Building Understanding: Using Math Journals to Increase Learning in an Intermediate Classroom

Paul Lawrence Jonasse
The College at Brockport, Pauljonasse@gmail.com

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Building Understanding: Using Math Journals to Increase Learning in an Intermediate Classroom

by

Paul Lawrence Jonasse

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Master of Science in Education
Building Understanding: Using Math Journals to Increase Learning in an Intermediate Classroom

by

Paul Lawrence Jonasse

APPROVED BY:

[Signatures and dates]
Dedication

It is with great joy that I dedicate this project to my loving parents Theresa Gefell Jonasse and Lawrence Jonasse, from whom I have learned so much. You always believed in me, held my hand, showed me the way, and let me know that I was loved no matter what. Gracias con mucho gusto y amore.

Paul
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Chapter One: Introduction

The teaching and learning of mathematics have in the past, often been viewed as quite isolationist; something that is done by one’s self. The teacher instructs from the textbook, using the chalkboard or overhead projector, and the students sit alone with pencil and paper struggling to make meaning out of new and increasingly difficult concepts. My own memories of elementary math instruction are fairly positive: I had great support at home and generally mastered everything my teachers threw at me, even including operations with fractions. I can remember many of my classmates however, were not as enamored with their math experience as they struggled to make meaning out of all that our teachers threw at us. Indeed, I see it today still, children who have memorized certain rote math facts but do not understand what they are doing or why they are doing it when they divide, multiply, subtract, or add.

My rationale for this project arose from watching my students struggle with new mathematical concepts: word problems, fractions, and increasingly difficult types of division have totally baffled many of my students. For the first several months of the school year most of my students did very well in math class, but they struggled with increasing difficulty as the concepts have become more abstract. I have struggled along with them to figure out ways to help them better understand. This project has grown out of our struggles with teaching, learning, understanding, and the working together to improve them all.
I entered into this research with several guiding questions:

- Can the use of mathematics journals help students better understand word problems and other mathematical concepts?
- Are students able to learn by teaching themselves through writing and use writing as a tool for learning?
- Are mathematics journals useful as an evaluative tool?

I planned to utilize math journals in this project as a tool to assist my students in constructing mathematical meaning and that this method of construction would lead them to a greater conceptual understanding of mathematics and give them greater confidence in their problem solving abilities. I also planned to use the journals as an ongoing assessment tool, which would serve as a window into the students' confusion as well as their understanding, and thus be a guide for my daily teaching.
Chapter Two: Review of Literature

In this chapter I will briefly discuss the national standards movement in the United States, followed by a look at traditional mathematics instruction and the ongoing mathematics education reform movement. Following these will be a discussion of constructivism and social constructivism and related ideas about how children learn; which will then segue into a discussion of journal writing and its benefits.

National Standards Movement

Lynn Steen (1995), points out that historically “the U.S. has always favored local control over education. But by 1983, mounting evidence of failures of U.S. education moved the authors of *A Nation at Risk* (1983) to recommend strengthened requirements, rigorous standards, and higher expectations for *all* students” (p. 38). The National Commission on Excellence in Education (cited in Steen, 1995) stated that “the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people” (p. 39). Their findings further lamented the state of education in the United States, when compared to the rest of the world, especially concerning science and mathematics. The commission cited persistent downward trends in achievement for the previous 15-20 years in both mathematics and science.

In 1989 President George Bush called a meeting of the nation's Governors to set national goals for education. This meeting laid the foundation for national curriculum standards: One of the goals urged that all students demonstrate
competency in challenging subjects including English, history, science, mathematics, and geography. Yet another of the goals declared that by the year 2000 the U.S. should be first in the world in mathematics and science education. Four years later these goals were written into legislation (Steen, 1995).

Independently, but also in response to the 1983 publication of A Nation at Risk, the National Council of Teachers of Mathematics (NCTM) began to develop the nation's first mathematics education standards. These voluntary standards were produced not by the federal government, but through a multi-year consensus-building effort led by the nation's mathematics teachers and mathematicians. The authority of these standards rests not on governmental mandate, but on the evidence and logic invoked by the standards themselves. Published in 1989, the NCTM Standards quickly became the nation's premier example of educational standards—a set of public expectations, rooted in research and practice that are intended to raise the academic achievement of all students (Steen, 1995). Worth noting is the fact that these Standards are regularly being revised and updated as the NCTM actively seeks out current research because they believed that they should need to be periodically examined, evaluated, and revised in order to reflect current research and thus remain relevant (NCTM, 2000).

With the March 1989 publication of The Curriculum and Evaluation Standards for School Mathematics, the NCTM set the course for the reform movement in mathematics. The Standards presents five wide-ranging goals for all students: "(1) that they learn to value mathematics, (2) that they become confident
in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically” (p. 5).

Traditional Mathematics Instruction

Mathematics instruction in the United States has gone through three major paradigm shifts since the beginning of the 20th century. Until the early 1960’s the predominant method of instruction was rooted in behaviorism and the belief that there is a connection formed in the mind between a question, say 2+2, and the correct response, 4. Behaviorists generally saw children as blank slates that just needed to be filled up with information in order to learn. Teachers would transmit information to their students who would, in theory, absorb the information like a sponge. Drilling and repetitive practice was the generally taught and accepted method for mathematics instruction (Knuth, & Jones, 1991; Van deWalle, 1994; Shirley, 2000; Wilson, 2003).

Heibert (1996) points out that through much of the last century our mathematics curricula have been heavily influenced by concerns of preparedness for the workplace and life outside school, and that problems were used as a way of practicing applications. Heibert goes on to state that these same methods of teaching persist, even with the great pressure that exists for change. He says that “after a decade of mathematics reform in the 1960’s the Conference Board of Mathematical Sciences (1975) found that ‘teachers are essentially teaching the
same way they were taught in school’... And, in the midst of current reforms, the average classroom shows little change” (p. 77).

Heibert (1999) describes a typical mathematics classroom as one that may look rather familiar to many of us:

Most characteristic of traditional mathematics teaching is the emphasis on teaching procedures, especially computation procedures. Little attention is given to helping students develop conceptual ideas, or even to connecting the procedures they are learning with the concepts that show why they work (p. 11).

Van de Walle (1998) concurs, stating that in the past “school mathematics was focused almost entirely on the skills of pencil-and-paper computation” (p. 4). He also says that it was commonplace to cite one’s incompetence in math, implying that at the time it didn’t much matter because “most jobs required little more than basic computational skills” (p. 4).

Heibert (1999) further cites the findings of the video study of 1999’s Third International Mathematics and Science Study (TIMSS) saying that “for 78% of the topics covered...procedures and ideas were only demonstrated or stated, not explained or developed. And 96% of the time that students were doing seatwork they were practicing procedures they had been shown how to do” (p. 11). How are these children to actually learn mathematics, much less apply mathematics appropriately to a given situation when they are working mainly in the procedural realm?
Heibert (1999) concludes by saying that compared with the curricula in other countries, the US curriculum is

Relatively repetitive and unfocused, and undemanding... [and] provides few opportunities for students to solve challenging problems and to engage mathematical reasoning, communicating, conjecturing, justifying, and proving ... much of the curriculum deals with calculating and defining, and much of this activity is carried out in a rather simplistic way” (p.11-12).

When one considers the rigorous standards put forth by NCTM, combined with the problems exposed by TIMSS, mathematics education in the United States clearly has a long way to go in order to achieve these high standards.

Reforming Mathematics Education

Around the country there are numerous examples of outstanding mathematics instruction, and while some of these programs are very encouraging, there is still much work to do. The rapidly growing global interdependence and increasingly technological nature of the world requires that citizens possess a much greater level of mathematical and technological competence than was necessary in the past. The movement for change is very strong because:

- Mathematical literacy is one of the most important keys to quality of life both for today and in the future (NCTM, 2000).
- Mathematical competency will allow for equal access and equity for all children as they grow to adulthood (NCTM, 2000).
- According to the National Assessment of Educational Progress (2000), American students perform passably on low-level cognitive skills such as
computation, but generally have difficulty with higher-order skills like problem solving and application.

- Also according to NAEP, American students consistently perform at a lower level than students from other countries (NCTM, 2000).

As stated in the previous section, one of the greatest problems in mathematics education is the fact that many classrooms continue to teach by rote skills and memorization, rather than through experience and problem solving. The Mathematical Sciences Education Board (1998) described the ideal mathematics classroom, saying that it "should provide practical experiences in mathematics skills that form a bridge to real life experiences, real world applications, and actual adult responsibilities; this will necessarily involve going well beyond rote memorization of fact families and multiplication tables to include reasoning and problem solving skills" (http://www.tenet.edu/tek/math/resources/lookfor.html).

Kamii (1994) argues that "children should reinvent arithmetic because logico-mathematical knowledge is the kind of knowledge that each child can and must construct from within...if they are to understand today's algorithms" (p. 33). She goes on to state that the problem with the teaching of algorithms is that they "force children to give up their own numerical thinking, they 'unteach' place value and hinder children's development of number sense" (p. 33).

Van de Walle (1998) also says that our world of the twenty-first century is "increasingly complex and dominated by quantitative information in every facet of the economy, [and] mathematical thinking has become indispensable in even the
most ordinary jobs” (p. 4), while Shaw and Blake (1998) call attention to the fact that the needs for mathematics have changed, “varied, complex problems face people today...thinking and problem solving skills are needed more than routine computations” (p. 4).

Countryman (1992), in describing the change in prevailing attitudes toward mathematics instruction states that

Mathematicians have described a shift in contemporary mathematics away from emphasis on number and space and toward pattern and application....The intelligent citizen of the twenty-first century needs to know how to analyze data, how to reason in probabilistic situations, and how to make choices. Students at all levels need experience in identifying the kind of answer they want in a given situation. Students need to know and to understand the advantages of different methods of obtaining answers. They need to know when to guess, when to use pencil and paper, when to use a calculator, how to recognize an answer and whether the answer makes sense (p. 8-9).

NCTM (2000) also states that

Students should come to view algorithms as tools for solving problems rather than as the goal of mathematics study. As students develop computational algorithms, teachers should evaluate their work, help them recognize efficient algorithms, and provide sufficient and appropriate practice so that they become fluent and flexible in computing...(The) standards reinforce the dual goals that mathematics learning is both about making sense of mathematical ideas and about acquiring skills and insights to solve problems (p. 143-4).

Van de Walle (1998), in discussing the types of changes that we must strive for, describes mathematical thinking, saying “that it involves the ability and the habits of reasoning and solving problems...having number sense-intuition about
numbers, their magnitudes, their effects in operations, and their relationships to real
quantities....” (p. 4). Kamii goes so far as to argue that algorithms are harmful to
children, saying that “when children are made to follow algorithms, they have to
give up their own ways of thinking numerically, [thus] children obey teachers by
giving up their own thinking” and following the lead of the teacher (p. 33).

A Tale of Two Classrooms

Having discussed previously how many of us were educated (or not)
mathematically, I will present two vignettes in order to further illustrate one of the
problems inherent to traditional mathematics education. Deborah Schifter (1993)
contrasts a traditionally taught fourth grade mathematics class with one in which
the construction of strong conceptual linkages is stressed. In the former, she meets
a student who can confidently use the mathematical algorithms for division, but
upon further examination was found to have no idea what the actual concept of
division was all about, nor what it really meant. The child was proudly able to
divide 24,682 by 5, and when asked to divide 32 by 5, provided the correct answer
of $6\, r2$. But, when asked to explain $r\, 2$, she was only able to respond, “remainder
two” (p. 7). The child was hesitant to use manipulative blocks to show the
division, and was unable to show how to do the problem. When asked to use the
blocks to show what she had done she ended up with three groups with six blocks
and two groups with seven blocks and didn’t know what to do with the extras.

In the other classroom with the focus on construction of meaning and
concept connections the dominant paradigm is that the understanding of math is
primarily one of “concept constructions and active interpretation” (Schifter, 1993, p. 8). Where teachers are not the source of mathematical truth, the keepers of the keys, if you will, but rather they are creators of problem solving situations which intend to engage students in the problem solving process which utilizes appropriate strategies.

Indeed, the NCTM (2000) says that

Problem solving is the cornerstone of school mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited. Students who can efficiently and accurately multiply but who cannot identify situations that call for multiplication are not well prepared. Students who can both develop and carry out a plan to solve a mathematical problem are exhibiting knowledge that is much deeper and more useful than simply carrying out a computation. Unless students can solve problems, the facts, concepts, and procedures they know are of little use. The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems (p. 181).

Constructivism

Since the mid-1980s, constructivism has played a major role in mathematics education, and constructivist approaches to learning - which are supported in the two NCTM Standards documents - are beginning to influence the teaching of mathematics. Two hallmarks of the constructivist position (Van de Walle, 1998) help guide effective mathematics teaching and learning. First, constructing knowledge is a highly active endeavor on the part of the learner (Baroody, 1987). Second, constructing and understanding a new idea involves making connections
between old ideas and new ideas. Teachers might help make this connection by asking reflective questions such as the following which were developed by Cook and Rasmussen (1991):

- Does this idea fit with what you already know, and if so, then how?
- Is this problem in any way like other problems or situations you've experienced?
- What is it about this problem that is reminiscent of other problems that you have seen?

