Describing Transformations: A Math 8 Unit Plan Aligned to the New York State Common Core and Learning Standards

Jessica Stam

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Describing Transformations:

A Math 8 Unit Plan Aligned to the New York State Common Core and Learning Standards

Jessica L. Stam

The College At Brockport: State University of New York
Describing Transformations

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Introduction

The recent implementation of the Common Core State Standards (CCSS) has been challenging for teachers and students alike. One such example is the increase of the eighth grade Congruent and Similar Figures section from two to six standards (Bromley, Jovell, & Sobolewski, 2011b). This thesis addresses the paradigm shift from the National Council for Teachers of Mathematics (NCTM) Standards to the new CCSS using an eighth grade transformation unit plan and two of the eight mathematical practices. Many of the practices are incorporated into this curriculum allowing students the opportunity to discover properties of transformations.

Mathematics teachers have had to develop new materials to implement the CCSS, and as with any paradigm shift, it takes time to adapt materials appropriately. According to Alan Dessoff, the former 2010-2012 president of NCTM, district curriculum directors need to assist their teachers with implementing the CCSS and with meeting the requirements in the new eight mathematical practices. Prior to CCSS, the NCTM had five process standards; problem solving, reasoning and proof, communication, representation, and connections, and when comparing these with the eight mathematical practices of the CCSS one could conclude that they are similar. Both had practices focused around problem solving, reasoning, communicating, and modeling. The representation process standard was split into two CCSS mathematical practices; model with mathematics and use appropriate tools strategically. The connecting process standard also was split into two CCSS mathematical practices: look for and make use of structure and look for and express regularity in repeated reasoning. The CCSS also introduced a new mathematical practice: attend to precision.
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The CCSS high school standards are divided into five major categories. Each individual category consists of an overall introduction and a list of topics that should be covered within the category. The Standards for Mathematical Practice are “important processes and proficiencies” that students should have related to the field of mathematics (New York State Education Department, 2011). The eight Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (New York State Education Department, 2011, p.5-7)

The new CCSS is the intended curriculum while what is actually taught is the implemented curriculum, and the attained curriculum describes what the students actually learned (Harvey, Cambron-McCabe, Cunningham & Koff, 2005). An educator should focus on minimizing the differences between the intended and implemented with the goal of the attained curriculum demonstrating an increase in student performance on state and/or national assessments. It is challenging to implement new curriculum into instruction. Robelen (2012) recalled a conversation with a 15-year teaching veteran regarding the CCSS and stated, “One big challenge, she said, is figuring out how to reach the deeper level of math understanding the standards espouse” (p. 28). According to Alberti (2013), “one of the biggest risks we currently
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Face is full-speed implementation without an understanding of the changes that the standards require” (p.24). Hence, the purpose of this thesis is to inform educators of the differences between the two different sets of standards and to model a transformation unit that is aligned to the new CCSS.

Literature Review

To understand the new Common Core curriculum, it is beneficial to first understand the history behind the mathematics education standards. Beginning as early as 1893, there existed the basic traditional sequence of mathematics courses that we have today. This sequence begins with basic algebra, continues with geometry and advanced algebra, and then concludes with trigonometry (Angus & Mirel, n.d). However, around this same time, there began a questioning period focused on the level of importance and significance that mathematics education plays in high schools (Angus & Mirel, n.d). “Specifically, these critics argued that the traditional course of study in the high schools was elitist or “aristocratic” because it allegedly focused on preparing students for college and thus neglected the needs of the increasing numbers of non-college-bound students” (Angus & Mirel, n.d, p.444). Later there was the widespread view that students should take fewer mathematics courses in high school (Angus & Mirel, n.d). This idea grew greater in the 1930’s and 1940’s and the amount of mathematics courses required for high school graduation continued to decrease. Throughout the late 1960’s and early 1970’s, students were allowed a great deal of choice when it came to mathematics education. This helped students to avoid taking difficult mathematics courses. During the 1970’s, most high school students were only required to complete one course of mathematics (Angus & Mirel, n.d). In the years following 1982, the field of mathematics education began to see the trend of extensive choice in mathematics courses to reverse. According to Angus and Mirel regarding this trend reversal:
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perhaps we can now … devote all our energies to developing those multiple instructional strategies that will be necessary to give all American young people realistic access to the kinds of knowledge that both effective citizenship and worthwhile work will require in the twenty-first century (p.485).

