to Equations” (Found in Appendix B) worksheet and covers both day 10 and 11. This will not be due until day 12, but students should be instructed to begin working on this homework after day 10. This will ensure that if they have any questions on the material covered on day 10 that they can ask it on day 11. There is not any homework due on day 11.

**Day 12 – Wednesday, September 13, 2013**

**Lesson:** Connecting Patterns, Equations and Graphs

**NYS Common Core State Standard(s):**
- Algebra– Creating Equations – Create Equations that Describe Numbers or Relationships
  - A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Algebra– Reasoning with Equations and Inequalities – Represent and Solve Equations and Inequalities Graphically
  - A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a straight line).

**Objective:** By the end of the lesson the students will be able to-
- Use tables, equations and graphs to describe relationships and relate these findings for linear equations.
- Identify solutions of a two-variable equation.

**Instruction:** The class will start by go over the answers for the homework on “An Introduction to Equations.” This will be reviewed on Day 13 so do not spend longer than 5 minutes on this in the beginning of class. **Homework answer:** 1. False 2. True 3. Open 4. Yes 5. No 6. Yes 7. $4x + (-3) = 8$ 8. $115d = 690$ 9. 13 10. 5 11. 6 12. Between -4 and -5 13. An expression describes the relationship between numbers and variables. An equation shows that two expressions are equal. An expression can be simplified but has no solution. 14. -6 15. 0
Hand out the “Patterns, Equations, and Graphs” packet (Found in Appendix B), and students will use patterns with ribbons and kites to familiarize themselves with two-variable equations. We show the students a table that connects the relationship between the number of kites and the total number of ribbons on the kites’ tail. The purpose of this exercise is to get the students to induce how many kites we could have that would make us have 275 ribbons. If every kite has 5 ribbons on its tail then we can notice a pattern. Following the pattern we can figure out that if we have 275 ribbons we can have 55 kites. Let’s use this knowledge to further the student’s knowledge.

Next we will identify solutions of a Two-Variable Equation. You can use an equation with two variables to represent the relationship between two varying quantities. A solution of an equation with two variables x and y is any ordered pair (x, y) that makes the equation true. Do Example 1 as a class, and then have the students do Example 2 individually. It should only take 2-3 minutes then go over as a class.

Example 1: Is (3, 10) a solution of the equation y = 4x?

\[
y = 4x \\
10 = 4(3)
\]

10 ≠ 12 - we see that (3, 10) is not a solution of y = 4x

Example 2: Is the ordered pair (5, 20) a solution of the equation y = 4x?

\[
y = 4x \\
20 = 4(5)
\]

20 = 20 - we see that (5, 20) is a solution of y = 4x

We can represent the same relationship between two variables in many different ways. Let’s try with a real-world situation. Both Carrie and her sister Kim were born on October 25th, but Kim was born 2 years before Carrie. How can you represent the relationship between the girl’s ages in different ways?

First show them to make a table with the values that are given. We know that Kim was born 2 years before Kim so when Carrie was 1, Kim was 3, etc.

| Carrie's and Kim's Ages (years) |
|---|---|---|---|---|---|---|---|---|---|
| Carrie | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Kim    | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

Now help the students write an equation for the relationship. First we have to set out variables. Let’s say that x = Kim’s age and let’s say that y = Carrie’s Age. From the table we just made we can see that y is always 2 greater than x (because Carrie is 2 years older than Kim). Therefore we can say write an equation y = x +2, which represents the difference between Carrie’s and Kim’s age.

Finally, we can graph the relationship. We will do this on the coordinate place only displaying quadrant 1. We will put Kim’s Age along the y-axis and Carrie’s age along the x-axis.

Next you will have the students try making a table, equation and graph in pairs for “Noticing Patterns” sections making connections between stars and how many points they have.
The students should be given 10-15 minutes to complete this problem, and then choose three pairs of students (one to make the table, one for the equation, one for the graph) to put their solutions on the board. The answers should look like the following:

**Table –**

<table>
<thead>
<tr>
<th># of Stars</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Points</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

**Equation –** Let \( y = \) stars, \( x = \) points; therefore we can say \( y = 5x \).

**Graph –** Make sure the students label their axis correctly and plot the linear equation correctly.

**Assessment/Closure:** The students have a “Homework” section of the “Connecting Patterns, Equations and Graphs” handout (Found in Appendix B). Make sure to answer all students questions about today’s lesson and inform them that tomorrow’s lesson will be a review class, and the Unit Test will be the following day. Students were told in the beginning of the Unit that this was the date of the Unit Test. This is to ensure that the students have ample time to do any make up work that they may have missed due to absences, or to get extra help outside of class.

**Day 13 – Thursday, September 14, 2013**

**Lesson:** Unit Review – Stations in Groups

**NYS Common Core State Standard(s):**

- Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
  - N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
- Algebra – Seeing Structure in Expressions - Interpret the Structure of Expressions
  - A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.
- Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
  - A.SSE.1.a – Interpret parts of an expression, such as terms, factors and coefficients.
- Algebra – Creating Equations – Create Equations that Describe Numbers or Relationships
  - A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Algebra – Creating Equations – Create Equations that Describe Numbers or Relationships
  - A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Algebra – Reasoning with Equations and Inequalities – Represent and Solve Equations and Inequalities Graphically
  - A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a straight line).
**Objective:** By the end of the lesson the students will be able to –
- Represent quantities, patterns, and relationships through writing algebraic expressions and equations, making tables, and graphing.
- Define all the key vocabulary from Unit 1 – Connecting Patterns and Equations

**Instruction:** Tables will be set up into 4 different groups (depending on how large your class, split up the class in advance into 4 even groups to ensure this activity goes smoothly). Go over the homework from Day 12 lesson, only take about 5 minutes. You will need as much time as possible to do ensure that each group of students gets to complete each station. The answers for the homework is 1. Yes 2. No 3. Yes 4. Yes 5. $y = \frac{1}{3}x$

<table>
<thead>
<tr>
<th># of sides</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td># of triangles</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Answer any questions from the students then explain that they will be doing review in stations for this lesson. They will move from station to station with their group and spend 10 minutes at each station. (Make sure you keep time on some sort of timer. Let the students know when they have 5 minutes left, and then 1 minute left. They may not move to the next station until the full 10 minutes have elapsed). Hand out the “Review of Unit 1” packet (Found in Appendix B) that the students must use throughout their group work in stations to record their answers and show their work. Make sure to give explicit directions as to which station each group is starting at, and the order they will be moving. Start the clock and let the students start their first station. If necessary you can shorten the time to ensure students finish each station just make modifications based on the needs of your class. As the students are completing the stations the teachers should circulate around the room and help each group as needed.