Constructing knowledge requires reflective thought. A major assumption of constructivist theory (Van deWalle, 1998) is that children do not enter a learning situation as empty vessels or blank slates to be filled with ideas. The key to effective teaching (and learning) is helping children to be active, reflective thinkers so that their minds will be working and forming relationships, making connections, and integrating concepts and procedures.

Indeed, as far back as 1929, John Dewey “placed great faith in scientific (and ordinary) methods of solving problems” (cited in Heibert, 1996, p. 14). He referred to the methods by several names including the “experimental practice of knowing” [and] “reflective inquiry” (cited in Heibert, 1996, p. 14). He believed reflective inquiry was the key to moving beyond the distinction between knowing and doing… “To the extent that we could use the method of reflective inquiry, we would be acting intelligently” (cited in Heibert, 1996, p. 14).
Van de Walle (1998) suggests seven ways teachers might structure lessons to promote reflective thought, and he encourages you to think of ways to add to the list:

1. Create a mathematical environment.
2. Pose worthwhile mathematical tasks.
3. Use cooperative learning groups.
4. Use models and calculators as thinking tools.
5. Encourage discourse and writing.
7. Listen actively (p. 34).

Also, networks or "cognitive schemas" that exist in the learner's mind are the principal determining factors for how an idea will be constructed. These networks are the product of both constructing knowledge and developing mathematical concepts (http://www.ncrel.org/sdrs/areas/issues/content/cntareas/math/ma3know.htm).

Thus, we use what we know and add on to it in order to construct new meaning in an ongoing cycle of learning.

The constructivist approach to education and human development became more widely accepted by the late 1970's. This is when we began to see a shift in thinking about mathematics instruction. The view of mathematics changed from that of numbers and computation to mathematics as problem solving (Knuth & Jones, 1991; Kamii, 1994; Shaw & Blake, 1998; Van de Walle, 1998).
Educational researchers began to look more at content area learning and problem solving.

Knuth & Jones (1991) and Charlesworth (2000) say that as a result of the work of cognitive research, there is now a better understanding about the nature of and importance of understanding. Understanding has come to be thought of as an active process of constructing connections. To understand is to be aware of the connections between mathematical facts, procedures, and concepts. Furthermore, it means building relationships between existing knowledge and new learning so that it has a cumulative and expansive effect whereby children build upon previous knowledge. Metaphorically it is like the construction of a building or new home. You begin with a foundation and build upward from the ground, with the bricks below supporting the bricks above. Without a solid foundation upon which to build, the structure will collapse.

Knuth & Jones (1991) also point out that it is important to teach for understanding in mathematics for the following reasons:

- Understanding is necessary in order to retain knowledge and have the ability to apply it to newly encountered situations.
- Research shows that teaching for understanding does not lead to a decrease in skill ability; rather it leads, in the long run, to greater skill capability because of the ability to apply the knowledge as opposed to parroting back facts.
• Skilled application and performance is wholly dependent upon understanding.

Understanding of mathematics is primarily the construction of concepts and the ability to actively interpret those constructions (Schifter & Twomey-Fosnot, 1993; Shaw & Blake, 1998). Students need to have the ability to build new mathematical knowledge through solving problems, applying and adapting a variety of appropriate strategies to solve problems and be capable of reflecting on the process of mathematical problem solving (NCTM, 1989).

Charlesworth (2000) also states that “understanding occurs when a mathematical concept or procedure becomes a real part of the mental structure” (p. 4). That is to say that the student is able to determine when to use a particular strategy or formula to solve a problem with which they are faced, just as posited by the NCTM (2000) in the Principals and Standards. Conversely, Charlesworth states that “understanding is not present to any great degree when mathematics is learned as isolated skills and procedures. Understanding develops through interaction with materials, peers, and supportive adults in settings where students have opportunities to construct their own relationships...” (p. 4)

Stein (2001) points out that mathematical understanding comes from internal networks that contain multiple and strong connections among mathematical ideas, among different representations of the same idea, and between mathematical concepts and procedures are considered to represent more sophisticated mathematical understanding than are...those with few, non-reinforced
linkages and linkages that are not multiply connected to more than one piece of information (p. 113).

Stein further states that insofar as mathematical understanding is concerned, since “new knowledge builds on prior knowledge...individuals with well-structured, rich, and coherent networks are able to more readily build bridges to newly encountered information, thereby allowing them to integrate and understand the new information more easily” (p. 113). In other words, we really should be sure that we are teaching mathematics for understanding, and not just the ability to memorize formulas; by learning the trade, rather than learning the tricks of the trade.

Nuthall (2002) sees the role of the teacher to Engage with the existing knowledge beliefs and skills of the students and by setting challenging problems or posing significant questions to engage...the knowledge building practices...{which} requires the teacher to be constantly monitoring the ways in which the students are interacting with each other... constantly focused on the ways students use reasons and evidence to support their views. Part of the role of the teacher is to avoid providing students with knowledge or solutions when it is possible for them to work them out for themselves (p. 47-8).

In other words, Nuthall implies that perhaps teachers are too quick to give students answers rather than allowing them the time to work ideas out on their own.

Social Constructivism

Charlesworth (2000) discusses the work of Lev Vygotsky. He believed that socialization played a key part in the development of children, and is best known for the concept of the zone of proximal development. Vygotsky (1978) said that
The zone of proximal development is the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (p. 86).

He fostered the idea that the work that children did together was “even more indicative of their mental development than what they can do alone” (p. 85).

Vygotsky's *zone of proximal development* has many implications for those in education. One of them is the idea that human learning presupposes a specific social nature and is part of a process by which children grow into the intellectual life of those around them. According to Vygotsky (1978), an essential feature of learning is that it awakens a variety of internal developmental processes that are able to operate only when the child is in the action of interacting with people in his environment and in cooperation with his peers. The implication is that each child has a range of complexity in which they are able to operate, from what they are able to do independently to what they are able to do with the assistance of a more capable peer or an adult.

**Journal Writing**

Journal writing has many champions throughout education and journals have come to be widely used from the primary grades through graduate schools. Routman (1994) believes the benefits of journal writing are significant for both students and teachers. She argues that journals promote fluency in both reading and writing, encourage risk taking, provides a safe and private place to write,
promotes thinking and makes it visible as well as helping develop proper written
language conventions (p. 199-200).

The NCTM (2000) Standards state that:

- In grades 3-5 students should use communication as a tool for
  understanding and generating solution strategies (p. 193).

- Much of the work in these grades should be focused on reasoning
  about mathematical relationships (p. 187).

- Mathematical reasoning develops in classrooms where students are
  encouraged to put forth their own ideas for examination (p. 187).

- Students need to explain and justify their thinking and learn how to
  detect fallacies and critique others' thinking (p. 187).

- They need to have ample opportunity to apply their reasoning skills
  and justify their thinking in mathematics discussions (p. 188).

With such a strong emphasis on communicating ideas, reasoning about
relationships, explaining and justifying thinking, it just makes good sense that
writing would be one way to accomplish these goals. These new standards are also
evidenced on the standardized tests given (at least in New York) by their insistence
on written justification of student answers.

Armstrong (1994) states that “keeping a personal journal involves students
in making ongoing written records related to a specific domain....they can also
incorporate multiple intelligences by allowing drawings, sketches, photos,
dialogues, and other non-verbal data” (p. 68). This can be especially helpful for

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students who are not particularly verbal, but conversely can pose a roadblock for
the student who is reluctant to write.

In addition to preparing students for state assessments, journals can give a
teacher a closer look at their students on a personal level. Chapman (1996)
discovered that his students, through their journals, showed mathematical
awareness that had been concealed by low grades. And, by utilizing journals in the
classroom, it allows the teacher a realistic way of listening to each student
individually. Borasi and Rose (1989) state that:

The journals certainly allowed the teacher as a reader to get to
know students individually and to realize their specific
problems and difficulties—whether they were of a cognitive or
affective nature. As a consequence, the teacher became more
aware of the individual needs of each student, and could
respond better to them—both individually and corporately
(p. 358).

Marilyn Burns (1993) argues that writing in math class adds an “important
and valuable dimension to learning by doing” (p. 13). Countryman (1992) says that
when students ask her to show them what writing has to do with math class, she
argues that writing “is math... it's fine to get the right answer, but what good is that
answer if you can't explain it to anyone?” (p. 2).

In addition, Stein (2001) adds that mathematical argumentation is an
excellent pedagogical tool because “when students take up positions and defend
them, they have the opportunity to develop and practice the skills of conjecturing
and mathematical justification in [the] authentic setting” (p. 129) of their
mathematics journal. Furthermore, she states that as they “become invested in their
point of view, they learn how to garner and present the necessary evidence to convince others that their claim is warranted" (p. 129).

Countryman (1992) states that furthermore,

> When students use language to find out what they think about mathematics, the result is often surprising. ‘The very fact that [the teacher] assigned writing was enough to make me start thinking,’ one student said of his calculus class. ‘We all thought this writing was ridiculous at first, but I’ve come to see it as the most important thing we did all year.’ The connection between writing and mathematics became obvious and important to this student because of the writing and the mathematics that he did that year (p. 7).

Journals also give students the ability to experiment and test out ideas. Ostrow (1995) comments on the importance of this ability:

> I have learned that children need to experiment with numbers, come up with their own algorithms, test those strategies and come up with other methods, and eventually discover the most efficient way for themselves...What is easy and efficient to a child may differ from the traditional way we were taught (p. 65).

I think this is important, and it reflects back to Kamii’s (1994) notion, presented above, that sometimes a student may have a way of arriving at a solution that uses a non-traditional method, but works for them. Who are we to tell them they are wrong because they don’t conform to the standard method of doing things?

In her book *Writing to Learn Mathematics* (1992), Joan Countryman argues that “Writing can provide opportunities for students to construct their own knowledge of mathematics... [It gives] them a chance to practice inferring, communicating, symbolizing, organizing.... Writing helps students make sense of
mathematics. Mathematics helps students make sense of the world” (p. vi-vii).

Isn’t this ultimately what we are after: Helping kids to understand the world?

One can easily conclude from these arguments that journal writing provides students with the opportunity to build and expand upon their cognitive schemas, scaffolding ideas into understanding by providing them with the occasion to engage in metacognitive mathematical discourse in which they, with the help of their peers and teachers, arrive at a fuller, more conceptual understanding of problem solving through mathematics.

Summary

The research cited above clearly shows that mathematics education in the United States is in need of reform if we as a nation wish to continue to be world leaders in education. The lessons of the constructivists suggest that we need to have classrooms where teachers and students work together to construct true conceptual understanding, moving past rote memorization and more toward application and synthesis of ideas. Journal writing, by its very nature appears to be a natural tool to help achieve the goals outlined by the NCTM in its *Principals and Standards*.  

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Chapter Three: Methodology: Gathering and Analyzing the Data

In this chapter I will briefly talk about how my students and I entered into this project together. I will outline our discussions leading up to the actual starting date, how the journals were set up, and how they were implemented. I will also discuss the mandated curriculum, as well as student demographics, and finally will present the problems used in this project and the dates on which they were assigned.

Getting Started

My students and I launched this project together on March 29, 2004. They were anxious and excited to begin, as we had been discussing the project for several weeks. They were especially intrigued at the notion that they were going to be teaching me in this process. Although I often discuss with my students how I learn from them, I think they were feeling different about this because it was "official," and for college.

We began by revisiting the fact that it is important that students be able to explain how they arrived at answers. Indeed, many of my students think that I'm crazy when I tell them that the explanation is more important than the answer. I always tell them that I'd often rather see a problem with no answer, but an explanation of how the student tried to solve a problem than to see a correct answer with no explanation. I tell them that if I can see how they are thinking, I can better figure out where they need assistance, but if they only have an answer, correct or not, I have no idea if they understand what they did, or if they just got lucky and
guessed, or if their older sister solved the problem for them. We also discussed the notion of constructing knowledge, or as I put it to them, that it was possible for them to teach themselves by using the writing process to draw upon what they knew to help them learn how to solve newer and more difficult problems. I tried to stress to them that this journal writing project was about learning, and that their journal grades would be based on their making an honest effort to engage the work.

"Fluting Mushrooms"

To better illustrate the idea of learning by doing more clearly to the students, one day I brought a mushroom and a paring knife to class with me so that I could share with them something that I had wanted to learn to do many years earlier that had taken me a really long time to master. I proceeded to quickly carve a lovely spiral design on the top of the mushroom. One would normally carefully cook this mushroom and use it as a garnish. The students were in awe, and everyone wanted to know how it was possible to do that. I must admit that I am quite proud of the fact that I was able to master this task because it took me several hundred tries to do so. I shared all of this with them (they were disappointed that there was only one mushroom) and stressed to them that the point of all this was to show them that you can get good at doing things you find difficult, but that the key to success is not giving up and continuing to try. I told them that if they wanted to be successful at problem solving, if they wanted to be successful at just about anything, the key to their success was showing up each day, giving their best
effort, and trying to learn from their mistakes. With enough practice and perseverance they would be “fluting mushrooms” before long.