The CCSS was created to provide a set of standards that are common across the United States (US) that prepares Americans for adult life with the intent that the content of instruction would improve (Schmidt & Burroughs, 2013).

Leading up to the CCSS, “the Trends in International Math and Science Study (TIMSS) and other international studies concluded that mathematics education in the US was “a mile wide and an inch deep”” (Alberti, 2013, p.26). The CCSS were designed to cover a smaller array of material and to cover the topics in more depth. According to Schmidt and Burroughs in reference to the CCSS, “the new math standards will address two long-standing problems in U.S. education: the mediocre quality of mathematics learning and unequal opportunity in US schools” (Schmidt & Burroughs, 2013, p.54). Ehren et al. (2012) stated the CCSS are built upon existing standards and are designed to ensure consistency and quality between the country’s varying educational systems.

One goal of the CCSS is to prepare high school students for college and career. Some educators have confused the CCSS with the idea of being told how to teach and what to teach, therefore, taking away freedom within the field of education. As Phillips and Wong stated, “the Common Core of Standards, however, points state policy making in the right direction without imposing rigid specifications about how states should use them” (Phillips & Wong, 2010, p. 38). Educators are expected to implement these practices within their lessons. Overall, the CCSS
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have been summarized through the Common Core State Standards Initiative Mission Statement which states,

“The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy” (Ehren, Blosser, Roth, Paul & Nelson, 2012, p.11).

**Unit Plan**

The unit plan detailed is focused on the topic of transformations and was implemented in a New York State (NYS) classroom. In the New York State Common Core Learning Standards (NYSCCLS) for mathematics there is a cluster entitled “Geometry,” which is often represented using 8.G. The Math 8 Unit in transformations is aligned to the following standards from the Geometry cluster (New York State Education Department [NYSED], 2013b, p.48):

8.G.1 - Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.

8.G.2 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations;
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given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

The lesson plans have many important aspects. The first is the key vocabulary terms that are needed for students to fully understand the mathematics content. It is essential for teachers to spend time building mathematics vocabulary. Second, the lesson plans include activities that will occur throughout the lesson and it addresses the common misconceptions that the students may have. Prior to delivering a lesson, it is important for educators to predict the areas that students may struggle with and develop possible responses to help correct the misconception. Finally, the following lesson plans incorporate a variety of assessment methods that the teacher would use to determine if the students understood the material that was presented.

The majority of the unit plan is original material. However, some ideas have been adapted from the Engage New York’s module two and module three. These lessons have been scaffolded and various questions have been added to help students fully understand the content. In addition, some problems have been adopted from the McGraw Hill’s Glencoe textbook series. The lessons provided were intended for fifty-minute class periods. The unit spans over twelve days which includes a review day and a day for testing. The table presented below outlines the unit plan:
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**Describing Transformations**

### Day One

**Standards:** NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   - a. Lines are taken to lines, and line segments to line segments of the same length.
   - b. Angles are taken to angles of the same measure.
   - c. Parallel lines are taken to parallel lines.

**MP Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

**Lesson Title:** Introduction to Transformations

**Lesson Materials:**
- Transparency Paper
- Wet Erase Marker
- Note Packet
- Pencil

**Key Vocabulary:**
- Transformation
- Translation
- Rotation
- Dilation
- Reflection
- Pre-Image
- Image
- Plane

**Classroom Activities:**

**Teach:**
1. Teacher will work with students to develop a “student friendly” definition of transformations.
2. Students will explore the various transformations that could occur.
3. Teacher will introduce math vocabulary to the students that match the descriptions that they gave during the exploratory challenge.
4. Students will work with a partner to notice transformations.
5. Students will end with a review on graphing points on a coordinate plane.

**Students could struggle with:**
- Exploratory Challenge: Looking at all four pictures at once.
- Give these students post-it notes to cover up some of the images.