**STATION 1 – Adding, Subtracting, Multiplying and Dividing with Manipulatives**
For this station the students will be given a set of problems and have to use the manipulative blocks on the table to figure out the problem set. This will help students visualize the numbers and how positive and negative numbers relate to the operations. Inform the students that they should use the single manipulative blocks for adding and subtracting, and then the larger groups of blocks for multiplying and dividing. The different colored blocks (red/black) should be used to signify positive (black) and negative (red) blocks.

**STATION 2 – Identifying Properties of Real Numbers**
For this station the students will be given scissors and tape and have to identify the properties shown on their paper. (The very last page of the packet should be torn off since this is what they will be cutting up.) The students should help each other figure out which expression represents which property of real numbers.

**STATION 3 – Evaluating Expressions using Order of Operations**
For this station the students are given a set of problems that they must evaluate using PEMDAS! The students must write each step that they perform (Order of Operations) as they evaluate each expression. For example if they are given $4^2 + 4$ they should say first we will
evaluate the exponent to get 16 + 4, then we will add to get our simplified answer of 20. This will ensure that students can explain their thought process for their problem solving.

STATION 4 – Recognizing Patterns Representing Relationships as Equations, Tables & Graphs

Students will be given a set of problems in which they must decide whether the given ordered pair is a solution of the corresponding equation for that question. Students then must complete one real-life situation in which they must recognize a pattern between two variables and represent the relationship as an equation, on a table and on a graph.

Answers for the Review Packet:
Station 1: 1. 5, 2. 13, 3. -13, 4. -60, 5. -4, 6. 40, 7. -9, 8. -11
Station 3: the equation is \( y = 2x \) with the corresponding graph and the following table.

<table>
<thead>
<tr>
<th># of houses</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of windows</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Station 4: 1. \( \frac{8}{3} \), 2. 123, 3. 16, 4. 225, 5. -56

Assessment/Closure: Once the students have finished each station, go over the answers out loud as a class. (Make sure to check and correct the students work as you are circulating to make sure they leave class with all of the correct answers.) Ask students if they have any questions about anything from Unit 1. Students will receive an informal assessment grade for their participation in the station activity and how well they worked in groups. Instruct students to go over their note packets and homework from Days 1-12, their quiz, and review packet (. The Unit Test will be next class.

Day 14 – Friday, September 15, 2013

Lesson: Unit Test

NYS Common Core State Standard(s):
Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
N.RN.3 – Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
Algebra – Seeing Structure in Expressions - Interpret the Structure of Expressions
A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.
Number and Quantity – The Real Number System – Use Properties of Rational and Irrational Numbers
A.SSE.1.a – Interpret parts of an expression, such as terms, factors and coefficients.

Algebra– Creating Equations – Create Equations that Describe Numbers or Relationships
A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Algebra– Reasoning with Equations and Inequalities – Represent and Solve Equations and Inequalities Graphically
A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a straight line).

**Objective:** Today’s lesson will be a cumulative test on the material from Unit 1. Students will demonstrate their individual understanding of Connecting Patterns and Equations through completing the Unit 1 Test (Found in Appendix B).

**Instruction:** Students may have the first 5 minutes to ask any last minute questions they might have on any of the material from Unit 1. Once there are no more questions instruct students that all they may have on their desk is a calculator and pencil. Hand out the Unit 1 and students may start completing the test.

**Assessment/Closure:** Students may hand in their test once they are finished and grab the handout that will be their homework. The homework will be an activity that introduces them to the next unit, “Solving Equations.”
Answer Key for Unit 1 Test:
1. two or more than the product of -12 and t
2. \( \frac{p}{97} \)
3. 54
4. \( -\frac{5}{6} \)
5. false (examples vary)
6. yes (work needed for full credit)
7. \( -\frac{4}{5}, -\frac{7}{8}, -\frac{13}{16}, \frac{7}{4} \)
8. 16
9. rational numbers
10. distributive property
11. no (work needed for full credit)
12. \( p = 11t \) (graph should have this line);
187 players on 17 teams.

Players in Soccer League

<table>
<thead>
<tr>
<th># of teams</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of players</td>
<td>11</td>
<td>22</td>
<td>33</td>
<td>44</td>
</tr>
</tbody>
</table>

13. 104°F
14. false (examples vary)
15. -5ab - 2
Chapter 4

Validity of Curriculum Project

The content validity for the curriculum project is addressed through the implementation of the unit on Connecting Patterns and Formulas. Content validity is a measurement used to define the domain of what is being measured (Rungtusanatham, 1998). Two veteran teachers were asked to implement this curriculum in their classrooms in order to establish a measure of the content validity of the curriculum. The Algebra I curriculum and was taught by two veteran teachers in September, 2013. The two 9th grade mathematics teachers each taught in Central New York suburban high schools, which is where they implemented the unit plan.

Immediately after the unit plan was implemented the two teachers reflected on the unit, and completed a questionnaire (Appendix A) that answered the following questions:

1. What is your general feelings toward the Algebra I Unit on Connecting Patterns and Formulas? Please include both positive and negative thoughts.
2. What were the most successful aspects of the unit? Least successful aspects?
3. Did you feel that you had enough time to complete the unit? Was it too much time?
4. What, if anything, would you change about the unit? Give specific examples.

The questionnaire was used by the author of the curriculum design in order to measure the content validity of the unit. Once the reflections were completed, submitted and reviewed by the author, the author of the unit noted these recommendations for possible changes to the unit in the future. The presentation and discussion of the questionnaire is discussed in Chapter 5.
Chapter 5

Reflections and Conclusions

The inspirations for the design of the curriculum project were the paradigm shift from the NCTM standards to the CCSS, and the prospect of creating a design that evoked creativity, deep meaningful learning among students and actively engaged students. The resources used were inspired by the NYS CCSS shift and included pacing recommendations (Pearson, 2012). Within the resources the section on modeling in the mathematics classroom was highly stressed, and by implementing visual aids, media, and technology within the lessons the curriculum design sought to expand and elevate the resources (Pearson, 2012; NYSED, 2013).