Logistics

The journals were set up in basic spiral notebooks and had a preface page which I wrote using several sources. It read as follows: Not all problems can be solved in the same way. Listed below are some of the many problem-solving strategies you might find helpful for different kinds of problems.

- Compute using addition, subtraction, multiplication, or division.
- Draw a picture, model, or diagram.
- Use or make tables, charts, or graphs.
- Write a number sentence.
- Break the problem into parts.
- Look for patterns.
- Act it out.
- Guess and check.
- Work backwards.
- Use mental math.

After you solve each problem you must explain step-by-step how you solved the problem. It is important that you are able to explain how to solve problems. Be sure that you use proper math language (i.e. subtract, not take-away).
Beginnings

On day one, after we read the preface together and discussed it, I had the students begin by writing a letter to me in which they were to tell me their impressions of math in general, and our class, in particular. I asked them to freely share their likes, dislikes, joys, frustrations, and other comments. My students are quite honest when expressing their feelings. I was pleased to read their comments, as most expressed how much they really enjoyed math class. I was especially pleased at how positive the comments were, even from those who had been struggling, some a great deal. Most of the students said that they especially enjoyed playing fractions Bingo and Quizmo. Martin wrote that he learned “how to multiply, add and subtract fractions” and that “nothing is hard.” Dennis let me know how much he likes doing the one-hundred multiplication problem warm-ups because he likes to see how fast he can finish them (In fact, he and Martin like to race to see who can finish first). Shameika told me that “math class was going well” and that she likes the lessons “because they can sometimes be fun.” Then there is my personal favorite, Jennifer, who let me know that she “likes doing homework and stuff because it is fun and helps me get smarter.”

Some of the students also expressed displeasure with themselves as well as me. Aliyah wrote that she needed help with “fractions and decimals because I just don’t get them.” Sarah wished that math was “a little more challenging” and also expressed her desire for more freedom on the weekends, saying “Why do we have to have homework on the weekends? The weekends is [sic] our break from
school.” Kenneth added that “the thing I like most about math is learning new things...most hardest thing is probably Roman Numerals, it is sort of confusing.”

I was glad to read that they mostly remained positive and affirmative in their belief that they would ultimately be successful. Although many of the students who expressed having difficulty with a particular concept mentioned fractions and decimals, everyone remained upbeat and positive. I think that it is extremely important that children, indeed anyone, have a positive attitude when struggling with learning something. It is much more difficult to be successful at something if you do not expect to succeed. After reading their letters I was left with the feeling that I was teaching a group of students who believed in themselves and were ready to work hard to improve themselves.

Prior to using the journals we did generally start class with a problem solving activity, although it did not have the journaling/justification component. Once the journals had been implemented, we would begin math class, which was approximately one hour each day, with the problem solving journals. The problems were either copied and stapled into their journals, or written on the overhead, in which case the students would copy it into their journal before solving it.

Though I did have the constraints of having to teach the Saxon Math curriculum (Saxon Math 6/5, 2004) I tried to give them as much time as they needed to work through the problems. Generally this was not more than 20 minutes per class period. Initially this took longer, as the students had to adjust to
this change in their scheduling. Occasionally there were a few problems that took up the majority of a class period. On the occasions when this did happen, I allowed it to take over the class because I saw the children working through some difficult processes, which I saw as very productive, and thus worthwhile; the students were asking excellent questions and building their mathematical understanding.

Generally, I presented the problems and let the students work through them. When the students had questions we discussed them and sometimes discussed ways to work through the problems. As the students worked, I would circulate and observe, taking notes, asking questions, asking guiding questions where needed, and encouraging those who quickly gave up. I would also engage the students in conversation when they initiated it. This turned out to be especially helpful to those who had difficulty generating written explanations to justify their answers. Many of my students are more comfortable with spoken language than they are with the written word. By guiding the students and having them give verbal explanations they were able to gain more confidence in their writing, and thus take more chances in their journals.

The problems were worked through on the pages of the journals and once a solution was arrived at the students were then asked to explain how they worked through the problems as thoroughly as possible. I allowed the students to share ideas and discuss the problems with each other. Some days the class worked furiously, but in near total silence as they worked through their ideas, and on other days the class was abuzz with discussion and excitement. Oftentimes after their
writing was completed we would discuss solutions and the methods used to arrive at them. This was especially a lot of fun when there were multiple solutions to problems. I tried to structure the class so that we would have ample time for discussion, but I did not want to do this by taking away from their active time of problem solving and justifying.

These journals were the main source of the data used. In addition to the journals, I took field notes, both during class and at the end of each day. Sometimes these notes would focus on processes and behaviors observed, other times they would be based upon conversations, between me and the students, as well as small group and full class discussions.

Once the data were gathered, I read each of the journals several times in order to immerse myself in the data and let the thematic elements jump out at me, rather than imposing them on the journals. As I began reading the journals I wanted to see what the students were thinking and if there were any particular foci that repeated themselves in multiple journals. As I read each journal I wrote on post-it notes about what I saw in each, whether they were engaged or not, if they were writing good detailed explanations of their strategies, what strategies they used, and, whether they were actually able to solve the problems or not. I also made notes about particular problems that I thought might make good material for writing about in the analyses.

After reading each of the journals and taking these notes I posted the notes on the cover of each journal and separated them into three piles. One, contained
those journals which I thought would provide good subject material for this project; another was for those journals which were possibilities, and the third I deemed unusable for a variety of reasons. Ultimately, I chose the three journals which I analyzed because I thought that they would provide the broadest look at the different levels of learning and understanding within the class.

Once I had narrowed down my choices (to about a dozen), I re-read them once again and began to jot down similar themes. The initial themes that jumped out at me after the third readings were as follows:

- There was great hesitation (especially at the beginning) on the students' part to explain their solutions. They don't want to explain, they just want to do. They love doing math, but not writing about it.

- A lot of the students would rather have an incorrect answer fast than taking time to explore other possibilities. (Again this was especially true at the beginning).

- There was a great deal of careless mistakes caused by hurrying. The students don't always use the tools that are available to them (what they know), but their strategies do improve over time.

- Some of the students do use a variety of strategies.

- Many of the students have difficulty with written expression.

- Children with a good conceptual understanding of mathematics were able to solve and justify a wider range of problems on a consistent basis.
I chose to base my analyses on Nicholas, Sarah, and Theresa’s journals because I felt that they best represented the cross-section of the themes I found to be prevalent throughout the class, as well as the fact that they were representative of what I thought was the higher as well as the lower level of achievement within the class. Finally, I chose them because I thought that they would be the most interesting to write about. Nicholas, as it turned out was the most enjoyable to write about because I feel as if I really learned a lot about him in particular and about students struggling with mathematics in general.

Curricular Issues

Before moving on, I would like to say a word about the curriculum used at this school. My school uses Saxon Math which is heavy on isolated skills, and relatively short on problem solving. Saxon says that they are “the nation’s best selling and most thoroughly researched skills-based mathematics program for grades K-12… [That] incorporates 20 years of research and classroom experience” (Saxonpublishers.com).

Though the Saxon curriculum does have a problem solving component to it, many of the problems are simplistic and one dimensional, requiring little actual problem solving beyond simple algorithmic function. The students found them somewhat repetitious and often neither enjoyed them nor fully engaged with them. I chose the problems for this project based on several criteria. I wanted good problems that, in most cases, had multiple methods for solving. I also wanted to give them a good variety of different types of problems. I drew the problems from
several sources. Some were from the “problem library” that I was given by Dr. Lynae Sakshaug who taught my methods course in mathematics at SUNY Brockport, some came from the Saxon curriculum, while others came from internet resources, and still others I made up.

Problems with multiple solution strategies allow students freedom to think in various ways and permit the formulation of ideas which lead to the students working through the problem which then gives them ownership of the problem. When problems are given where the answer/methodology is easily determined, and the students didn’t have to invest much, they are simply doing algorithms, which is what they didn’t like about the Saxon curriculum problems. Also, choosing problems with multiple solution strategies made the problems more accessible for different children. Finally, in choosing problems I wanted to give the students challenges, but at the same time I did not want to give them problems that would frustrate them to the point of giving up. Although this did indeed happen on occasion, for the most part a high percentage of the students were engaged and actively working problem solvers on most days.

The Saxon math curriculum focuses heavily on skills and for the most part their problem solving curriculum is rather pedestrian and unchallenging. According to their literature, the fifth grade curriculum is an integrated program of 120 daily lessons and 12 activity-based Investigations. Students are tested after every fifth lesson, and all tests are cumulative. Concepts are introduced incrementally and are continually practiced throughout the problem sets. After
using this curriculum for several years, my colleagues and I all have the same issue with Saxon, aside from the fact that it is algorithm heavy, they introduce new concepts and skills too quickly (a new concept each day, often in a seemingly random order), and even though the students are not assessed on new concepts until they have practiced them through ten lessons, there is a new concept or skill introduced each day. The students often feel like they are unable to become familiar with, much less master, a skill before they are introduced to a new one.

Whereas Saxon does introduce skills and concepts “incrementally” as advertised, at times they seem like their sequence was determined by a random shuffle of the I-Pod. One day’s lesson might be on multiplying three digit numbers, and then the next day could be division with a zero in the quotient, followed by the naming of polygons of various sides. Occasionally there are two or even three new skills introduced in a single lesson. The students often point out the fact that they get confused by so many new skills; I’m not surprised. But, the curriculum must be taught in order because of the way the assessments are structured, with a written assessment every five lessons which will cover all of the lessons taught up to the fifth previous one. That is to say that after lesson thirty-five there is an assessment which will cover all material up to and including lesson thirty. Although the administration at my school was supportive of this project, there was a certain amount of pressure on me and the other staff to move through the curriculum and “teach all of the lessons in the book.”
Just to give an idea of the daily plan, let’s take a look at the Saxon lesson structure. Each daily lesson consists of the following:

- **Warm-Up** (8-10 minutes)
  Students take a quick Facts Practice Test to increase their proficiency with basic operations; solve several mental math problems; and complete a problem-solving exercise by using such strategies as making lists, drawing pictures, working backward, and guessing and checking.

- **New Concept(s)** (10-15 minutes)
  The teacher presents the new concept(s) and works several examples with the class.

- **Lesson Practice** (5-10 minutes)
  Students solve problems that cover the new concept(s).

- **Mixed Practice** (20-30 minutes)
  Students solve problems that provide practice on previously introduced concepts as well as the new concept(s).

The Saxon daily lesson plan may look reasonable, but when it comes to executing it in the classroom it is extremely difficult to accomplish. There are the inevitable questions that must be answered, and how can anyone expect that five to ten minutes of practice, immediately following the introduction of a new concept or skill is sufficient before sending the students to complete thirty problems of homework which cover not only the new skill but also all of the ones previously
covered. I find that Saxon is all about “covering” a lot of concepts and skills in a rather dubious order, rather than providing the context for understanding, application and synthesis.

Demographics

The administration at my school decided that we would group the students at our grade level into three groupings based on their “ability”: proficient (high), basic (middle), and novice (low). The group that participated in this study was the proficient group, which is to say that they were deemed the highest performing 1/3 of the fifth grade class. This was determined by using the Saxon Math placement test, and beginning of the year assessment, which consisted of a twenty question assessment well as their score on the Metropolitan 8 standardized test taken at the end of the previous school year. Teacher input was also considered, as I looped with many of these children, having taught them in fourth grade the preceding year.

Of the students involved in this project 20 were African-American, 2 were Hispanic, and 2 were Caucasian. There were 13 females and 11 males.

The Problems

Date

3-29-04 Write a letter to me telling me how you like math class this year. Tell me what you like and or don’t like, also include anything that you might find particularly difficult.
There were 6 frogs on each lily pad in the pond. There were 18 lily pads in the pond. What is the total number of frogs in the pond?

You have seven coins that equal $1.00. What are the coins?

I am a number less than 100. My units digit is a 4. The sum of my digits is an odd number. My tens digit is a multiple of 3. Who am I?

Monica is thinking of a number. Can you guess her number using these clues? It is a three digit number. Its digits are in descending order. It is divisible by three. The hundreds digit is two more than the ones digit. Add the three digits and it will give you a dozen. What is the number?

Jeff has less than 30 marbles. When he puts them into piles of 3 he has no marbles left over. When he puts them into piles of 2 he has 1 left. When he puts them into piles of 5 he has 1 left. How many marbles does he have?

Amanda is 21 years older than Sue. The sum of their ages is 57. How old is Amanda? How old is Sue?

Serena had 265 frogs. All but 187 died. How many frogs does Serena have left?

Todd has 8 sets of cards. There are 24 cards in each set. How many cards total does Todd have?
If Tiffany has $5.00 and buys 4 balloons for 35¢ each and 2 pencils that sold 2 for 16¢. How much did she spend? How much change did she get back?

I am thinking of a number that is the square of 20% of 25. What is the number?