**Ticket Out the Door:**

On a half sheet of paper, describe to an elementary student what the key vocabulary terms below mean:
- Transformation
- Translation
- Rotation
- Reflection
- Dilation

**Homework:** None

**Assessments:**
- Formative Assessment: Ticket Out the Door
- Summative Assessment: Unit Test

**Resources Used:**
Engage NY – 8th Grade Math – Module 2
Transformation: An operation that maps a figure on to a new figure
What it really means: __________________________________________________________

Exploratory Challenge
1.) Describe what kind of transformation will be required to move the figure on the left to each of the figures (1-3) on the right.

From the original to (1): _________________________________________________________________
From the original to (2): _________________________________________________________________
From the original to (3): _________________________________________________________________

Discussion Questions:
Given two figures, how do you determine if they are the same size without measuring them?
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________

Why do we move things around on a plane?
________________________________________________________________________________________
________________________________________________________________________________________
Transformations occur on a ________________.
- A plane is a flat surface that is infinitely large.

An_________________ of a figure is the copy of a figure after a transformation has been used.

We have discovered that there are 3 main ways to move objects on a plane and keep the size and shape the same. These three ways are:

1) __________________________________________________________________________________________________

2) __________________________________________________________________________________________________

3) __________________________________________________________________________________________________

You Try
Draw a figure on your paper and name it. Draw its image under some transformation. Use your transparency paper to help you. Draw your new image somewhere else on the paper. Describe how you moved the figure. Use a complete sentence and key math vocabulary you have learned throughout this lesson.

With a Partner:
Draw a new shape and label it. Perform two transformations to the image. Draw the new shape elsewhere on the paper. Trade papers with your partner. Your partner has to describe to you how you moved your image.

Partner’s Response: To get from the figure to the image of that figure, you…
Coordinate Plane

😊 The _____________________ is the horizontal line.
😊 The _____________________ is the vertical line.
😊 Both lines intersect at the ____________________.
😊 The plane is divided into four _________________.
😊 An ___________________________ gives the coordinates and location of a point.

Find the coordinates of the indicated point:

A:   I:   G:  
C:   J:   N:  

Name the letter at each ordered pair:

(6, 9):   (-4, -5):   (-7, 6):
(-5, 0):   (1, 5):   (7, 2):

Name the quadrant or axis on which each point lies:

B:   (7, 2):   (-7, 6):
(8, -3):   E:   (-9, -9):
**Standards:** NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

**MP Mathematical Practices**
4. Model with mathematics
5. Use appropriate tools strategically

**Lesson Title:** Abstract Translations

**Essential Question:** What does it mean to translate an object?

**Lesson Materials:**
- Transparency Sheet
- SMART board
- Dry Erase Marker
- Note Packet

**Key Vocabulary:**
1. Slide
2. Translation
3. Vector
4. Segment
5. Plane
6. Distance Preserving
7. Rigid Motion

**Classroom Activities:**

**Teach:**
1. Teacher will introduce vocabulary to the students through use of guided notes.
2. Teacher demonstrates how to use transparency paper to translate the image.
3. Teacher completes questions one through three with students.
4. Students work in groups to complete classwork activity and “What have we learned…” section.
5. Teacher assists students in completing “Key Characteristics” section

**Students could struggle with...**
1. Understanding how far to move the shape
2. Understanding the vocabulary

**Ticket Out the Door:**
On a half sheet of paper, students will describe what a translation is. Students should incorporate as much of the key vocabulary as possible.

**Homework:**
Day Two Homework Worksheet

**Assessments:**
Formative Assessments:
1. Teacher Observations
2. Ticket Out the Door

Summative Assessments:
1. Homework 2.2
2. Unit Test

**Resources Used:**
Engage NY – 8th Grade Math – Module 2
**Vocabulary:**
A ________________________ is one of the simplest transformations that would map figures onto each other. The transformation that slides a figure is called a __________________________.

**TRANSLATE**

A segment in the plane is called a ____________________. It has a starting point and an end point. The arrowhead points to the vector’s endpoint.

---

1) Use your transparency sheet to trace the line segment AB and point P. Translate the image along line segment AB.

What observations do you notice?

2) Use your transparency to translate the line, L, and the two points along the vector shown below.
Describing Transformations

3) Translate the three following figures along the vectors shown below.

When we label the points on our image, we label them with the same letter and an apostrophe. We call this apostrophe ________________.