In addition to aligning the curriculum to the NYS CCSS standards the curriculum design also includes real-life application problems in most of the lesson plans throughout the unit in order to elevate the students learning of the content. By applying mathematical modeling throughout the unit students can construct real-life situations to show their deeper understanding, which is one of the standards described in the NYS CCSS (Pearson, 2012; NYSED, 2013). One of the main reasons for the paradigm shift from the NCTM Standards to the CCSS was to elevate the mathematics learning standards and education in the United States to better compete with the mathematics education in other countries (NCTM, 2000; NCTM, 2009; Woolard, 2012; NYSED, 2013).

Modeling in the classroom is one of the main components in addition to sense making of problems, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, strategically using tools with precision, looking for patterns and structure when formulating solutions, and effectively looking for patterns in calculations (NCTM, 2009;
Pearson, 2012; NYSED, 2013). The curriculum design focuses on the eight components of the NYS CCSS with hopes that students receive a deep level of learning in the classroom.

**Findings: Comments from Questionnaire**

The curriculum design was implemented in two Central New York suburban 9th grade classrooms by two veteran teachers. Upon completing the implementation of the curriculum design both of the teachers from the Central New York suburban classrooms wrote a reflection and answered the questionnaire (Appendix A).

After reviewing the responses and reflections from the two participating teachers, the author of this curriculum project was pleased with the feedback from the teachers that implemented the curriculum. Both teachers had similar experiences, and successfully completed the unit in the allotted 3 weeks without many issues with the curriculum design. The teachers felt that the curriculum was very easy to follow, and that they did not need to make too many modifications throughout the 3 week unit. Teacher 1 used the Day 1 activity in order to get to know the students and said:

My students loved the ‘Getting to Know Each Other’ activity on the first day of school. I was able to make my example before the first day of classes and the students really enjoyed being creative. It was a great ice-breaker and got the freshman comfortable and eager for what was in store for the rest of the year (Direct quote from Teacher 1’s response from questionnaire, found in Appendix A).

Each of the teachers found that the pre-assessment was a great indicator to which students retained knowledge from middle school and those who were a little behind, which was the intention of this activity.
Both teachers were fortunate enough to have Smart Board technology in their classroom, which made it much easier to project the hand-outs on the board and the board while they taught the lessons. Teachers 2 changed the way that she graded the homework, and collected homework each day for a grade instead of merely checking it with the suggested format of Below Average Participation (-), Average Participation (✓), and Above Average Participation (+). Teacher 2 found that this helped the students take homework seriously and really ensured that he knew the students’ strengths and weaknesses throughout the unit.

At the end of the unit both cooperating teachers said that the students in all of their classes performed at high levels. Only one of the teachers included the class averages for her students. She explained that her students, this year, performed at a much higher level than those of past years. She further reported that she teachers three sections of 9th grade Algebra 1 and the class averages at the end of Unit 1 were 92, 86, and 82. She also included that the students were challenged by the real-world application problems, but through practice and modeling during the unit the student’s performed proficiently on the Unit Test. One of the teacher’s commented on how impressed he was with the student responses on the critical thinking questions on the Unit Test. He expressed having concerns with including that on the test, but was pleasantly surprised when he received responses that were creative, connecting to real-life scenarios, and accurate.

All around both cooperating teachers said that they would continue to use this curriculum design in their classroom, and follow the general design to elevate their curriculum throughout the entire year of Algebra 1. Teacher 2 explained, “I have been struggling to ensure that I have been correctly aligning my lessons with the new CCSS, and this unit plan in addition to the resources supported by the design have opened my eyes to the possibilities in the classroom.” Teacher 1 commented:
I am a very technologically savvy person and really like to incorporate technology in my classroom as a way to enhance modeling mathematical content. This curriculum design incorporated the Smart Board and a few websites that would elevate learning, but I wish it had a little bit more. That was really the only component I would add to make this great curriculum design even greater.

**Conclusions: Final Thoughts and Reflection**

After reviewing the thoughts, comments, and concerns of each of the cooperating teachers it is clear that the curriculum design was instructionally successful; more so for one of the teachers than the other. The main focus for this design was to ensure that the curriculum aligned with the CCSS, which both teachers expressed was achieved, but one of the areas that needed improvement was to include more activities utilizing a wider range of technology. The constructive criticism from both of the cooperating teachers can be taken into account and used to add activities throughout the curriculum design to elevate the students learning while still following the CCSS.
References


Appendix A

Questionnaire for teachers who implement the Unit Plan on Connecting Patterns and Equations for Algebra 1.

Instructions: Please write a reflection about your experience using the Unit Plan on Connecting Patterns and Equations created by Kristina R. Graziadei for the Algebra I classroom curriculum. Feel free to add anything else you experienced that might not be covered by merely answering the following questions.

1. What is your general feelings toward the Algebra I Unit on Connecting Patterns and Functions? Please include both positive and negative thoughts.

2. What were the most successful aspects of the unit? Least successful aspects?

3. Did you feel that you had enough time to complete the unit? Was it too much time?

4. What, if anything, would you change about the unit? Give specific examples.
Appendix B

Unit Assessments: Handouts, homework, quiz, and tests for Unit Plan on Connecting Patterns and Equations for Algebra 1. (p. 47-82)
**Facts About ________________**

**My Birthday is ________________**

Directions: In each box, draw pictures and explain in one sentence a little about yourself for that category. Please be creative!! Be prepared to share with the class once everyone is done.

<table>
<thead>
<tr>
<th>Photo of Yourself</th>
<th>Favorite Hobbies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Favorite Candy</th>
<th>Dream Job</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fun Fact</th>
<th>Fun Fact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Pre-Algebra Review Packet

Directions: Read each question, then circle the letter of the correct answer. Make sure you show your work in the side margins so you can explain your work.