Jordan, Jamal, Tyrone, and Dave shared a pizza. Jamal had 3/8, Jordan had 1/4, and Tyrone had 1/8. How much was left for Dave? Draw a picture and write your answer as a fraction.

Write about something in math class that you find difficult.

Write as many ways as you can think of to make 36.

If Neil, Joshua, and JR are given 54 pencils to share, and they divide them evenly, how many will each boy get?

Julie ran 675 yards in two minutes. At the same rate, how many yards will she run after six minutes? How many will she run after seven minutes?

Luigi bought 6 boxes of nails for $1.79 each and a hammer for $23.99. If he paid with a $50, how much change did he receive?

Paddy Hill School had a goal of collecting 400 cans in April. This is how many collected so far: 179 (week 1), 116 (week 2), and 84 (week 3). How many do they need to collect in week 4 in order to meet their goal?
The pool opens at 9:30. It takes Tom 35 minutes to ride there. What time must he leave if he wants to arrive 20 minutes early?

Mr. Swenson wants to tile his bathroom floor. Each is 12 square inches. If the room measures 8’x12’, how many tiles will he need?

A restaurant has 5 tables for 4 people, and 4 tables for 6 people. What is the greatest number of people that the restaurant can seat at one time?

What are fractions? How are they useful?

List, use pictures, etc. to show as many different ways to make $1.00 as you can.

James bought 6 Yu-Gi-Oh cards on Monday. On Tuesday he bought twice as many. If this pattern continues, what is the total number of cards he will have bought by Friday?

Charlie is making a necklace of beads with a red, white, blue pattern. If the pattern continues, what color will the hundredth bead be?

Fernando can carry six bags of fertilizer at one time. If 4 bags weigh 60 pounds, how much do 6 bags weigh?

Junior was going to have a party as soon as he collected 100 bottles. He started with 27; on Monday he collected 15, on
Tuesday he collected twice as many as on Monday. How many does he need to reach his goal?

5-21-04  34 of the town’s 86 firefighters march in a parade. What fraction of the firefighters marched? What fraction did not?

5-24-04 Chester was given a large bag on cubes. He was instructed to build a 15-step staircase. If it takes 6 blocks to build a 3-step staircase (picture), how many blocks will be needed to build the 15-step staircase?

5-25-04 Three painters can paint 6 fences in 3 hours. At this rate, how many fences can 7 painters paint in 5 hours?

5-27-04 What is the greatest number of tables that can be used to serve 55 people so that each table has a different number of guests?

5-28-04 If you were to list all the numbers from 1 to 999, how many times would the digit ‘8’ appear?

5-31-04 Charles was given a $100 gift certificate. If he could buy 3 games for $25, how many could he buy with the whole $100?

6-1-04 There is a number machine that performs the same operation on every number you enter. If you enter a 6, it returns 18. If you enter an 11, it returns a 33. If you enter a 14, it returns a 42. What is the rule for this machine?
6-2-04  Two cars are 60 miles apart. At 9:00AM, they start driving
towards each other at 20 miles per hour. At what time will they
meet each other?

Summary

Having now discussed how and why this project came to be. It was with
great anticipation that I began this project in earnest on March 29, 2004. As I have
shown, the mandated curriculum was a major contention in implementing problem
solving and journal writing into math class. It was fortunate that we had a one hour
to one hour and fifteen minute block of time in which to put them into practice.
Having read the problems there is but one place to go, and that would be on to the
analytical phase.
Chapter Four: Analyses

"...why can’t we just do the problems instead of wasting all this time explaining our answers?" Conor, age 10

Introduction

In this chapter I will first provide an overview of some of the commonalities that I found across the class, and the emergent themes which came from reading the students’ journals. Then I will briefly discuss why I chose the three students whose journals I analyzed and how I used the emergent themes to perform the analyses. Finally, I will present, individually, the analyses of Nicholas, Sarah, and Theresa’s journals.

Initially there were several threads which stood out amongst all the data. First and foremost was a great reluctance on the part of many of the students to do any writing in math class. There was even greater hesitation on their part to explain their answers. They just wanted to solve the problems and be done. After much discussion during the first week, I got most of them to buy into the notion that I might be better able to help them learn if I could understand their difficulties, and if they could show me, through their writing about why they did particular things I could then be better able to teach them. I found that there were a number of children who would rather have an incorrect answer on their paper quickly, than take the time to explore a variety of ways of solving problems.

A second thread that was prevalent in many of the children’s journals was their difficulty with problems that had multiple conditions. Most of the students
were able to easily solve many of the problems that had only one condition, such as "the pencil problem", which reads as follows: If Neil, Joshua, and JR are given 54 pencils to share, and they divide them evenly, how many will each boy get? Most everyone was able to figure out a strategy for solving this problem. However, many of the students found the more involved, multi-step problems were definitely a challenge. For instance the rate problems, such as the "Julie ran" problem and the "Fernando" problem, were especially perplexing for many:

- Julie ran 675 yards in two minutes. At the same rate, how far will she run after six minutes?
- Fernando can carry six bags of fertilizer at one time. If 4 bags weigh 60 pounds, how much do 6 bags weigh?
- Two cars are 60 miles apart. At 9:00AM, they start driving towards each other at 20 miles per hour. At what time will they meet each other?

Every single child in my class wrote that they loved to do math, but at the same time many of them did not like to write about it. I found that this was especially true of those who love to do algorithmic mathematics. For many children, especially those, like my students, who go through elementary school mathematics with a procedure based curriculum, such as Saxon, mathematics is strictly about the four basic functions: addition, subtraction, multiplication, and division, with a bit of fractions and decimals thrown in for good measure and spiced up with a handful of geometry and pinch of basic algebra. Indeed, many of
my students, having spent their early school years in just such a system, were quite reluctant to actually write in their journals. But over time this too changed. Only a few of them maintained their resistance and did little to no writing over the 10 week period, and not so incidentally these same children rarely went past the basic function questions on the weekly assessments mandated by the Saxon Math program.

I would like to posit here that I also believe that their reluctance to work out the “word problems” may stem, at least in part, from difficulties with language. Indeed, quite often in their journals the language is not clear and the students’ processes seem convoluted, but understanding comes through and perhaps most importantly, it allows for an excellent look into the thought processes that students go through when attempting to solve problems. This was especially true of Nick’s work, which I will look at in detail below. There is definitely a great need to work on clarity of meaning and making correct word choices.

I did notice a good deal of haste in their work and a lack of willingness to check answers and written solutions for sense and correctness, especially in the early part of the project. There was also a great deal of difficulty in solving problems where there are multiple conditions which must be satisfied. Some of the students developed the ability to use a variety of strategies, and over time their choices and numbers of students choosing better strategies improved. In fact, by the end of the ten weeks there was a great deal more willingness to check answers and try different strategies if their first one did not work.
As I read the students’ journals I was struck by how well many of them illustrated the ways in which the children think mathematically. Therefore, the main focus of the following analyses will be to show and discuss these children’s mathematical thinking. Let’s dive in.

Nicholas

I jokingly refer to Nick as the algorithmic animal because he was one of the students most troubled by this project. He loved to work with algorithms, and would do page after page of them given the opportunity. He did not like to write and was very reluctant to do so. In fact, given only a cursory glance, Nick’s journal appears to be that of a lazy disengaged student. He, along with many of my other students has a great deal of difficulty expressing his ideas in writing. Only by getting to know them and interpreting their meaning is a person able to understand what Nick and his classmates truly know.

On some of the easier problems, such as the “frogs” problem one, Nick was able to do well. The problem reads as follows:

There were 6 frogs on each lily pad in the pond. There were 18 lily pads in the pond. What is the total number of frogs in the pond?

Nick knew to multiply the 18 and the 6, and his answer was correct, yet his explanation was simply to state that “I X 18 by 6 to get 108 is the total number of frogs in this pond.” This was a very common type of explanation, where the student would simply justify his or her answer by saying “this is what I did, so it is correct.” Now, Nick’s answer, in this case, is indeed correct. He used an
appropriate method to arrive at his answer, but either lacks the appropriate language facilities necessary to explain it more fully, or it is quite possible that he believes he has provided absolute proof and a legitimate mathematical explanation. I did work with him on expanding his justifications, but he never really got to why his answers were correct, which indeed, quite often they were. They simply restated the steps he used to arrive at his answer, although it is also likely that it has as much to do with his understanding that to do mathematics means to do algorithms. It may be that he’s explaining in the only way he sees as legitimate; yet I’m sure that his work would please the folks at Saxon Math.

Another good example of this occurs when Nick attempts to solve problem number four:

Monica is thinking of a number. Can you guess her number using these clues? It is a three digit number. Its digits are in descending order. It is divisible by three. The hundreds digit is two more than the ones digit. Add the three digits and it will give you a dozen. What is the number?

There are several conditions which must be met for the solution to this problem, yet Nick chooses one of these conditions, that the sum of the digits is equal to 12, meets it and concludes that his answer, 381 is correct. He does meet all of the conditions for the problem except for one: that the digits need to be in descending order. His justification reads as follows: “I got my answer by adding $3 + 8 + 1 = 12$ so that’s how I got my answer....” After a little prodding I got him to check it and
he responded “it is divisible by 3, the hundred’s digit is two more than the one’s and they equal twelve. Upon further conversation on the matter he seemed agitated that I was not happy with his answer, so I explained that it was not a matter of happiness, but the fact that you must meet ALL of the conditions of a problem in order for it to be correct. Though I tried to mollify him by expressing my pleasure with the work that he had done, but that there was yet another step to take in order for it to be perfect; by this time he was frustrated and he shut down.

I believe that there were more conditions than Nick was ready to deal with at this point. I don’t know that this was as much a factor in his inability to write about his solution so much as the fact that he was just not ready to deal with multiple conditions. I would venture that this has a lot to do with his learning of math to this point which seems to be heavily weighted in the procedural, (this was Nick’s first year at the school, so I’m unsure what curricula he had been using previously), rather than the conceptual. In this case, the problem is out of Nick’s zone of proximal development, so his coping mechanism was to shut down. I would also guess that Nick had not been exposed to these types of problems before.

He bounced right back though, and continued to give a good effort each day. I was impressed by his ability to work through the simple algebra problem I gave them a few days later:

Amanda is 21 years older than Sue. The sum of their ages is 57. How old is Amanda? How old is Sue?
Nick immediately attacked this problem, starting by adding $57 + 21 + 21$, though once he saw that their sum was 99, he regrouped and using the fact that the sum of their ages was 57, he added pairs of numbers, starting with $27 + 30$ and manipulating them by increasing one and decreasing the other through nine different pairs of addends until he met both conditions of the problem and arrived at the correct answer of 39 and 18. Quite the good strategy! Though he offered no written explanation, his process is obvious. He used the condition that the sum of their ages was 57. He showed that he was able to deconstruct that number and manipulated the numbers in a reasonable and logical fashion until he was able to meet the other condition as well. He was able to use one of his strengths, the addition algorithm to help him strategize his way to the solution. Indeed, he was quite happy as he plugged along adding and adding. The big difference between this problem and the previous one, for Nick, anyway, was the fact that there were only the two conditions to reckon with rather than the five in the Monica problem.

Perhaps what we are seeing at work here is social constructivism at work. Previously, Nick was unable to solve the “Monica” problem due to its multiple conditions. It is possible that through our interactions on that problem that Nick was able to fathom the idea of multiple conditions, and thus able to work through a solution to this problem, where he was unable with the previous one. However, Nick provided no justification for his solution, which harkens back to the point made previously about Nick’s reliance on algorithms (again evidenced in his
solution here), and his possible belief that showing the algorithms used constitutes all the proof one might need.

When Nick attempted the “number machine” problem, which was one of the last problems assigned, he did make a better attempt at explaining and justifying his answer. The problem reads as follows:

There is a number machine that performs the same operation on every number you enter. If you enter a 6, it returns 18. If you enter an 11, it returns a 33. If you enter a 14, it returns a 42. What is the rule for this machine?

To solve the problem Nick drew a machine with the six, eleven, and fourteen going in and the corresponding products coming out. He writes “The rule is that it [the machine] multiplies by 3. \(6 \times 3 = 18\), \(11 \times 3 = 33\), \(14 \times 3 = 42\), so the rule is multiplies by 3’s.” I was surprised to find that he wrote the following justification for his answer: “I got my answer by putting a \(12 \times 3 = 36\) then putting 13 in [and] x by 3 and 39 come out so I [sic] rule is multiplying by 3’s.” I think that this is a big step for Nick. Although he is still relying on algorithms, we can clearly see that he is moving beyond what is given, and actually checking his rule by applying it to other numbers. Again, he uses the multiplication algorithm to arrive at his answer, a good strategy considering that it is obvious to him that the machine multiplies by three.
About a week later I posed a problem that I think serves to show just how clear certain problems are for him to solve and how automatic some processes have become to Nick. The problem reads:

I am thinking of a number that is the square of 20% of 25. What is the number?