Point A after a transformation would be called ____________.
Point B after a transformation would be called ____________.
Point S after a transformation would be called ____________.

Key Characteristics

1) A translation maps a point to a point, line to a ________________, a segment to a segment, and an ________________ to an angle.

2) A translation preserves ________________ of segments.

3) A translation preserves ________________ of angles.

When translating along a vector, each point moves the ________________ ________________ as the vector.

Since a translation preserves lengths of segments and measurements of angles, we say that transformation is ________________ ________________.

**Any transformation that is ________________ ________________ is called a ________________ ________________.
Describing Transformations

**Class Work Activity:**

The diagram below shows figures and their images under a translation along $HI$. Think about what you know about translations. Use the original figures and the translated images to fill in the missing labels for points and measures.
1) Name the vector in the picture below.

2) Name the vector along which a translation of a plane would map point A to its image $T(A)$.

3) Is Maria correct when she says that there is a translation along a vector that will map segment $AB$ to segment $CD$?

4) Assume there is a translation that will map segment $AB$ to segment $CD$ shown above. If the length of segment $CD$ is 8 units, what is the length of segment $AB$? How do you know?

5) Translate the plane containing Figure A along $\overrightarrow{AB}$. Complete a sketch of the image of Figure A by this translation. Mark points on Figure A and label the image of Figure A accordingly.
### Standards: NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

### MP Mathematical Practices

1. Make sense of problems and persevere in solving them.

### Lesson Title: Transformations on a Coordinate Plane

### Essential Question: How do you translate a figure on a coordinate plane?

### Lesson Materials:
- Note Guide
- Pencil

### Key Vocabulary:
- Translation
- Slide
- Segment
- Preserved

### Classroom Activities:

**Teach:**
1. Teacher guides students through note packet.
2. Students practice individually.
3. Students complete ticket out the door.

**Students Could Struggle With:**
1. Remembering which coordinate comes first in a coordinate point.
2. Reading a graph

### Ticket Out the Door:

Students will have to complete two translation problems on a half sheet of paper. While they complete the problems, they will have to describe their steps using sentences.

### Homework:

Homework Day Three Worksheet

### Assessments:

**Formative Assessments:**
- Ticket Out the Door
- Teacher Observations

**Summative Assessments:**
- Homework 2.3
- Unit Test

### Resources Used:
- Translation Picture - [www.regentsprep.org/regents/math/geometry/gt2/Trans.htm](http://www.regentsprep.org/regents/math/geometry/gt2/Trans.htm)
- McGraw Hill Glencoe Textbook Series
A translation is simply a slide – no flipping, spinning, or resizing…just moving.

What two properties are preserved when an object is translated?

____________________ and ____________________

Every point of the shape must move:

- The same _______________
- The same _______________

Examples:

1. Translate segment AT 2 units right.

<table>
<thead>
<tr>
<th>Original</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(-4, 1)</td>
<td></td>
</tr>
<tr>
<td>T(-1, 3)</td>
<td></td>
</tr>
</tbody>
</table>

2. Translate segment AT 4 units down.

<table>
<thead>
<tr>
<th>Original</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(-4, 1)</td>
<td></td>
</tr>
<tr>
<td>T(-1, 3)</td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

Notation

Translations can be written different ways...

- When a point A is translated 3 units to the right and 5 units up:
  \[ A(x, y) \rightarrow A'(x + 3, y + 5) \quad \text{OR} \quad T(3, 5) \]

- When a point is translated 4 units to the left and 6 units down:
  \[ B(x, y) \rightarrow B'(x - 4, y - 6) \quad \text{OR} \quad T(-4, -6) \]

Examples:

1. Graph the triangle C(-7, 3), A(-2, 3), and R(-2, 8). Then translate CAR by \( T(5, 1) \).

2. Graph the points F(3, 4), R(6, 4), O(8, 6) and G(4, 6). Then translate FROG by \( T(-4, -6) \).

3. Under the translation \( (x, y) \rightarrow (x + 3, y - 2) \), find the new coordinates of (2, 5).

4. Under the translation \( (x, y) \rightarrow (x - 5, y - 9) \), find the new coordinates of (1, 1).
Describing Transformations
Name: ________________________________________   Date: __________
Math 8 – Class Period ___________________________   Day Three Homework

Translations

Draw the image of the figure after the indicated translation.