1. Which equation represents the phrase “seven more than a number is 37?”
   A. $7 - x = 37$  
   B. $x + 7 = 37$  
   C. $37x + 7$  
   D. $7x + 37$

2. Which set below is the domain of $\{(1, -4), (2, 8), (3, -1), (4, 0), (0, 5)\}$?
   A. $\{1, 2, 3, 4\}$  
   B. $\{-4, 8, -1, 0, 5\}$  
   C. $\{-4, 8, -1, 0, 5\}$  
   D. $\{0, 1, 2, 3, 4\}$

3. The Rodgers have chickens and cows on their farm. If there are 23 animals and a total of 74 legs, how many of each type of animal are there?
   A. 4 chickens, 19 cows  
   B. 19 chickens, 4 cows  
   C. 14 chickens, 9 cows  
   D. 9 chickens, 14 cows

4. What is $43.2 \times 10^4$?
   A. $432,000$  
   B. $4,320,000$  
   C. $0.0432$  
   D. $0.00432$

5. Erica had $50 she put into a savings account. If she saves $15 per week for one year, how much will she have saved altogether?
   A. $780$  
   B. $50$  
   C. $830$  
   D. $35$

6. What equation do you get when you solve $2x + 3y = 12$ for $y$?
   A. $y = -\frac{2}{3} + 4$  
   B. $y = \frac{3}{2} + 12$  
   C. $y = -2 + 12$  
   D. $y = 4 - 3x$
7. What is the formula for the circumference of a circle solved for \( r \)? (Recall \( C = 2\pi r \))

A. \( r = C \cdot 2\pi \)  
B. \( r = \frac{C}{2\pi} \)  
C. \( r = 2\pi \)  
D. \( r = \frac{C\pi}{2} \)

8. What is the solution of \(-3p + 4 < 22\)?

A. \( p < -6 \)  
B. \( p < 18 \)  
C. \( p > -6 \)  
D. \( p > -18 \)

9. Which two whole numbers does \( \sqrt{85} \) fall between?

A. 9 and 10  
B. 41 and 42  
C. 8 and 9  
D. 42 and 43

10. Which of the following expressions is equivalent to \( \frac{4^3}{4^6} \)?

A. \( \frac{1}{4^3} \)  
B. \( \frac{1}{4^2} \)  
C. \( 4^3 \)  
D. \( 4^2 \)

11. What is the simplified form of the expression \( \frac{6+3^2}{(2^3)(3)} \)?

A. \( \frac{1}{2} \)  
B. \( \frac{5}{8} \)  
C. \( \frac{1}{3} \)  
D. \( \frac{5}{6} \)

12. A 6 ft tall man casts a shadow that is 9 ft long. At the same time, a nearby tree casts a 48 ft shadow. How tall is the tree?

A. 72 ft  
B. 45 ft  
C. 36 ft  
D. 32 ft
13. Your grades on four exams are 78, 85, 97, and 92. What grade do you need on the next exam to have an average of 90 on the five exams?

A. 100  C. 98
B. 92    D. 90

14. What is the equation of the line in the diagram below?

A. \( y = \frac{3}{4} + 4 \)
B. \( y = -x + 3 \)
C. \( y = \frac{4}{3} x + 4 \)
D. \( y = \frac{4}{3} x + 4 \)

15. What is the ordered pair of the solution of the system of equations graphed below?

A. (1.5, 1)
B. (1, 1.5)
C. (1, 3)
D. (2, 3)
Variables and Expressions

Key Vocabulary:
- **Quantity** – anything that can be measured or counted
- **Variable** – a symbol, for example a letter, which represents the value of a quantity
- **Algebraic Expression** – a mathematical phrase that includes one or more variables
- **Numerical Expression** – a mathematical phrase involving numbers and operation symbols, but no variables

Let’s Write Expressions!
*Remember... an expression is a group of numbers, symbols and operators (+, -, etc.) that show the value of something.*

Let’s try one together...

Example 1: What is an algebraic expression for 23 more than a number \( n \)?

Model →

|---------------n+23----------------------|

\( n+23 \) is the algebraic expression

Now you try!

Example 2: What is an algebraic expression for 18 less than a number \( n \)?

Model →

The algebraic expression is ________________.

Example 3: What is an algebraic expression for 8 times a number \( n \)?

Model →

The algebraic expression is ________________.

Example 4: What is an algebraic expression for the quotient of a number \( n \) and 5?

Model →

The algebraic expression is ________________.
*CHALLENGE QUESTION*

Example 5: What is an algebraic expression for 3 more than twice a number \( x \)?

The algebraic expression is ________________.

**Let’s use our words!**

Example 6: What word phrase represents the algebraic expression \( 3x \)?

\( \rightarrow \) 3 times a number \( x \)  \( \rightarrow \) the product of 3 times a number \( x \)

(Either of these answers are acceptable)

Example 7: What word phrase represents the algebraic expression \( 5x + 8 \)?

__________________________________________________

**Describing Patterns in the Real World**

Example 8: Give any regular polygon, you can draw a segment from any one vertex to the other vertices. Each of these segments cut the regular polygon into non-overlapping triangles. Using the table below, give the rule and algebraic expression that describes the pattern.

<table>
<thead>
<tr>
<th>Number of Sides of the Regular Polygon</th>
<th>Number of Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4 – 2 = 2</td>
</tr>
<tr>
<td>5</td>
<td>5 – 2 = 3</td>
</tr>
<tr>
<td>6</td>
<td>6 – 2 = 4</td>
</tr>
<tr>
<td>( n )</td>
<td>?</td>
</tr>
</tbody>
</table>

**Homework:**

1. Classify each of the following expressions as numerical or algebraic.
   a. \( 7 + 4 \)  
   
   ________________

   b. \( 4x – 19 \)  
   
   ________________

2. Write the algebraic expression for the quotient of \( n \) and 20. ________________

3. Write the word phrase for the algebraic expression \( 3y + 14 \). ________________
4. Use the table to decide whether $49n + 0.75$ or $49 + 0.75n$ represents the total cost to rent a truck that you drive $n$ miles.

<table>
<thead>
<tr>
<th># of Miles</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$49 + (0.75 \times 1)$</td>
</tr>
<tr>
<td>2</td>
<td>$49 + (0.75 \times 2)$</td>
</tr>
<tr>
<td>3</td>
<td>$49 + (0.75 \times 3)$</td>
</tr>
<tr>
<td>n</td>
<td>?</td>
</tr>
</tbody>
</table>

The algebraic expression that best represents the table is

5. Cheryl writes a word phrase “the quotient of $h$ and 8” to describe the expression $\frac{8}{h}$. Explain the students’ error and write the correct word phrase.

6. John and Sidney are wrapping Christmas present at the same pace. John starts first. Once John has finishes wrapping 2 presents, Sidney starts wrapping presents. Write an algebraic expression that represents the number of boxes Sidney will have wrapped when John has wrapped $p$ presents.
Evaluating Expressions
(Order of Operations)

Key Vocabulary:

**Power** – a number that represents repeated multiplication containing an *exponent* and a *base* number. *(Example - 9^4 where 9 is the base and 4 is the exponent, also represented by 9 x 9 x 9 x 9)*

**Simplify** – replacing a numerical or algebraic expression with its single or lowest numerical value.

**Evaluate** – replacing each variable in an algebraic expression with a number, then simplifying the expression using the Order of Operations.