Nick started with 5 and when I asked him why, he looked at me like I was trying to trick him and said, “Well, it’s twenty percent of 25 isn’t it?” So I left him to finish. He then squared 5, showing $5 \times 5 = 25$ on his paper, excellent work. Once again, he falters in his explanation, only because of his difficulty with written language. Nick obviously knows how to compute percentages in his head, albeit simple ones and he clearly understands the concept of squaring as well, yet, if assessed by his explanation he seems to be confused at best. Nick writes: “I got my answer by figuring out the square route [sic] of 20% of 25. By multiplying $5 \times 5 = 25$. 5 is the square number of 25 from 20%.”

Without having discussed the problem with Nick, I wouldn’t think he knew what he was doing; however, our conversation cleared that up and let me know that we need to work on clarity of meaning in compositional skills. This issue comes up regularly with my students. I do try to be sure to regularly engage my students in mathematical discourse, both with me and with their fellow students, sometimes as a class and other times in small groups. I believe that this is extremely important. If I expect my students to be able to discuss mathematics in meaningful ways, in addition to modeling mathematical discourse, I need to provide them with
opportunities do so on their own and put forth arguments to back up their positions. It is quite likely that Nick believes that he is explaining his thinking, and I cannot be certain that any of them have ever had to think about their work in math class before.

Another common error that I found in abundance was the lack of accounting for multiple items in a problem. For instance, in the "Luigi" problem students are presented with a problem where they would typically multiply and add, then subtract to find the answer, yet the fact that they needed to multiply, or add repeatedly was lost on most of them. Here's the situation:

Luigi bought 6 boxes of nails for $1.79 each and a hammer for $23.99.

If he paid with a $50, how much change did he receive?

Almost all of the students did some form of subtracting the $1.79 and the $23.99 from the total of $50.00, totally ignoring the fact that there were six boxes of nails, not one. In this case, Nick's justification for his answer is fairly well written, but missing the accounting for five of the six boxes of nails. When I directed Nick and his classmates to go back and check their work, many of them came back baffled. Nick's response is as follows: "I got my answer by subtracting $50.00 minus $23.99 which equals $26.01 and I subtracted that by $1.79 which equalls [sic] $25.22 which equals [sic] my answer for Luigi's change that he got back from the cashier." Nick has the right idea, even though he didn't subtract correctly, he just doesn't carry it through completely. I don't think that Nick is able to articulate why he took these steps because he is so wedded to the notion that mathematics is all
about performing algorithmic functions, and is likely used to solving problems that are much more straightforward, one-step to the solution kinds of problems, which require using one algorithmic function and then we're done and out in time for lunch.

Feeling pressed by the constraints of having to cover curriculum, after a few minutes I showed the class the correct solution. In retrospect, I think that the class may have been better served by my having them collaborate to see if maybe they forgot something or if there might be another way of breaking the problem down. This problem within the problem might likely be more easily solved by the visual learner, who may suggest drawing a graphic representation of the problem, which should lead them to draw each of the six boxes of nails, thus solving the problem within the problem. This would be a good strategy for them to use in their journals.

I think that this raises an important point here about the messages teachers send about what it means to do mathematics and to talk about mathematics. As I pointed out earlier, Heibert (1996, 1999) says that teachers are still teaching mathematics strongly rooted in procedures and algorithms and not giving much credence to helping children develop problem solving skills, with the implication that they (the teachers) are doing so because that is how they were taught. If children are taught at an early age that mathematics is all about procedures it can easily become ingrained in them, and the Saxon curriculum certainly reproduces and reinforces such notions.
I find that there is a great deal of reluctance by many of my students to look beyond their first effort, not just in Nick’s particular class, but in general. There seems to be a pervasively apathetic attitude that if something wasn’t done right the first time, then it really isn’t all that important. Perhaps it is a coping mechanism which is used to avoid feeling stupid, choosing indifference over looking bad in front of their peers; or possibly because the emphasis has been on following an algorithm, not on problem solving? This calls into question what preconceived notions these students have about problem solving specifically and mathematics in general.

Another instance of this indifference surfaced with the restaurant problem which is also relatively straightforward, yet many of the students, including Nick, made the error of missing the fact that there is an implied “each” when referring to how many people can be seated:

A restaurant has 5 tables for 4 people, and 4 tables for 6 people. What is the greatest number of people that the restaurant can seat at one time?

Many of my students simply added the numbers, arriving at a total of 19 people possibly being seated. Nick put his own little spin on it, adding the 5 and the 4 for the total number of tables, which is correct, but he also strolls right past the implied “each,” adds the 4 and 6 people and arrives at the tiny bistro total of 10 patrons. As mentioned above, he was reluctant to even look at the problem, saying “that’s the answer” (it worked so well on the squaring problem, didn’t it?). When we read his justification together, he agreed that it didn’t really make sense: “I got
my answer by adding $5 + 4 = 9$ tables. $6 + 4 = 10$ for 10 people. So the restaurant can hold ten people with 9 tables for each person." I asked about 9 tables for each person, hoping that it would spur him to action to see where he went wrong. Even though he agreed that there must be something wrong with his explanation and possibly his whole answer, Nick just would not go back and look at the process to try and troubleshoot. I suggested that he might want to try drawing a diagram of the restaurant to see if that might help him, but he insisted that it wasn’t necessary because he could see it in his head how many tables there were. But traditional mathematics education isn’t about such logical reasoning by individuals—it’s about following procedures.

This is one of the problems that we did go over, in depth, as a class. When presented with the evidence that no one person needed 9 tables and then prodded to draw a graphic representation there was a mass illumination of light bulbs over their collective heads.

Summary

Nick has a genuinely positive attitude about mathematics in general and math class in particular. I was able to see a change in his attitude toward problem solving from the beginning and the end of the ten weeks we used the journals. At the beginning of the ten weeks Nick was very difficult to engage in this "weird" way of doing math. Nick did begin to engage the problems rather early on and by the end of the project he was at least attempting to provide a more thorough justification of his responses as evidenced in the "number machine" problem. I
think that over time Nick became more comfortable with what we were doing and became willing to take a chance at using words to justify his answer.

I firmly believe that Nick is relatively common in his conception of mathematics as generally procedures and algorithms. Math is one discipline in school where he has had more success than in others because he has developed a fairly good number sense and is good at repetition and memorization.

Nick struggles with problems like the restaurant problem above because the notion of math as algorithms is so solidly ingrained in him that he sees numbers and like a great number of my students, just wants to do something with them. The Saxon curriculum teaches students to look for key words or phrases to let them know what function to perform, and off they go, adding like crazy. When students learn to focus on the key words they learn that they themselves do not have to think beyond “each means you multiply.” This is a clear example of why you want to learn the trade and not just the tricks.

The restaurant problem is a good example of this because of the implied \textit{each} which is not there. So Nick and all the algorithmic animals see that they are looking for a total, and thus start adding without taking the time to see that \textit{each} of the five tables seats four people and \textit{each} of the four tables seats six.

I believe that Nick would benefit from continuing to solve problems and practice learning to justify his reasoning, orally at first, as he did with the restaurant problem above, and working to transition oral proof to written proof. This may be a good method for engaging a child, such as Nick, who is generally unwilling to do
writing of any sort, by getting him to write about something (a good oral justification for a math problem) that he would undoubtedly be proud of. In this way Nick could improve his language skills by writing and revising justifications of math problems. Perhaps he just may start to like writing after all.

Nick would also benefit from continuing to problem solve and justify his answers because it will help him to see that math is far more than just the four basic functions. During the ten weeks, Nick became more willing to examine his work, to accept that his first strategy toward a solution may be wrong, and more eager to try to find ways to better solutions. After our group discussion of the restaurant problem Nick was upbeat, positive, and seemed to grasp where he went wrong with the problem; but more importantly: he cared. If children are to be successful they have to care whether they are or not.

Sarah

Sarah is a confident and articulate ten year old who lists math among her favorite subjects in school. She asks for extra practice homework, and after I taught a lesson on basic algebraic function, she almost daily asks for new problems to solve. Sarah is extremely self-motivated. She grasps new concepts relatively easily and is eager to learn as well as demonstrate her mastery. She also enjoys staying after school in order to provide extra help for some of her struggling classmates. She is, indeed, one of those children who are a true pleasure to teach.

Her journal is well written and she does an outstanding job of justifying her answers in a thoughtful and articulate manner on a regular basis. I really enjoyed
talking to her about problem solving as she seemed to possess a wisdom and confidence that were unusual for a child her age. For example, in response to this “I am a number” problem:

I am a number less than 100. My units digit is a 4. The sum of my digits is an odd number. My tens digit is a multiple of 3. Who am I?

She quickly finds the answer and formulates a thorough justification of it.

Although this is not a particularly difficult problem, Sarah does an excellent job of walking us through her thought process and shows us how well she understands what she does and why. She clearly shows how she works through what she decides is step 1, writing: “What number plus 4 equals an odd number? 4 + 1 = 5, 4 + 3 = 7, 4 + 5 = 9, 4 + 7 = 11, 4 + 9 = 13” [this last one is circled, denoting that it is correct]. She goes on to her next step, saying that “because 9 is a multiple of 3, and we know the digit in the tens place is a multiple of 3. So, 94 [bold, hers] is the number because the ones digit is a 4, also 9 + 4 = 13 and that’s an odd number.”

Her explanation is reasonably thorough and shows that she is able to choose good strategies. She underlined “my units digit is a 4,” so she knows that she will have to add a number to the 4 and have the sum equal an odd number, so she makes her list of possible choices and then uses the other condition, that the tens digit be a multiple of 3, to eliminate the other choices in her list.

I walked past her desk and upon seeing how quickly she finished; I challenged her to see if it was the only answer that fit all the criteria. She quickly looked at her list of sums and then wrote the following: “It can be 34 [also]
because $3 + 4 = 7$ and that's an odd number too. The ones digit is 4 and 3 is a multiple of 3 so it has to be 34. She not only demonstrates clarity, but she is rather concise in her explanation, even though she leaves out the part about it having to be a two-digit number, she is likely satisfied because her answer is a two-digit number.

Many of the students had difficulty finding a place to start working with that problem, although most were able to derive a solution (interestingly, most came up with the 9 and not the 3 in the tens place). We discussed the multiple conditions of the problem and I suggested that they begin with one of the conditions and then work on meeting the other conditions. Once given clarification about what was meant by "the sum of my digits" and "my tens digit is a multiple of 3," they began writing and figuring and deriving solutions.

Thus empowered, they asked if I would give them a similar problem, which I did the next day with the "Monica" problem:

Monica is thinking of a number. Can you guess her number using these clues? It is a three digit number. Its digits are in descending order. It is divisible by three. The hundreds digit is two more than the ones digit. Add the three digits and it will give you a dozen. What is the number?

Sarah gave an even better explanation to accompany her solution than she did for the frogs/lily pads problem. Especially noteworthy is her usage of correct mathematical terminology. I have found that this is an especially difficult bridge for intermediate students to cross. They tend to hang onto terms such as "take
away” and “times” rather than subtract and multiply. I say this, not as a necessarily bad thing, but rather an illustrative point showing that children are able to utilize the proper mathematical discourse if we teach them to. The justification of her answer, 543, reads as follows: “5, 4, and 3 have a sum of 12, because a dozen is 12. 543 can be divided by 3 and have a quotient [sic] of 181 (she shows this by using the algorithm for short division). Its numbers are in descending order. Its hundreds digit is a 5 and its ones digit is 3, so 5 – 3 have the difference of 2 so 5 is 2 more than 3. 543 is also a 3 digit number.” Sarah clearly demonstrates the ability to satisfy multiple conditions in a problem, and she does so convincingly, showing her understanding in her well written justification.

Sarah did an equally fine job with a multi-step problem, the “Balloons and Pencils” problem, which I presented to the class about two weeks later:

If Tiffany has $5.00 and buys 4 balloons for 35¢ each and 2 pencils that sold 2 for 16¢. How much did she spend? How much change did she get back?

Though Sarah had no difficulty with this problem, I used this problem and the class’s struggle with it to reinforce the importance of taking one’s time, and being certain that you take the time to understand the question and whether there are multiple conditions, before trying to do it, or at the very least, check them again when you think you have solved it.

As Nick did with the Hammer and Nails problem, many of the students read the problem as if the pencils cost 16¢ each rather than 2 for 16¢, therefore their
answers would be incorrect. Sarah however, caught the subtlety. She wrote her answer as such: “Tiffany spent $1.56 on 4 balloons and 2 pencils. Tiffany got $3.44 as change.” Below is the work she used to arrive at her answer:

\[ \begin{align*}
2 \\
\cdot 0.35 \text{ balloons cost} \\
\times 4 \text{ how many balloons} \\
\$1.40 \text{ for 4 balloons} \\
+ \cdot 0.16 \text{ for 2 pencils} \\
\$1.56 \text{ total cost of products} \\
\hline \\
491 \\
\$5.00 \text{ how much money she had} \\
- \cdot 1.56 \text{ how much money the products cost} \\
\$3.44 \text{ how much money she got in change}
\end{align*} \]

Not only did she label her work in a clear and efficient manner, but she offered the following explanation: “I multiplied 35 X 4 because she bought 4 balloons for 35¢ each. I added 16¢ because 2 pencils are 16¢. I subtracted $1.56 from $5.00 because I wanted to find out how much she [Tiffany] got in change. This is a great example of a child reasoning and using algorithms correctly in order to solve the kind of problem, though somewhat simplistic, that we as people may have to solve.
on a regular basis. I do believe that Sarah has the ability to be a good comparison shopper.