1. 3 units right and 2 units up

2. 5 units right and 3 units down

3. 2 units left and 1 unit up

4. 4 units left and 2 units down

Graph the figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

5. $\Delta FGH$ with vertices $F(1, 3)$, $G(2, 4)$, and $H(3, 2)$; translated 3 units left and 1 unit down

6. rectangle $PQRS$ with vertices $P(-4, -1)$, $Q(0, 1)$, $R(1, -1)$, and $S(-3, -3)$; translated 2 units right and 3 units up
### Day Four

**Standards:** NYSCCSS

**8.G Understand congruence and similarity using physical models, transparencies, or geometry software.**

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

**MP Mathematical Practices**

4. Model with mathematics
5. Use appropriate tools strategically

---

**Lesson Title:** Abstract Reflections

**Essential Question:**
What does it mean to reflect an object?

**Lesson Materials:**
- Mira
- SMART Board
- Note Packet
- Pencil

**Key Vocabulary:**
1. Mira
2. Translation
3. Reflection
4. Image
5. Line of Symmetry
6. Distance Preserving
7. Rigid Motion

---

**Classroom Activities:**

**Teach:**
1. Teacher goes over introduction vocabulary and ten demonstrates exercise one.
2. The students complete exercise one and have it checked by the teacher.
3. Students work to complete exercises one through six using Miras. The students will record observations as they reflect images.
4. Teacher will discuss the vocabulary term “line of symmetry” with students.
5. Students will complete exercise seven.
6. Students and teacher will complete summary notes
7. Student will complete ticket out.

**Students could struggle with:**
1. Placing the Mira in the correct place
2. Remembering the definition of Rigid Motion and Distance Preserving
3. Making observations of images compared to pre-images

---

**Ticket Out the Door:**

On a half sheet of paper, ask the students to describe what it means to reflect an object

**Homework:**

Day Four Homework Worksheet

**Assessments:**

**Formative:**
1. Teacher Observations
2. Ticket out the Door

**Summative**
1. Unit Exam
2. 2.4 Homework Worksheet

---

**Resources:**


Engage NY – 8th Grade Math – Module 2 – Page 48 - 50
Another transformation that we discovered in class was the transformation that flipped the figure.

The transformation that flips the figure is called a ____________________________.

**Reflections Using the Mira**

**Exercise One**
Use the Mira to put the child on the swing. Then reach behind the Mira and trace the child on the swing.

Observations:
Describing Transformations

**Exercise Two**
Find the image of \( \triangle ABC \) reflected about the line \( l \). Label the new triangle appropriately.

Observations:

**Exercise Three**
Find the image of \( \triangle ABC \) reflected about the line \( l \). Label the new triangle.
Describing Transformations

**Exercise Four**
Find the image of quadrilateral $ABCD$ reflected about line $l$. Label the new points.

**Exercise Five**
Find the image of pentagon $ABCDE$ reflected about line $l$. Label the new points.
Describing Transformations

**Exercise Six**
Let there be a reflection across line $AB$. Reflect $\triangle CDE$ and label the reflected image.

1. Using the diagram above, what is the measure of $\text{Reflected } \angle CDE$?

2. Using the diagram above, what is the measure of $\text{Reflected line } CE$.

A line of ___________________________ is a line that divides a figure so that it is the same shape on both sides of that line.

**Exercise Seven**
Use the Mira to find lines of symmetry of the following shapes.

1. Square
2. Triangle
3. Star
Describing Transformations

**Summary**

Reflections preserve _______________________ and _____________________.

Since reflections preserve shapes, we say that they are ____________________
______________________________.

Distance preserving transformations are called ____________________
______________________________.

The distance of every point in the figure to the line of reflection is the
______________________________ as the distance from every point in the
______________________________ to the line of reflection.
In the picture below, $\angle DEF = 56^\circ$, $\angle ACB = 114^\circ$, $AB = 12.6$ units, $JK = 5.32$ units, point $E$ is on line $L$ and point $I$ is off of line $L$. Let there be a reflection across line $L$. Reflect and label each of the figures, and answer the questions that follow.

What is the size of reflection($\angle DEF$)? Explain.