Simplify each of the following expressions. Show the repeated multiplication for each example.

Example 1: \(5^6 = \)

Example 2: \((1.4)^3 = \)

Example 3: \(\left(\frac{4}{9}\right)^4 = \)

Simplify each of the following expressions. Show your work. (Remember PEMDAS)

Example 4: \((6-5)^2 \times 8 = \)

Example 5: \(3 \times 7 - 3^3 = \)

Example 6: \(12 - 25 \div 5 = \)

Example 7: \(\frac{4+3^4}{7-2} = \)

Evaluate each of the following expressions for \(x = 4\) and \(y = 2\).

Example 8: \(x^2 + x - 12 \div y^2 = \)

Example 9: \(89 - 2x^2 + 18y = \)
For Homework:

Directions: Complete the follow questions and evaluate the real-world algebraic expressions.

1. Write an algebraic expression for the amount of change you will get when you pay for a purchase $p$ with a $20 bill. Make a table to find the amounts of change you will get for purchases of $11.59, $17.50, $19.00, and $20.00.

2. An object’s momentum is defined as the product of its mass $m$ and velocity $v$. Write an algebraic expression for the momentum of an object. Make a table to find the momentums of a vehicle with a mass of 1000 kg moving at a velocity of 15 m/s, 20 m/s, and 25 m/s.
### Real Numbers (Number Line)

**Classifying Numbers**

Directions: For the following chart write down the definition and example of the key term for that row.

<table>
<thead>
<tr>
<th>Key Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radicand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perfect Square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Element of a Set</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rational Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irrational Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inequality</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Some numbers can be more than one type of number...

The Real Number Line...
The real number line is used to graph and order real numbers from least to greatest. Let’s try to put a few sets of numbers on a number line. Classify the sets of numbers.

Steps to follow-
1. Classify the set of numbers.
2. Organize the numbers from least to greatest.
3. Plot on the number line and label each number.

Set #1 \{9, 1, 5, 0\} - ________________________

Set #3 \{\sqrt{2}, 0.4, -1.4, \frac{2}{3}\} - ________________________
Day 5

**Real Numbers and the Number Line**

Directions: You will create a number line that lists numbers from greatest least to greatest. Your number line should include:

2 – Whole Numbers: _____  

2 - Natural Numbers: _____  

2 – Irrational Numbers: _____________  

2 – Rational Numbers: ______________  

Put numbers in order from least to greatest:

Plot numbers on the Real Number Line and Label:
Properties of Real Numbers

Directions: Fill in the definitions for the Properties of Real Numbers. Include the algebraic expression and an example.
Let’s Practice!
Name the property for each statement.

1. $85 + 4 = 4 + 85$  
2. $0.5 \times 1 = 0.5$  
3. $J + 0 = J$  
4. $68 \times 0 = 0$

5. $653 \times 348 = 348 \times 653$  
6. $10 \times (-1) = -10$

7. $21 + 6 + 9$  
8. $0.1 + 3.7 + 5.9$  
9. $8 + (9t + 4)$  
10. $(12 \times r) \times 13$

Deductive Reasoning and Counterexamples...

For the following examples decide if the statement is true or false using deductive reasoning. If the example is false give a counterexample.

Example 1: For all real numbers $a$ and $b$, $a + b = a \times b$.

Example 2: For all real numbers $x, y,$ and $z$, $x \times (y \times z) = (x \times y) \times z$.

Example 3: For all real numbers $a$, $a \times 0 = a$. 
Write an algebraic expression for each word phrase.

1. a number \( p \) divided by 9. _______
2. 2 less than the product of 5 and \( n \). _______

Write the rule for the follow table of information as an algebraic expression and in words.

3. The table shows how the total cost of a field trip depends on the number of students. What is a rule for the total cost of the ticket?

<table>
<thead>
<tr>
<th>Field Trip</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Students</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>60</td>
</tr>
</tbody>
</table>

Simplify each expression.

4. \( 24 \div (3 + 2^2) \)  
5. \( \sqrt{144} \)

Evaluate the following expression for the values \( a = 2 \) and \( y = 4 \).

6. \( (4a)^3 \div (b - 2) \)

Name the subset(s) of real numbers to which each number belongs. Then order the numbers from least to greatest.

7. \{\sqrt{105}, -4, \frac{4}{3}\}

8. Name the property shown in the following equation.

\[ (5 + 8) + 11 = 5 + (8 + 11) \]

9. How many natural numbers are in the set of number from \{-10, ..., 10\}? Explain your answer.

10. Use the Commutative Property of Multiplication to rewrite the expression \((x * y) * z\) in two different ways.
Adding and Subtracting Real Numbers

Rules for Adding and Subtracting Real Numbers

1. If the signs are the same keep them the same and add.
   Ex. $2 + 29 = 31$; $-9 + -8 = -17$

2. If the signs are different take the sign of the larger real number and subtract.
   Ex. $-2 + 30 = 28$; $40 + (-79) = -39$

Let’s Try Using a Number Line!

Use a number line to model adding and subtracting real number.

3. $9 + 3 = \text{?}$
   4. $(-8) - 4 = \text{?}$

For Extra Practice... and Loads of Fun!

Log onto the following website for extra practice:

http://www.ictgames.com/mummyNumberLine/mummyNumberLine.html

On this website you use a number line to add and subtract real numbers. Read the directions at the bottom on the page of the website to remind yourself how to play. Let’s try some in class!!

Homework:

Directions: Find the sum or difference of each question. Remember, you can draw a number line if that is helpful!

1. $2 + 5 = \text{?}$
2. $-3 + 8 = \text{?}$
3. $-13 - 7 = \text{?}$
4. $8.5 - 7.6 = \text{?}$
5. $36 - (-12) = \text{?}$
6. $\frac{1}{6} - \frac{3}{4} = \text{?}$
7. $5.1 + (-0.7) = \text{?}$
8. $-9 + -2 = \text{?}$
9. A stock’s starting price per share is $51.47 at the beginning of the week. During the week, the price changes by gaining $1.22, then losing $3.47, then losing $2.11, then losing $.98, and then gaining $2.41. What is the ending stock price?
**Multiplying and Dividing Real Numbers**

Look at the following chart and answer each of the problems. Look at the patterns between multiplying positive and negative numbers. After you finish filling out the chart think about these two questions:

- What is the sign of the product of a positive number and a negative number?
- What is the sign of the product of two negative numbers?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2 * 3 = 6</td>
<td>-2 * 3 = -6</td>
<td>-2 * -3 = 6</td>
</tr>
<tr>
<td>5 * 10 = ___</td>
<td>5 * -10 = ___</td>
<td>-5 * -10 = ___</td>
</tr>
<tr>
<td>4 * 8 = ___</td>
<td>-4 * 8 = ___</td>
<td>-4 * -8 = ___</td>
</tr>
</tbody>
</table>

**Multiplication Rules:**
1. The product of two positive numbers is ____________.
2. The product of a positive number and a negative number is ____________.
3. The product of two negative numbers is ____________.