Though Sarah did outstanding work on a daily basis, there were some problems with which she had difficulty and there was some clear evidence of her building on her existing knowledge and learning from her experiences. There were several ratio problems that I gave the students to work with, and though Sarah solved half of the next problem correctly, she shows the importance of paying close attention to all the details of your work. I call this one the “Julie ran” problem:

Julie ran 675 yards in two minutes. At the same rate, how many yards will she run after six minutes? How many will she run after seven minutes?

Sarah chose an excellent strategy to help her solve this problem; she created tables, which I have recreated below:

<table>
<thead>
<tr>
<th>Yards</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>675</td>
<td>2</td>
</tr>
<tr>
<td>1350</td>
<td>4</td>
</tr>
<tr>
<td>1675</td>
<td>6</td>
</tr>
<tr>
<td>5400</td>
<td>8</td>
</tr>
<tr>
<td>10,800</td>
<td>10</td>
</tr>
<tr>
<td>21,600</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yards</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>337 1/2</td>
<td>1</td>
</tr>
<tr>
<td>1012 1/2</td>
<td>3</td>
</tr>
<tr>
<td>1687 1/2</td>
<td>5</td>
</tr>
<tr>
<td>3037 1/2</td>
<td>7</td>
</tr>
<tr>
<td>5737 1/2</td>
<td>9</td>
</tr>
</tbody>
</table>

In addition to the tables, she also used a division algorithm which shows her dividing 675 by 2, in order to find how many yards Julie ran in one minute, which is 337 1/2. I was intrigued by her quickness in coming to this conclusion, so I
asked her about it. She said, "I need to find out how far she runs in one minute in
order to find out how far she runs in seven minutes, so I can add the distance she
runs in one minute to the distance she runs in six minutes. If I know how far she
runs in one minute I can figure out how far she runs for any number of minutes as
long as it is the same rate."

Wow! What a great explanation, unfortunately, when I went back to check
her journal, she had not written any sort of an explanation for her solution to this
particular problem. She must have figured that she had explained herself well
enough orally and that further elaboration was not necessary. I think some of the
children came to value these journals as opportunities for them to show what they
know, but others definitely had the attitude that it was just more work that Mr.
Jonasse wanted them to do.

Upon close examination of the tables above, one can clearly see that her
solution idea is sound, but she was careless in her execution of the solution. For
her answers to the questions, she wrote:

After 6 minutes she ran 1925 minutes [sic].

After 7 minutes she ran 3037 1/2 minutes [sic]

Although her answer for the first question is correct, the table that she created
breaks down after the six minute mark. She began by doubling the distance for 2
minutes from 675 yards to 1350 yards to arrive at the distance for 4 minutes. Then
she doubled that number once again, instead of adding just adding another 675
yards (or multiplying the 675 X 3) to find the distance for 6 minutes. Although she
did catch her mistake on the distance for 6 minutes and corrected it, the rest of the
table shows the distance being doubled every two minutes instead of increasing by
the appropriate 675 yards.

She had the same difficulty with the other table. She began by dividing the
675 yards by 2 in order to find the distance for one minute and she correctly added
675 to 337 1/2 in order to find the distance for 3 minutes, and again to find the
distance for 5 minutes, then for some reason she added her 1350 yards to that,
which would give the correct distance for 9 minutes though, not 7 minutes. I would
hypothesize that the problem here is not confusion on Sarah’s part; I would say that
it is a case of carelessness and hurriedness in the execution of her plan that was her
downfall. She was quite pleased with herself that she was able to formulate a
coherent method so quickly (many of her classmates were moaning about this
problem) and she wrote furiously to finish. I think that a few of the students were
having a competition between themselves to see who could complete their
responses fastest. I think that traditional mathematics places far too much value on
being “fast.” Children pick up on this and those who are fast quickly become the
ones who are considered “good at math” by their peers.

Summary

I would say that from talking to her and from the bulk of her written work
Sarah has a solid grounding in mathematics and mathematical thinking. However,
like many children she can make careless mistakes because she is in such a hurry at
times to complete her work. Her response to the “Balloons and Pencils” problem
shows that she is able to identify subtle distinctions in multi-step problems such as
the fact that there are multiple items discussed, but only a per unit price given.

Likewise, the oral explanation of the method she used to solve the “Julie
ran” problem shows that she understands that there is the extra step involved in
finding how far she ran for a time period outside of a multiple of two minutes, but
she drops the ball in her haste.

Her journal shows that she uses a variety of strategies to solve problems;
she draws pictures, constructs tables and charts, is capable of breaking multi-step
problems into component parts, identify patterns and make educated guesses. She
is not limited to a few “tricks’ in her bag of tools and approaches every problem
with the confidence that she will figure it out.

I think that the journal supported Sarah’s mathematical meaning making by
giving her the opportunity to put her thoughts on paper, see them, rethink them, and
make revisions, which can be difficult to achieve in the mind alone. Her writing
seemed to build her confidence by showing her that she really did know what she
was talking about  As evidenced in the “I am a Number” problem, as well as the
“Monica” problem, Sarah worked through to her solution, clearly explained her
thought processes and when encouraged to look for other solutions she eagerly
attacked the enterprise.
Theresa

Theresa is a very interesting study. She generally does well in math class. She is very good at computation, using algorithms appropriately, and organizing information as well as being fairly adept at problem solving. She, like a good number of her classmates, is a visual learner and she often utilizes pictures and graphics to assist her in arriving at solutions. She is willing to take chances and, unlike many of her classmates, is not frustrated by having to work through multiple attempts to arrive at a correct solution. I noticed this in her solution to the “frog” problem:

There were 6 frogs on each lily pad in the pond. There were 18 lily pads in the pond. What is the total number of frogs in the pond?

She wanted to make sure that she covered all of her bases as far as explaining her solution. Theresa directly identified the problem as an “Equal Groups” problem, as they are called in the Saxon mathematics curriculum. This case called for her to use the multiplication algorithm, which she did, but she also decided to draw the lily pads and the frogs as well. Her illustration shows each of the 18 lily pads occupied by 6 frogs. Her explanation simply states that “eighteen times six frogs equals 108 frogs.” Her explanation isn’t as complete or sophisticated as her illustration. I think she drew the picture because she likes to “see” the solution. I also believe that she was using the picture as justification for her answer.

Many of Theresa’s solutions were accompanied by pictures. Perhaps this is how she was taught to solve problems, or it could be that she just likes to draw. On
the one hand, I think that if a child is able to show clearly with a drawing or other illustration that they understand and are able to solve a problem that it should constitute a reasonable mathematical explanation, but conversely there is also the need to be able to express proof through written language, and like it or not the children will be expected to do so on the state assessments. A drawing, however, is a good starting point, because it shows that the child understands mathematically what they are doing; expository justification would be a goal to work toward.

I was especially pleased to see how confidently Theresa arrived at solutions to many of the problems, such as the following:

You have seven coins that equal $1.00. What are the coins? She quickly arrived at her solution: 3 quarters, 3 nickels, and 1 dime. To check on her work she drew a picture of the coins, added them up and wrote “yes!” to confirm her answer. I would again venture to guess that Theresa’s predilection for graphics led her to draw the picture, and her mastery of mental mathematics allowed her to add up the sum of the coins in her head.

Likewise, she was just as quick to arrive at solutions for both of the problems I presented in which the students were given a short time to list multiple solutions to seemingly simple problems such as the following:

Write as many ways as you can think of to make 36. I wanted to give this problem just to see how much of a handle the students had on the many ways we can arrive at different numbers, and I purposely chose 36 because it is a simple square and has many factors. Most of the class followed a
similar pattern where they stuck to an addition pattern in which they changed each
of the addends by increasing one and decreasing the other (i.e. \(35 + 1 = 36; 34 + 2
= 36\ etc.) or used a subtraction pattern in which they similarly increased the
subtrahend while decreasing the minuend (i.e. \(37 \ - \ 1 = 36; 38 \ - \ 2 = 36\)). This is
not to suggest that I considered those responses to be wrong, but a bit less
sophisticated than I was expecting. Theresa however, after listing the first 10 of the
aforementioned sums of 36 using the standard addition assortment, decided that she
would flex her mathematical muscles and use multiple functions as well as
multiplication. She produced the following list:

11. \(100 - 64 = 36\)
12. \(100 + 5 - 69 = 36\)
13. \(35 + 6 - 13 = 36\)
14. \(34 + 3 - 1 = 36\)
15. \(30 - 29 + 35 = 36\)
16. \(30 + 6 - 29 + 7 = 36\)
17. \(4 \times 9 = 36\)
18. \(2 \times 2 + 32 = 36\)
19. \(6^2 = 3\)

I was pleased to see her branch out and try some different ways, such as
three different functions in one equation, as well as the multiplication examples. I
was especially pleased that she remembered how to square, as the Saxon
curriculum does not spend a lot of time dealing with squares and square roots at
this grade level. This shows me that she has not only a good grasp of the concept, but an interest in accumulating mathematical knowledge as well. This particular problem required no justification beyond listing possible solutions.

I was a bit surprised that she did not include any examples with division, and when I mentioned this to her she looked at me disappointed, as if I was less than pleased with her effort, and she quickly rattled off "72 ÷ 2 = 36; 144 ÷ 4 = 36...." I assured her that I was anything but disappointed in her, but that I was simply wondering about its omission. She smilingly accepted my reassurance as I moved on to the next student.

I think many if not most children tend to look to their teachers as great repositories of knowledge and purveyors of truth. Often when a child is questioned they assume it is because they have done something wrong. Perhaps this is because they are conditioned to this in the primary grades, and it is perpetuated in the intermediate grades by many teachers. It is unfortunate that "wrong" answers are not more often examined in order to see what the student is thinking. I try to combat this by working to build my student’s confidence in themselves by questioning correct as well as incorrect responses in class. I try to have students orally justify their responses whether they are in a book circle or discussing solutions to math problems. If we don’t teach our children to be confident, and articulate, and support their thinking then we are failing them.

Given a similar task, Theresa again showed her mathematical agility when given the following problem:
List, use pictures, etc. to show as many different ways to make $1.00 as you can.

She came up with one of the most varied lists in the class. She included those solutions which were relatively common; after all, these are economically savvy fifth graders. She listed the following eleven ways to make a dollar:

1. 100 pennies
2. 20 nickels
3. 1 silver dollar
4. 4 quarters
5. 10 dimes
6. 5 dimes, 2 quarters
7. 3 quarts [sic], 5 nickels
8. 50 pennies, 2 nickels, 4 dimes
9. 60 pennies, 3 nickels, 1 quarter
10. 2 half dollars
11. 2 nickels, 2 quarters, 4 dimes

The thing I like most about this list as well as the deconstruction of the number thirty-six is that they both are really great examples of Theresa’s good strong number sense and especially her ability to confidently manipulate and substitute numbers in order to arrive at a solution. For instance, I am impressed by the fact that she chose 50 pennies, 2 nickels, and 4 dimes, rather than the more pedestrian 50 pennies and 5 dimes (or 2 quarters). I think this shows that she is trying to set
herself apart from her classmates by not choosing the first solution that comes her way, and perhaps looking for solutions that may not be so common. Although this particular problem doesn’t involve writing per se’, it does show that she is actively engaged in trying to find numerous solutions to problems, at least in this case where it should be obvious that there are at least several different combinations which will yield the desired outcome.

Providing a good counterpoint to Theresa is Nicholas, who stopped at three solutions to this same problem. This is not to say that if pressed he could not have come up with more, but he had more than one solution, and therefore he was relatively content with that. Where Nicholas is to some extent limited, perhaps even stuck, by his somewhat limited problem solving skills, and heavy dependence upon algorithms in order to arrive at solutions, Theresa’s excellent number sense allows her to be able to quickly substitute and arrive at multiple solutions to problems.

Her great number sense is also evident in her solution to the “Amanda and Sue” problem which reads as follows:

Amanda is 21 years older than Sue. The sum of their ages is 57.

How old is Amanda? How old is Sue?

Theresa used the following steps to arrive at and justify her solution:

1. 57 ÷ 2 = 28 r 1
2. 29 + 47 = 76
3. 19 + 37 = 56
4. $19 + 38 = 57$

5. $18 + 39 = 57$

6. $39 + 18 = 57$ years old

7. $39 - 18 = 21$ years older

How old is Amanda? 39 years old

How old is Sue? 18 years old

Here again, her solution shows her excellent number sense. She uses step one to find half of the sum of their ages, by dividing 57 by 2, which is 28 remainder 1. She added the remainder of 1 to the 28, because she said she knows that there can’t be a remainder when talking about two peoples’ ages. Then she started to play guess and check with the numbers. In step two she takes a stab at the solution by approximating the twenty-one year difference in their ages by adding nearly twenty to the halved sum of their ages from step one, sees she is way off by twenty on the sum of their ages, and so subtracts 10 from each to move closer to the correct sum and moves to step three Since Amanda is twenty-one years older than Sue, she took the half and subtracted ten from one and added ten to the other which she knew would at least get her close.