What if the length of reflection($JK$)? Explain.

What is the size of reflection($\angle ACB$)?

What is the length of reflection($AB$)?

Two figures in the picture were not moved under the reflection. Name the two figures and explain why they were not moved.
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<th>Lesson Title: Reflections on a Coordinate Grid</th>
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<tr>
<td>3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td>
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</tr>
<tr>
<td>1. Teacher will work with students, focusing on information discovered the lesson before, to fill in the key ideas notes.</td>
<td></td>
</tr>
<tr>
<td>2. Teacher will demonstrate Exercise One with the students.</td>
<td></td>
</tr>
<tr>
<td>3. Students will be asked to complete exercises two through four with their group.</td>
<td></td>
</tr>
<tr>
<td>4. Teacher and students will work as a group to come up with algebraic rules for reflecting on a coordinate plane.</td>
<td></td>
</tr>
<tr>
<td>Students May Struggle With:</td>
<td></td>
</tr>
<tr>
<td>• Understanding the algebraic rules to reflections</td>
<td></td>
</tr>
<tr>
<td>• Applying algebraic rule without a coordinate grid</td>
<td></td>
</tr>
<tr>
<td>Ticket Out the Door:</td>
<td></td>
</tr>
<tr>
<td>Students will write a paragraph about reflections. A list of the following vocabulary words and terms will be written on the board. The students will need to incorporate these words in their description of reflections.</td>
<td></td>
</tr>
<tr>
<td>Vocabulary Words/Terms:</td>
<td>Homework:</td>
</tr>
<tr>
<td>Shape</td>
<td>2.5 Homework Worksheet</td>
</tr>
<tr>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>Distance Preserving</td>
<td></td>
</tr>
<tr>
<td>x-axis</td>
<td></td>
</tr>
<tr>
<td>y-axis</td>
<td></td>
</tr>
<tr>
<td>coordinate y=x</td>
<td></td>
</tr>
<tr>
<td>Resources Used:</td>
<td></td>
</tr>
<tr>
<td>Pictures - <a href="https://www.mathsisfun.com/geometry/reflection.html">https://www.mathsisfun.com/geometry/reflection.html</a></td>
<td></td>
</tr>
<tr>
<td>McGraw Hill Glencoe Textbook Series</td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

Name: _________________________________      Date: ____________
Math 8 – Class Period _________________      Day Five Notes

Reflections on a Coordinate Plane

A transformation that flips the figure over a given line is called a _________________.

Key Ideas:
Reflections preserve _________________ and _________________.

The distance of point \( A \) to the line of reflection is ________ ___________ distance as point \( A' \) to the line of reflection.

To reflect a shape over a line of symmetry:
1. Measure the distance of each point to the line of symmetry
2. Measure the same distance on the other side of the line
3. Connect the new dots!

Exercise One
Graph the points \( A(2,2), B(5,3), \) and \( C(3,6) \). Then reflect these over the \( y \)-axis and label your new figure.

<table>
<thead>
<tr>
<th>Original Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mirror Line
Mirror Line

1. 2. 3.
Describing Transformations

**Exercise Two**
Graph the points A(-3,-6), B(-4,-1), and C(-5,-5). Then reflect these over the x-axis and label your new figure.

<table>
<thead>
<tr>
<th>Original Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exercise Three**

a) Graph the triangle with vertices D (-5, 2), E (-3,5) and F (-1,2)

b) Reflect ΔDEF in the x-axis and label it D’, E’, F’

c) Reflect ΔD’E’F’ in the y-axis and label it D” E” F”

**Exercise Four**
Reflect the triangle over the line y = x. Use your Mira to check your solution.

<table>
<thead>
<tr>
<th>Original Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (-5, 1)</td>
<td></td>
</tr>
<tr>
<td>B (-3, 5)</td>
<td></td>
</tr>
<tr>
<td>C (-1, 3)</td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

**More Practice**

Graph figure CDEF with vertices C(0, 2), D(3, 4), E(4, 2) and F(1, 1). Then reflect the figure over the x-axis.

Graph figure RST with vertices R(-4, 4), S(-2, 3), and T(-3, 1). Then reflect the figure over the y-axis.