**Inverse Property of Multiplication:** For every nonzero real number $a$, there is a number that $a \left(\frac{1}{a}\right) = 1$, where $\frac{1}{a}$ is the *multiplicative inverse*.

Ex. The multiplicative inverse of -4 is $\frac{1}{-4}$ because $-4 \left(\frac{1}{-4}\right) = 1$

The multiplicative inverse is an example of a *reciprocal*. A reciprocal is a nonzero real number of the form $\frac{a}{b}$ is $\frac{b}{a}$.

**Division Rules:**
1. The quotient of two real numbers with different signs is ____________.
   Ex. -20/5 = -4
2. The quotient of two real numbers with the same sign is ____________.
   Ex. -20/-5 = 4; 20/5 = 4
3. The quotient of 0 and any nonzero real number is ____________.
   Ex. 0/20 = 0
4. The quotient of any real number and 0 is ____________.
   Ex. 20/0 = undefined
Dividing Fractions - *Remember to multiply by the RECIPROCAL, multiply fractions across (numerators * numerators/denominators * denominators), then SIMPLIFY!!!*

1. \( \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \)
2. \( -\frac{9}{10} \div \frac{4}{5} = \)

**Let’s Practice!**

Evaluate the product and quotient of each of the following expressions.

1. \(-8 \times 20\)  
2. \(6 \left(\frac{1}{4}\right)\)  
3. \((-\frac{2}{9})^2\)  
4. \(48 \div 3\)  
5. \(-8.1 \div 9\)  
6. \(\left(\frac{3}{4}\right) \div \left(\frac{1}{4}\right)\)  
7. \((-\frac{12}{4}) \div \left(\frac{1}{8}\right)\)  

Simplify each of the following radicals.

8. \(-\sqrt{\frac{25}{81}}\)  
9. \(\sqrt{169}\)  
10. \(-\sqrt{16}\)
The Distributive Property

The distributive property can be used to simplify the product of a number and a sum or difference. This is a property of real numbers that helps you simplify expressions. Here is an area model that shows $8(x + 5) = 8(x) + 8(5)$.

![Area model showing $8(x + 5) = 8x + 40$]

Algebraic and Numerical Examples of the Distributive Property

- $a(b + c) = ab + ac \quad \rightarrow \quad 7(3 + 2) = 7(3) + 7(2)$
- $(b + c)a = ba + bc \quad \rightarrow \quad (3 + 2)x = 3x + 2(x)$
- $a(b - c) = ab - ac \quad \rightarrow \quad 20(4 - 5) = 20(4) - 20(5)$
- $(b - c)a = ba - ca \quad \rightarrow \quad (4 - 5)x = 4(x) - 5(x)$

Using the Distributive Property...

Use the distributive property to simplify the following expressions.

Example 1: $3(x + 8) =$

Example 2: $(5b-4)(-1) =$

Example 3: $\frac{4x-16}{3} =$

Combining Like Terms...

When simplifying expressions it is important to know when to combine parts of the expression that are alike, called “Like Terms”. In algebraic expressions a term is a number, variable, or product of a number and one or more variables. There are a few types of terms, such as a constant (a term that has no variable) and a coefficient (a numerical factor in front of a term). For example if we are given the following algebraic expression, $4x + (-5xy) - 12$, we can classify $4x$, $-5xy$, and $12$ as terms. More specifically, $4$ and $-5$ are coefficients, and $12$ is a constant.
Practice- Simplify the following expressions combining like terms. Try drawing different shapes around each of different types of like terms. (For example, circle any constant terms and put a box around each of the terms that are connected to an x)

1. $8x + 3x - 2$

2. $5x - 3 + 6x + 4$

Homework!

Use the Distributive Property to simplify each expression.
1. $6(a + 10)$  
2. $10(9 - t)$  
3. $(3 - 8c) 1.5$
4. $-(20 + 3)$  
5. $-(-2 - 9)$

Write each fraction as a sum or difference.
6. $\frac{2x + 7}{5}$  
7. $\frac{25 - 8t}{5}$

Simplify each expression by combining like terms.
8. $11x + 9x$  
9. $-n + 4n$  
10. $5 - 3x + y + 6$
An Introduction to Equations

An equation is a mathematical sentence that uses an equal sign (=). Equations are used to represent the relationship between two quantities that have the same value. An equation is only true if the quantities on each side of the equal sign have the same value. An equation can be classified as an open sentence if it contains one or more variables and may be true or false depending on the values of its variables.

Let’s classify equations as true, false, or open...

Example 1: $24 + 18 = 20 + 22$

Example 2: $6 \times 4 = 22$

Example 3: $2x - 10 = 40$

If an equations contains a variable then there is a solution of an equation, which is a value for that variable that makes the equation true. Next, model for the class how to find solutions of an equations.

Example 1: Is $x = 6$ a solution of the equation $32 = 3x + 12$?

- We want to substitute 6 for $x$ to get $32 = 3(6) + 12 = 18+12 = 30$. Since $32 \neq 30$, we can say that $x \neq 6$ and that $x = 6$ is not a solution of the equation $32 = 3x + 12$.

Let’s try writing an equation given a real-life situation.

Example 2: An art student wants to make a model of the Mayan Great Ball Court in Chichen Itza, Mexico. The length of the court is 2.4 times its width. The length of the student’s model is 54 in. What should the width of the model be?

- Let’s try 2.4 in, 22.5 in, and 11.25 in. First we must figure out what equation to use by reading through the word problem. $54 = 2.4w$, where $w$ is the width for the model. Now let’s plug each of the three terms into the equation and find which one is true.

Check (2.4 in): $54 = 2.4w$  
Check (22.5 in): $54 = 2.4w$  
Check (11.25 in): $54 = 2.4w$
Let’s try finding solutions using **mental math**. For examples 3 and 4 let’s find the solutions of the equation using mental math. Make sure to always check your answer by plugging it back in for the equation.