Step four finds her getting closer, but she notices that the difference in their ages at this stage is only nineteen years; thus in step five she makes the final adjustment so that the sum of their ages is 57 years and the difference between their ages is 21 years. When asked about this problem Theresa said that she found this problem to be “fun to do, but pretty easy.” She also said that she added the last two
steps only because she wanted to clearly show that she understood the problem and met all of its conditions. She also said that she had become aware, through our using the journals and discussing them in class, the need to be able to explain her answers clearly. She agreed that being able to do so was important and she thought that practicing this made her understand mathematics a little bit better, and that she had also grown to like writing these justifications. Though she, like Nick, did not offer a written explanation for her process, she was able to explain orally what she did and why it was correct. A lot of these students, to varying degrees, are wedded to the idea of algorithms as math, and like Nick, Theresa saw her extra step as “explaining” her answer. It is likely that she also thinks that showing her work is a sufficient mathematical explanation. I would have to say that I see this as a deficit as far as Nick is concerned because he does not have the language facilities to explain himself, though we see that at the end of the year he is at least trying to develop it. Theresa on the other hand has the language facilities, as evidenced by her oral justification; she just didn’t put it down on paper.

However, this is not always the case. Her solution and justification for the “Fernando” problem is clear and concise. Once again, here is the problem:

Fernando can carry six bags of fertilizer at one time. If 4 bags weigh 60 pounds, how much do 6 bags weigh?

Theresa uses three algorithms to arrive at her solution. She divides sixty by four for a quotient of fifteen which tells her how much each bag weighs. Then she multiplies fifteen by two to arrive at thirty which is the weight of two bags, and
finally she adds sixty to thirty for a sum of ninety pounds for six bags of fertilizer. Though there are other ways to solve this problem, Theresa’s method works well and she justifies it by saying “First I divided 60 by 4 to see how much each bag weighed.” This shows that she made perfect sense of the problem as stated and figured out how much each bag weighed. She continues: “Then I multiplied 15 x 2 to see how much the other two bags weighed (30 pounds) and then I added 60 + 30 because 4 bags are 60 pounds and 2 bags are 30 pounds (90 pounds [total]) This shows her using the good mathematical proof which I know she is capable of.

Although Theresa’s journal is filled with good writing, graphic representations and justifications, it is not without errors. For example in solving the “Serena” problem, she made the fatal mistake of not reading carefully. The problem reads:

Serena had 265 frogs.

All but 187 died.

How many frogs does Serena have left?

In Theresa’s solution she uses the subtraction algorithm correctly to subtract 187 from 265 which has a difference of 78, and she writes as justification, “the problem is asking you to find out how many frogs Serena had left after 187 of them died. She has 78 frogs left.” The critical mistake she, and most all of her classmates made is that they missed the word “but” in the second statement. I wrote the problem on three separate lines purposely to see if the students would catch the “but” but they did not. The solution, 187, of course, is given in the second
statement, but for some reason everyone seems to want to subtract those numbers. I think that her journal entry reveals that she didn’t notice the unfamiliar wording. The blame for this likely lies in the fact that traditional mathematics curricula, Saxon at least, do not challenge students in this way. As stated above, their problems tend to be relatively straightforward and not terribly challenging.

These “typical” problems as cited above by Heibert (1996&1999), NCTM (1989), Knuth, & Jones, (1991) and Shirley, (2000) do not allow students to develop conceptual ideas, and thus when faced with problems that require conceptual understanding, these students are left in the dust because they are stuck merely looking for the procedure to follow.

Another example is the “Julie ran” problem which Sarah worked through, at least conceptually, so nicely:

Julie ran 675 yards in two minutes. At the same rate, how many yards will she run after six minutes? How many will she run after seven minutes?

Theresa solves the first part of the problem easily enough by multiplying:

\[
\begin{array}{c}
21 \\
675 \\
x 3 \\
2025 \text{ yds.} = 6 \text{ minutes} \\
+675 \text{ yds} \\
2700 \text{ yds.} = 7 \text{ minutes}
\end{array}
\]

and she writes: “Because if you multiply 3 x 2 minutes, it equals 6 minutes, So I multiplied 675 x 3 and it equals 2025 yards. To get 7 minutes I added another 675 yards.” Her logic is much like the logic Sarah used to solve the same problem, and
she carries it through to find the solution for the distance ran in six minutes. However, like Sarah, she must have forgotten about the fact that the 675 yards is not for one unit of measure, but for two. Hence her solution falls apart. But, also like Sarah, Theresa shows that fundamentally, she understands how to solve this type of problem, she just failed to do so with this particular problem on this particular day. This problem is more rigorous that the typical Saxon problem in that it requires something different than the norm. Typically a Saxon problem might have you perform the first step of this problem, to find how far she will run in six minutes, which most students would accomplish by multiplying the 675 yards by 3. It is the second question which is so atypical in that it requires the student to find how far Julie will run in one minute by dividing and then adding it to the six minute total.

Theresa made a similar mistake on a problem dealing with area. The students were asked to solve the “Bathroom tiles” problem:

Mr. Swenson wants to tile his bathroom floor. Each tile is 12 square inches. If the room measures 8’x12’, how many tiles will he need?

To which Theresa responded, “You multiply 96 x 12 because 8 x 12 = 96 so you multiply 96 x 12 and it equals 1,152.” She also used the algorithm, which was executed perfectly, but unfortunately, poor Mr. Swenson will be buying enough tiles to do a dozen bathroom floors. Theresa obviously is confusing the formula for finding the area of the bathroom with the more practical question of how many tiles the man needs to buy. She further explained the process saying, “I multiplied 8x12
because that is what size the room is. Then I had to multiply the 96 x 12 because that is how many square inches each tile is.”

She even drew a nice picture which shows the 12’ x 8’ room with 96 tiles on it, but she somehow did not make the connection between the picture and her solution.

This problem does show me two things. Even though she is confused about the application of the formula for area, she does know how to use it, just not always when. She also shows me that she is able to correctly use the algorithm for double-digit multiplication. Thus I can actually use this one problem to make three separate skill area evaluations. But it also shows that she’s using the algorithm without thinking in a realistic way about her solution. Her picture shows the correct solution, but her conditioning to procedures causes her to conclude that she has to multiply by twelve again because she has the formula for area in mind rather than the simpler question of how many tiles he needs.

Summary

As we have seen above, Theresa’s journal provides another excellent look into the mathematical thinking and understanding of a very bright and self motivated young lady. She enjoys math class and approaches each new challenge with aplomb. She is an outstanding problem solver and like most of us, she makes a few mistakes on occasion.

Like Nick and Sarah, Theresa shows a good fundamental grounding in mathematical processes, she is able to solve some problems and justify her processes with efficacy. She shows that she is more adept with concepts than Nick,
but at times the quality is lacking in her explanations and she reverts, as in the
"Bathroom Tiles Problem," to using the procedures that she has learned previously.
Her justifications, at times resemble Nick's, as she uses her algorithms as her proof,
rather than exercising her linguistic talents by writing the sound mathematical proof
that I know she is capable of. But again, this is most likely due to the fact that she
has a mathematical background which over emphasizes procedures (I know that
she has been a student at this school for three years).

Though I know that she is a capable writer, as she has written some rather
LONG pieces for language arts assignments, for the most part her linguistic
abilities are not evident in her math journal. She does write a good justification for
the "Fernando" problem, but she is not consistent in this. I believe that with guided
practice over time she would easily develop the ability to write good sound
justification for solving problems.

Also like Sarah, her carelessness shows in the "Julie ran" problem, as she
finds out the distance for six minutes quickly, but neglects to see that the base unit
of time given is two minutes and not one. This carelessness is also evident in the
"bathroom tiles problem" as she failed to see that she solved the problem but
thought she had to go one step further because it seemed too easy for her. Likely
because she has been conditioned to see that kind of problem as an area problem
and knows that to find the area of a plane you simply multiply.

I think that with continued coaching and support Theresa will develop into a
first-rate problem solver and produce exemplary justifications on a routine basis.
She has a strong background in utilizing algorithms; indeed, she is most often able to automatically perform basic functions in her head. This combined with her strong oral skills and language proficiency in other areas should assure her success.

The main focus of my analyses was on how the children think mathematically, and especially what counts as a reasonable mathematical explanation. These analyses are rich and full of detailed investigations into these three children's mathematical thinking. Although this project did not produce the results I had expected, nonetheless, I did learn a great deal and would like to share some of those lessons with you in the final chapter.
Chapter Five: Lessons Learned and Pedagogical Ramifications

Give a man a fish and he eats for today; teach him to fish and he eats for a lifetime

- Chinese proverb

In this final chapter I will address how the writing in the students' journals was or was not what I expected as well as what I was able to glean from them. I will also discuss what I learned from this project and the ramifications for my pedagogical practice in the future.

Learning by Doing

Let me begin by looking at the guiding questions which I had at the beginning of this project. The first one was: Can the use of math journals help students better understand word problems and other mathematical concepts? I would have to say that after ten weeks this was not especially evident, but again, I raise the point that the research did take place during the last quarter of the school year, and that ten weeks may not really have been a sufficient period of time for this to develop. Were the journals to be implemented at the beginning of the school year and used regularly, I believe that the students would have, over time, started to construct deeper conceptual understandings, better problem solving skills and a more fully developed sense of mathematical reasoning, and thus the ability to justify their answers.

As I discussed in chapter two, Constructivism is rooted in the belief that knowledge is something that each of us accumulates by building upon what we have already learned and internalized. We then build connections and eventual
understanding from what we already know to newer concepts that we are trying to learn. And as Baroody (1987) pointed out, the construction of knowledge is an active endeavor which requires making connections between what you know and building upon that in order to construct new knowledge. Since many of my students have little experience with true problem solving, and their mathematical experiences are largely rooted in following procedures, it may have been asking a lot of them to learn how to solve multi-step problems with multiple conditions and then explain how they did it all over a short period of time. This is a complete inversion of their mathematical paradigm, especially when you consider the fact that most of my students have had several years of the Saxon curriculum. It is likely no mere coincidence that the student who was most capable of writing mathematical proof, Sarah, was new to the school this year and had not been taught using the Saxon.

Nick had an extremely difficult time with the notion of explaining his solutions, as I pointed out above; this is likely because he believed that his algorithms as solutions were all that was necessary to prove his case. Likewise, Theresa showed many of those same characteristics in her explanations; she occasionally wrote good, solid proofs, but she often fell into the same pattern as Nick. When students use algorithms as their proof, this necessitates interpretation and conjecture on the part of the teacher, which may or may not be true to the student’s intent. Meanwhile, Sarah showed the ability to write good proofs which makes it likely that whatever curriculum she had been exposed to, it is highly likely
one which places less emphasis on procedural understanding and more on conceptual understanding.

An excellent illustration of mathematical understanding is found in Sarah’s journal as she writes regarding the “I am a number” problem:

I am a number less than 100. My units digit is a 4. The sum of my digits is an odd number. My tens digit is a multiple of 3. Who am I?

She arrived at her answer and formulated a well written and thorough justification of it. As I pointed out in the analysis of her journal, although this is not a particularly difficult problem, Sarah does an excellent job of showing us her thought processes and makes it evident that she clearly understands what she is doing and why. With Vygotsky’s (1978) *zone of proximal development* in mind, it would have been beneficial to share Sarah’s journal with the class, with the idea that Sarah is the more capable peer and that students at the lower level of conceptual understanding could hopefully start to grasp the idea with the help of their more competent peer.

**Self-Teaching Through Writing**

The second guiding question was: Are students able to learn by teaching themselves through writing and use writing as a tool for learning? This was also not as evident as I had hoped it would be. These journals have shown me a lot about how children think mathematically and the processes they go through in order to problem solve, but at this point I have not seen a lot of support for their mathematical thinking in these journals. Again, I do believe that this is something
which would develop over time. Though I agree with Routman (1994) that writing promotes fluency and chance taking, there really was not a significant display of writing to learn. Sarah did have an epiphany while writing and shared that it helped her to figure out what was really going on with the problem she was trying to solve; this was really the only case of that happening. Again I would have to point to the contrived time frame of this project, and would like to see the results of a full year’s immersion in writing about mathematics.

There is evidence to support this assertion in Theresa’s journal. When examining her solution to the “Amanda and Sue” problem which reads as follows:

Amanda is 21 years older than Sue. The sum of their ages is 57.

How old is Amanda? How old is Sue?

Theresa showed several steps in which she used several algorithms to arrive at the correct answer, but then offered no written justification for it. This is ironic because it was on this day that she told me that she was coming to understand the importance of what we were doing, that is, writing proof in addition to solving problems, yet here she offered no written proof. She did explain to me orally what she was doing, but chose to not write it in her journal.