---

**Reflections on the Coordinate Plane**

**Reflection over the x-axis** (Notation: $r_{x\text{-axis}}$):

- What do you notice about the coordinates?

**Algebraic Rule:**

**Reflection over the y-axis** (Notation: $r_{y\text{-axis}}$):

- What do you notice about the coordinates?

**Algebraic Rule:**

**Reflection over y=x** (Notation: $r_{y=x}$):

- What do you notice about the coordinates?

**Algebraic Rule:**
1. Graph \( \triangle ABC \) with vertices \( A(2, 2), B(5, 4), \) and \( C(5, 1) \) and its reflection over the \( x \)-axis. Then find the coordinates of the reflected image.

2. Graph square \( ABCD \) with vertices \( A(-1, 2), B(2, -1), C(5, 2), \) and \( D(2, 5) \) and its reflection over the \( y \)-axis. Then find the coordinates of the reflected image.

The coordinates of a point and its image after a reflection are given. Describe the reflection as over the \( x \)-axis or \( y \)-axis.

3. \( B(1, -2) \rightarrow B'(1, 2) \)
4. \( J(-3, 5) \rightarrow J'(-3, -5) \)
5. \( W(-7, -4) \rightarrow W'(7, -4) \)

For Exercises 6–9, use the following information.

Triangle \( XYZ \) has vertices \( X(4, 2), Y(4, 4), \) and \( Z(0, 2) \).

6. What are the coordinates of the image of point \( X \) after a reflection over the \( y \)-axis?

7. What are the coordinates of the image of point \( Y \) after a reflection over the \( y \)-axis?

8. What are the coordinates of the image of point \( Z \) after a reflection over the \( y \)-axis?

9. Graph triangle \( XYZ \) and its image after a reflection over the \( x \)-axis.
### Standards: NYSCCSS

**8.G Understand congruence and similarity using physical models, transparencies, or geometry software.**

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

### MP Mathematical Practices
4. Model with mathematics
5. Use appropriate tools strategically

### Lesson Title: Abstract Rotations

### Essential Question:
What does it mean to rotate an object? What key characteristics do rotations have?

### Lesson Materials:
- Tracing Paper
- Ruler
- Protractor
- Compass
- Pencil
- Note Sheet

### Key Vocabulary:
- Rotations
- Congruent
- Clock-wise
- Counter-Clockwise

### Classroom Activities:

**Teach:**

1. Students will work in groups to complete Rotation Discovery Activity One.
2. Teacher will facilitate a discussion with students about their findings from activity one.
3. Students will work in groups to complete Rotation Discovery Activity Two.
4. Students will summarize their findings in a paragraph for their ticket out the door.

**Students May Struggle With:**

1. Using a protractor and a compass.
2. Understanding directions to discovery tasks.

### Ticket Out the Door:

Students will write a paragraph of what they learned about rotations.

### Homework:
None

### Assessments:

Formative Assessment:
- Teacher Observations
- Ticket Out the Door

Summative Assessment:
- Unit Test

### Resources Used:
Figure on Coordinate Plane - [http://ltconline.net/greenl/courses/CAHSEE/Geometry/planeExam2.gif](http://ltconline.net/greenl/courses/CAHSEE/Geometry/planeExam2.gif)
McGraw Hill Glencoe Textbook Series
1. Place your tracing paper over the coordinate grid. Outline the x and y axis, the origin, and the figure.

2. Take your worksheet and turn it one quarter turn clockwise. Lay the tracing paper on the worksheet and line up the origins. The y axis of the tracing paper will lie on the x axis of the worksheet. Trace the figure on the tracing paper again.

3. The tracing paper now has the pre-image and the image after a one quarter turn clockwise. With a partner, answer the questions below:
   
   a. Is the pre-image congruent to the image? How do you know?

   b. What would change if we were to turn it half a turn clockwise? A quarter turn counter clockwise?

   c. Measure the distance from point A to the center and the distance from point A’ to the center. What do you notice?

   d. Repeat part c with another set of points. What do you notice?
Describing Transformations

1. Trace the triangle below and the center, point O, on your tracing paper.
2. Draw a line segment from point O to point A.
3. Use a protractor to draw ∠AOD so that it is 90°.
4. Use your compass to draw a circle with center at point O and radius the same size as the line segment from point O to point A.
5. Point A' is the point of intersection between line segment OD and the circle.
6. Repeat these steps with the other points.
7. What do you notice about the two shapes?