Example 3: \( x + 8 = 12 \rightarrow \text{Think, what number plus 8 equals 12?} \)

Example 4: \( \frac{a}{8} = 9 \rightarrow \text{Think, what number divided by 8 equals 9?} \)

Let’s try using a table to find a solution of an equation. For the following examples use a table and mental math to starting values.

**Example 5:** What is the solution of \( 5n + 8 = 48 \)?

<table>
<thead>
<tr>
<th>N</th>
<th>5n+8</th>
<th>Value of 5n+8</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5(5)+8</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>5(6)+8</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>5(7)+8</td>
<td>43</td>
</tr>
<tr>
<td>8</td>
<td>5(8)+8</td>
<td>48</td>
</tr>
</tbody>
</table>

From the table we can see that a solution for the equation 5n+8 is 8. Also, notice the pattern as n increases by 1 that the value of the equation goes up by 5.

**Example 6:** What is the solution of \( 25 – 3p = 55 \)?

<table>
<thead>
<tr>
<th>P</th>
<th>25-3p</th>
<th>Value of 25-3p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let’s use a table to **estimate a solution**. If we identify a pattern in the values of an expression we can make an estimate.

**Example 7:** What is an estimate of the solution of \( -9x – 5 = 28 \)?

- First we must find the integer values of x between which the solution must lie. 
  \(-9(0) – 5 = -5 \) and \(-9(1) – 5 = -14 \).
- So we can make the conclusion that as x become greater than 0 the value of the equation is going down. We want to find a solution that makes our equation 28 therefore we should start with -1 since it is lower than 0.
- Now let’s make a table.

<table>
<thead>
<tr>
<th>N</th>
<th>-9x-5</th>
<th>Values of -9x-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-9(-1)-5</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>-9(-2)-5</td>
<td>13</td>
</tr>
<tr>
<td>-3</td>
<td>-9(-3)-5</td>
<td>22</td>
</tr>
<tr>
<td>-4</td>
<td>-9(-4)-5</td>
<td>31</td>
</tr>
</tbody>
</table>

Example 8: Estimate the solution of $3x + 3 = -22$ using a table.

<table>
<thead>
<tr>
<th>X</th>
<th>3x+3</th>
<th>Value of 3x+3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Homework!

Determine whether each equation is true, false, or open and explain.

1. $85 + (-10) = 95$
2. $-8(-2) -7 = 14 – 5$
3. $4a – 3b = 21$

 Decide whether the given number is a solution of each equation. Show your work.

4. $8x + 5 = 29; x=3$
5. $5b + 1 = 16; b = -3$
6. $2 = 10 – 4y; y = 2$

Write an equation for each sentence or word problem.

7. The sum of $4x$ and $-3$ is 8. ________________________________

8. An athlete trains for 115 minutes each day for as many days as possible. Write an equation that relates the number of days $d$ that the athlete sends training when the athlete trains for 690 minutes. ________________________________

Use mental math to find a solution for the equation.

9. $x – 3 = 10$
10. $20a = 100$
11. $18 + d = 24$
Use a table to find the solution of the following equation.

12. $3.3 = 1.5 + 0.4y$

13. Explain the difference between an expression and an equation. Does a mathematical expression have a solution? Explain your answers.

______________________________________________________________________________

______________________________________________________________________________

14. Find the solution of the equation $x + 4 = -2$ either using mental math or a table. If the solution lies between two consecutive integers, be sure to identify those integers.

15. Find the solution of the equation $1 = -\frac{1}{4}n + 1$ either using mental math or a table. If the solution lies between two consecutive integers, be sure to identify those integers.
Connecting Patterns, Equations and Graphs

We will be identifying solutions of a Two-Variable Equation. You can use an equation with two variables to represent the relationship between two varying quantities. A solution of an equation with two variables $x$ and $y$ is any ordered pair $(x, y)$ that makes the equation true.

Example 1: Is $(3, 10)$ a solution of the equation $y = 4x$?

\[
y = 4x
\]

- substitute $3$ in for $x$ and $10$ in for $y$

\[
10 = 4(3)
\]

$10 \neq 12$  
- we see that $(3, 10)$ is not a solution of $y = 4x$

Now you try...

Example 2: Is the ordered pair $(5, 20)$ a solution of the equation $y = 4x$?

We can represent the same relationship between two variables in many different ways. Let’s try with a real-world situation. Both Carrie and her sister Kim were born on October 25th, but Kim was born 2 years before Carrie. How can you represent the relationship between the girl’s ages in different ways?

Let’s Make a Table!

<table>
<thead>
<tr>
<th>Carrie’s and Kim’s Ages (years)</th>
<th>Carrie</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrie</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Kim</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Let’s Write an Equation!

First we have to set out variables. Let’s say that $x =$ ___________________________ and let’s say that $y =$ ___________________________. From the table we just made we can see that $y$ is always 2 greater than $x$ (because Carrie is 2 years older than Kim). Therefore we can say write an equation ___________________________, which represents the difference between Carrie’s and Kim’s age.
Let's Graph it! (Always make sure to label your graph)

Noticing Patterns...

Now try using what we’ve learned about tables, equations and graphs to show the relationship between the number of houses and the number of points on the stars. For example, if there is only one star we have 5 points, two stars there are 10 points, etc. Draw a table, create an equation and then graph your solution.

Make a Table...

Write an Equation...
Graph it... (The whole graph will not fit on the graph, generalize)

Homework!

Tell whether the given equation has the ordered pair as a solution:
1. $y = x + 6; (0, 6)$
2. $y = -x + 3; (4, 1)$
3. $y = -4x; (-2, 8)$
4. \( \frac{x}{3} = y; (-10, -2) \)

Use a table, an equation, and a graph to represent the following relationship.
5. The number of triangles is \( \frac{1}{3} \) the number of sides.
UNIT 1 Review – Stations

Directions – You will have 10 minutes at each station to work in group and finish the problems corresponding to the station you are in.

STATION 1
Adding, Subtracting, Multiplying and Dividing with Manipulatives

Directions: You will use the manipulatives to evaluate the following expressions. Make sure when you are performing the operations of multiplication and division that you use the black manipulatives to represent positive integers and red manipulatives to represent negative integers. The single blocks will be useful when adding and subtracting, while you can snap longer groups of blocks to show multiplications and division.

Evaluate the following expressions using manipulatives and record the answers below.