It is possible that this is due to the fact that she had been learning using the Saxon math curriculum for several years (three at least). Even though she seemed to understand the importance of the work we were doing in this project, she was not conditioned to having to write proof, but was drawing on her Saxon background where justification and proof are just words in the dictionary, not something that
you might do. However, there were also several examples in her journal where she did solve problems and write sound justification for her answer, as evidenced in her solution to the “Fernando” problem. This showed me that she was more than capable, but perhaps it is to some degree a matter of overcoming the bad habits learned from being immersed in poor curriculum, and this kind of compensation can only happen over time.

Sarah clearly demonstrated her ability to write clear and concise mathematical proofs. Theresa has shown this as well, although less consistently; and Nick was even beginning to show signs of this by the end of the project. I find this very encouraging and it leads me to believe that had this project been launched in September and carried through the year, the results would have been much more positive in this regard as well.

**Mixed Messages**

As the NCTM (2000) states in the *Standards*, “Unless students can solve problems, the facts, concepts, and procedures they know are of little use. The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems” (p. 181). Doing this project with the constraint of having to simultaneously teach the Saxon curriculum made the project somewhat difficult. I was still responsible for covering the curriculum, but I did have some freedom. Since I saw the value in allowing the children time to work through the problems in their journals, I tried to give them plenty of time. Unfortunately, there were times when I had to hurry them through so that we could
return to the curriculum which was in no way reinforcing the lessons I was trying to teach them through the journals. Hence, the children were receiving mixed messages from me. One the one hand I was telling them to value problem solving and proof, what we were doing in the journals, but then we were jumping to the parallel universe of Saxon, where it was all about algorithms. On the whole, the students were far more engaged in trying to solve the problems I posed each day than they were in Saxon; although I must admit that there are those who would like to do speed drills, flash cards, and timed practice all day long.

Valuing Proof as a Way of Doing Mathematics

As pointed out in chapter two, NCTM (1989) has five wide-ranging goals for all students: "(1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically" (p. 5). If we are to imbue our children with the skills and the attitude to be successful mathematical thinkers and doers, we have to do so from the time they walk in the door. If we are to teach them to think, reason, and communicate mathematically, then it is imperative that we do it from the time they enter school, and regularly reinforce it as the way of doing mathematics. If we do this I have to believe that children would become confident and capable at it.
Nicholas was able to work through many of the problems which I gave him, but was unable to write the kind of justification that I wanted and New York State will expect. I think that it is fundamentally wrong to expect Nicholas to be able to do something that he hasn’t ever really been taught to do. It is obvious, to me anyway, that his mathematical learning has consisted of lots of algorithms and memorization of the times tables, so is it any surprise that when he solves a problem he uses an algorithm as proof of his answer? I think not. If we are going to stress algorithms, then children are going to use them. However, if we stress reasoning and justification over process then we are going to be teaching children to think, rather than perform. By the end of the ten weeks I was just beginning to see this happen with Nick. I would like to have had the opportunity to help develop this over the course of an entire school year.

I believe that it is imperative that children be capable of arguing a point and to express themselves in writing. Arguing mathematical proof is one very good way of teaching these life skills. Since this is not happening with any regularity in American classrooms, the only way that I can make a difference is to continue to teach using the NCTM guidelines in combination with the lessons of constructivism, by regularly giving my students good problems to solve and the time to solve them, as much as possible, within the curricular constraints of my school.

Also, perhaps administrators need to be pressured to examine other curricula on the market and choose materials that are more in line with the
NCTM's notion of a good mathematics program as described in the Standards. Then we will better serve our students as we prepare them for life in the twenty-first century. If, as NCTM (2000) states, that “mathematical literacy is one of the most important keys to quality of life” (p. 181), We are remiss in doing our jobs as teachers if we do not advocate for our students by demanding better curriculum materials so that we might teach them well.

Math Journals as an Evaluative Tool

Finally, the last guiding question was: Are math journals useful as an evaluative tool? On this question I would have to give an unequivocal yes. Throughout the time we worked on this project I was able to use the journals to evaluate student achievement for both skills and concepts. I learned that many of my students were sorely lacking in conceptual skills, but quite strong in process skills. This is evidenced throughout Nick’s journal, and especially so in Theresa’s journal on the “ways to make thirty-six problem.” She, and many of her classmates, were quite adept at deconstructing the number thirty-six using the “one more-one less” pattern I described in the analysis of her journal. Not only was I able to ascertain what they could do, but more importantly I was able to see what they could not do, not the least of which was problem solve and write proof.

Where do we go From Here?

Mine is reminiscent of the classrooms described by Heibert (1996); Knuth, & Jones (1991); Shirley, (2000); and Wilson, (2003); that were the cause of the national panic attack following the publication of A Nation at Risk (1983). Yet
here we are twenty plus years later and I have a classroom full of children who can add and subtract to beat the band, but for the most part are seriously deficient in conceptual application because far too many schools across the country continue to teach mathematics using the "traditional" methods even though they are not producing positive results. Parents argue "that's how I learned math; It was good enough for me." School boards and principals often bend to the demands of their constituency because they don't want to take the heat. Although we have made some progress in this country, we must continue to strive to realize the worthwhile goals outlined by NCTM.

This is a fight that is worth waging because, as Charlesworth (2000) pointed out, "understanding is not present to any great degree when mathematics is learned as isolated skills and procedures. Understanding develops through interaction with materials, peers, and supportive adults in settings where students have opportunities to construct their own relationships..." (p. 4). Hence, we need to create constructive opportunities in our classrooms by presenting real problems and granting the students real time to solve them.

There is a lot of confusion and a lack of understanding in a very high percentage of my students precisely because most of them have been taught in the "traditional" manner, which is not only a disservice to them, but to our society as well. We must get away from seatwork, timed tests, drills, and "mad minutes" and devote ourselves to teaching our children to problem solve, because as NCTM (2000) states "problem solving is the cornerstone of school mathematics; without
the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited” (p. 181). We must make every effort to see that our children are competent problem solvers who know how and when to use what algorithms as tools to help them solve problems, not as mathematics by themselves.

I work hard to counteract these deficiencies in my students every day. I will continue to try to help my students “un-learn” their dependence on algorithms and we work at solving problems and making sense out of numbers and their applications. It seems that we need the equivalent of a good methadone program to wean the algorithm monkey off the junkies’ backs.

I think that what I did see here through writing these analyses is a fundamental shift in my questioning, not only in my guiding questions as I have already discussed, but also in the types of questions that I ask children when they are in the midst of problem solving. I now tend to focus on helping them to clarify what the problem is asking them to do, rather than what they have to do to solve the problem. I have found that many of my students, either because of language deficiencies or just their own haste, aren’t always sure what question they are trying to answer: they just want to take the numbers given in the problem and plug them into an algorithm and get going.

This was evidenced throughout the children’s journals, but perhaps the best example of this was in Nick’s attempt at the “restaurant” problem. As you saw in chapter four Nick knew that his answer wasn’t correct, that he hadn’t met all of the
conditions outlined in the problem, yet he seemed to believe that because he had
done something with the numbers, plugged them into an algorithm, that his answer
must hold some sort of validity. Telling him what the answer is doesn’t help him to
understand conceptually how to go about solving this type of problem. If I want
Nicholas to really learn, I have to ask him questions that will allow him to uncover
the answer on his own.

As Kamii (1994) and Ostrow (1995) argue, children must be allowed to
arrive at their own conceptions of mathematics and functions; we must necessarily
so allow them to arrive at their own explanations. This means that while Nick and
Theresa use algorithms and pictures to justify their mathematical thinking, we
should accept them as interim proof while they learn how to better reason
mathematically and transfer that skill into writing. My job as the teacher does
require that I prepare them for the state assessments, but most importantly, it is my
job to help them in constructing the conceptual bridges between their mathematical
inventions and conventional strategies, representations, and concepts.

This squares with Nuthall’s (2002) conception of the teacher in a
mathematics classroom, which I believe is worth repeating here. He says that the
teacher’s role is to

Engage with the existing knowledge beliefs and skills of the
students and by setting challenging problems or posing significant
questions to engage...the knowledge building practices...{which}
requires the teacher to be constantly monitoring the ways in which
the students are interacting with each other... constantly focused on
the ways students use reasons and evidence to support their views.
Part of the role of the teacher is to avoid providing students with
knowledge or solutions when it is possible for them to work them out for themselves (p. 47-8).

In order to do so effectively, it is necessary for teachers to sometimes stay in the background and give their students the time to work through problems and arrive at their own solutions.

**Seeing the Forest**

As stated above, I found that in general there was also great deal of reluctance by many of my students to look beyond their first effort. There seems to be a nearly universal attitude that if something wasn’t done right the first time, then it really isn’t all that important. I mentioned the possibility of it being a coping mechanism used to avoid feeling or looking bad. I also would like to raise the notion that if our children are in traditional mathematics programs, where the emphasis is on process and algorithms rather than real problem solving, we can’t really expect them to suddenly be able to solve complex problems and then justify their answers. This calls into question what preconceived notions these students have about problem solving specifically and mathematics in general. If a child’s notion of mathematics and problem solving is limited to add...subtract...multiply...divide, then that is precisely what they will do. If their answer is wrong, even when it is obviously so, as it was to Nick with “the restaurant problem,” they have no other tools in their cognitive schema to help them, so they throw their hands up and “say uncle.”
Schifter & Twomey-Fosnot (1993), as well as Shaw & Blake (1998) argued that the understanding of mathematics is primarily the construction of concepts and the ability to actively interpret those constructions. This can only be accomplished by giving students time to work with and through problems. Also, the NCTM (1989) says that students need to have the ability to build new mathematical knowledge through solving problems, applying and adapting a variety of appropriate strategies to solve problems and be capable of reflecting on the process of mathematical problem solving.

In order to remedy this situation, evidenced by Nick's struggles and to a lesser degree Theresa's, I believe that it is necessary for them to continue to practice "fluting mushrooms" by writing about math and being taught, sometimes explicitly, what a good justification looks like. Their struggles are indicative of a problem much greater than two fifth grade students in Rochester, New York. It is a systemic problem which needs to be addressed not only by individual classroom teachers, but hopefully by school boards and superintendents as well.

If the social constructivists are right, and I believe that they are, we need to provide situations where students can work together in mixed ability groupings in order to support each other's learning. As their teacher, I see myself continuing to provide increasingly complex problems to solve and, allowing them the time necessary to live with and work through them. By modeling for the students what a good justification looks like they will begin to get an idea of how to go about writing one themselves. The logical step from algorithm as justification to good
written proof might be as simple as sharing ideas in small groups or whole group
discussions, and allowing the more capable students to lead the less capable and
help them to build the ability to do it unassisted. By working with the students
within their own zone of proximal development (Vygotsky, 1978, p. 86), they are
allowed the comfort of the familiar, while being stretched to learn the newer and
more complex. By having the more adept students, or even the teacher leading, it
is possible to build up the children's level of mathematical discourse and change
their thinking about what constitutes a reasonable explanation. By having the
students pose their arguments to each other they can build both the ability to do so
as well as the confidence in themselves to take even greater chances.

The argument that children need time to build understanding is a good one.
I believe that students need to take ownership of the process as well as the product.
As I pointed out in my analysis of Sarah's journal, there were some students who
came to value these journals as opportunities for them to show what they know, but
others definitely had the attitude that it was just more work that Mr. Jonasse wanted
them to do.

I am not sure if this stems from teachers having children do meaningless
busy-work or not, but I'm sure that it cannot help. The problem here is that not all
students value the same things, yet if I expect them to take this business of problem
solving and justification seriously then I have to find ways for the students to be
vested in them. It is possible that it might make all the difference in the world if
the children used their journals to support their talk with each other, as opposed to the focus on sharing their thinking with only me.

A good method for doing so would be to have the students work together in mixed ability groupings to solve problems and perhaps to even collaborate on writing justifications. In this way they can actively share their ways of mathematical thinking with each other, and even teach each other by helping their peers to build their understanding ever greater. Then I believe that it is necessary to have groups share ideas in a whole class setting so that there is sharing between groupings as well. This would likely be an effective tool for having the students take greater ownership over their journals and valuing them as true mathematical tools for problem solving as well as a place to express mathematical thoughts.

In conclusion, I'm unsure how successful I can be in asking the Nicholas's of the world to “un-learn” their dependence on algorithms, but I do believe that if I feed them a steady diet of good problems to solve, and model for them what good justifications look like, and engage them in true mathematical discourse every day while they are in my classroom, that I can, over time help them cross the bridge from procedural learners to bona-fide problem solvers and mathematical learners.
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Vita

The author Paul Lawrence Jonasse was born in Rochester, New York on

He attended Monroe Community College from 1989 to 1992 and received an Associate of Science Degree in Liberal Arts with Distinction and served in the capacity of Vice-President of the Phi Theta Kappa Honors Society. He then attended the University of Rochester from 1992 to 1994, graduating Magna Cum Laude with a Bachelor of Arts Degree in English. He began work toward a Master of Science in Education at the State University of New York College at Brockport in the Spring of 2000.