1. \(-5 + 10 = \) __________
2. \(9 - (-4) = \) __________
3. \(-8 - 5 = \) __________
4. \(-20 \times 3 = \) __________
5. \(12 / -3 = \) __________
6. \(8 \times 5 = \) __________
7. \(-10 + 4 - 3 = \) __________
8. \(5 - 10 - 8 + 2 = \) __________
### Station 2

**Identifying Properties of Real Numbers**

Directions: First look at the back of this pack and tear off the last page “Properties of Real Numbers.” You will use your scissors to cut out each of the properties. Using tape, place the correct Property of Real Numbers that is being represented by the expression in that box. (Do not cover up the expression with the name of the property)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2 + 7 = 7 + (-2)$</td>
<td>$6 \times (-1) = -6$</td>
</tr>
<tr>
<td>$4 + (3 + x) = (4 + 3) + x$</td>
<td>$8 + 0 = 8$</td>
</tr>
<tr>
<td>$7 (x + 4) = 7 (x) + 7 (4)$</td>
<td>$9 \times 0 = 0$</td>
</tr>
<tr>
<td>$8 \times 9 = 9 \times 8$</td>
<td>$2 \times (5 \times 4) = (2 \times 5) \times 4$</td>
</tr>
<tr>
<td>$-4 \left(-\frac{1}{4}\right) = 1$</td>
<td>$21 \times 1 = 21$</td>
</tr>
</tbody>
</table>
STATION 3
Evaluating Expressions using Order of Operations

Directions: Evaluate the following expressions. For each problem write down the steps that you do and which operation you are performing. (Example: First plug in 5 for d, next divide... etc.)

Evaluate each expression for c = 3 and d = 5.

1. \(d^3 \div 15\)
2. \((3c^2 - 3cd)^2 - 21\)

Evaluate each expression for p = 5 and q = -3.

3. \(-3q + 7\)
4. \((pq)^2\)
5. \(7q - 7p\)
STATION 4

**Recognizing Patterns Representing Relationships as Equations, Tables, and Graphs**

Directions: Tell whether the given ordered pair is a solution of each equation. Show your work.

1. $3x + 5 = y; \ (1, 8)$
2. $y = -2(x + 3); \ (-6, 0)$
3. $10 - 5x = y; \ (-4, 10)$

Directions: Complete the following real-life situation by recognizing the pattern and representing the relationship in a table, as an equation, and on a graph. Make sure to draw a diagram first to ensure you can visualize the problem.

There are a total of 4 different stores in the Amsterdam Shopping Plaza. Each store has 2 large windows in the front of the store to display each store’s merchandise. Show the relationship between the number of store and number of windows in the shopping plaza on a table, a graph, and by writing an equation.
**Properties of Real Numbers**

**Terms**

Directions: Cut each of these terms out and use for Station 2.

<table>
<thead>
<tr>
<th>Term</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MULTIPLICATIVE INVERSE</strong></td>
<td><strong>COMMUTATIVE PROPERTY FOR ADDITION</strong></td>
</tr>
<tr>
<td><strong>ADDITIVE IDENTITY</strong></td>
<td><strong>COMMUTATIVE PROPERTY FOR MULTIPLICATION</strong></td>
</tr>
<tr>
<td><strong>MULTIPLICATIVE IDENTITY</strong></td>
<td><strong>ASSOCIATIVE PROPERTY FOR ADDITION</strong></td>
</tr>
<tr>
<td><strong>ZERO PROPERTY FOR ADDITION</strong></td>
<td><strong>ASSOCIATIVE PROPERTY FOR MULTIPLICATION</strong></td>
</tr>
<tr>
<td><strong>ZERO PROPERTY FOR MULTIPLICATION</strong></td>
<td><strong>DISTRIBUTIVE PROPERTY</strong></td>
</tr>
</tbody>
</table>
**Day 14 – Unit 1 Test**

**Grade _____/50**

**Unit 1 – Connecting Patterns and Equations**

Directions: Read each question and answer on the line(s) provided. (If you have any questions please raise your hand and the teacher will come to your desk)

1. Write a word phrase for \(-12t + 2\) ______________________________ (2 pts)

2. Write an algebraic expression for the word phrase \(\text{the quotient of } p \text{ and } 97\). 2. _______ (2 pts)

3. Evaluate the expression: \(-xy^3 / (-4)\) for \(x = 2\) and \(y = 3\). 3. _______ (2 pts)

4. Simplify the expression: \(-\frac{25}{64}\) 4. _______ (2 pts)

5. Is the following statement true, false or open? Explain, and if false give a counterexample.
   \[For \ all \ real \ numbers \ a, \ b, \ and \ c, \ a(b + c) = ab + ac\]
   ___________________________________________ (4 pts)

6. Is the ordered pair \((2, -5)\) a solution to the equation \(4 + 3x = -2y\)? Show all your work.
   ___________________________________________ (4 pts)

7. Put the following numbers in order from least to greatest, and plot each number on the number line.
   \[-\frac{7}{8}, -\frac{7}{4}, -\frac{13}{16}, -1\frac{4}{5}\]  (3 pts)

8. Simplify the following expression: \(12 ÷ \frac{3}{4}\) 8. _________ (3 pts)
9. Name the subset(s) of the real numbers to which the following numbers belong to: -1.5, 0.4, $-\frac{2}{3}$, $\sqrt{4}$  
9. __________ (2 pts)

10. Identify the following property: $a(b + c) = ab + ac$  
10. __________ (2 pts)

11. Is the ordered pair (0, 1.2) a solution of the equation $y = (x - 1.2)(-3)$? Show your work.  
11. __________ (3 pts)

12. If there are $t$ teams in a soccer league, and there are 11 players on each team how can we represent the total number of players $p$ in the league? Make a table, write an equation, **AND** make a graph to describe the total number of players $p$ in the league. How many players are on 17 teams?

Draw a table (in the space provided): (3 pts)  
An equation is: _________________ (2 pts)

How many players are on 17 teams? _________________ (2 pts)
13. You notice that $10^\circ C = 50^\circ F$, $20^\circ C = 68^\circ F$, and $30^\circ C = 86^\circ F$. Use inductive reasoning to predict the value in degrees Fahrenheit of $40^\circ C$. (Hint: Making a table might be helpful!)

13. ________ (4 pts)

14. Is the following statement true or false? If false, give a counterexample.

   *If the product of three numbers is negative, then all the numbers are negative.*

   ____________________________________________________________________________________ (5 pts)

15. Simplify the expression $3ab + 4 - 8ab - 6$. (Hint: Remember to combine like terms)

15. ________ (2 pts)