Whole Class Summary

1. Any point and its rotation image are __________________ from their center of rotation.
2. A figure and its rotation image are __________________.
3. Rotations are __________________ ______________________________ transformations and therefore, a rotation is a rigid motion.
### Describing Transformations

**Standards:** NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

**MP Mathematical Practices**

1. Make sense of problems and persevere in solving them.

**Lesson Title:** Rotations on a Coordinate Grid

**Essential Question:**
What does it mean to rotate an object? What key characteristics do rotations have?

**Lesson Materials:**
- Note Guide
- Pencil

**Key Vocabulary:**
1. Rotation
2. Counter Clockwise
3. Clockwise

**Classroom Activities:**

**Teach:**
1. Teacher will assist students in filling in note section of guided notes.
2. Teacher and students will complete the example together and then discuss algebraic rules for rotating on a coordinate plane.
3. Students will work in groups to complete the seven practice problems.
4. Students will complete the Ticket Out the Door

**Ticket Out the Door:**
Students will apply algebraic rules to complete two different rotations.

**Homework:**
Day Seven Homework Worksheet

**Assessments:**

Formative Assessments:
- Teacher Observations
- Ticket Out the Door

Summative Assessments:
- Unit Test
- Day Seven Homework Worksheet

**Resources Used:**
McGraw Hill Glencoe Textbook Series
An image can be rotated two ways:

OR

What two properties are preserved when an object is rotated?

____________________ and ____________________

A 360° rotation is called a______ turn.

A 180° rotation is called a______ turn.

What is another way to describe a 270° rotation?

There are some rotations that would produce the same shape. Talk with a partner to figure out which rotations would produce the same shape in the same location. Record your discussion below.
Describing Transformations

Example: Graph A(2, 1), B(6, 1), C(5, 3), and D(3, 3).

Rotation Rules:

<table>
<thead>
<tr>
<th>Rotation</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Practice:
1. Graph R(3, -3), A(8, 1), T(6, -6) and rotate 90° clockwise around the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>90° clockwise</th>
<th></th>
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<tbody>
<tr>
<td></td>
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</tbody>
</table>
Describing Transformations

2. Graph M(3, 6), A(6, 6), T(1, 3), H(8, 3) and rotate 180° clockwise around the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>180° clockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

3. Graph C(-4, 6), A(-7, 3), and T(-2, 3) and rotate 90° clockwise around the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>90° clockwise</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

4. Graph Y(2, 3), O(3, 6), D(6, 6), and A(5, 3) and rotate 270° clockwise around the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>270° clockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
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</tr>
</tbody>
</table>
5. Graph S(-4, 2), N(-6, -3), O(-6, -6), and W(-2, -6) and rotate 180° clockwise around the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>180° clockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

6. Quadrilateral MNPQ has vertices M(2,5), N(6,4), P(6,1), and Q(2,1). Graph the figure and its image after a rotation of 270° counterclockwise about the origin. Then give the coordinates of the vertices for quadrilateral M’N’P’Q’.

<table>
<thead>
<tr>
<th>Original</th>
<th>270° counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
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<td></td>
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</tbody>
</table>

7. Graph the same quadrilateral. Rotate the quadrilateral 90° clockwise.

<table>
<thead>
<tr>
<th>Original</th>
<th>90° clockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rotations

1. Triangle $JKL$ has vertices $J(-4, 4), K(-1, 3),$ and $L(-2, 1)$. Graph the figure and its rotated image after a clockwise rotation of $270°$ about the origin. Then give the coordinates of the vertices for triangle $J'K'L'$.

2. Quadrilateral $BCDE$ has vertices $B(3, 6), C(6, 5), D(5, 2),$ and $E(2, 3)$. Graph the figure and its rotated image after a counterclockwise rotation of $180°$ about the origin. Then give the coordinates of the vertices for quadrilateral $B'C'D'E'$.

3. Triangle $RST$ has vertices $R(1, 1), S(1, 4),$ and $T(3, 1)$. Graph the figure and its rotated image after a clockwise rotation of $180°$ about the origin. Then give the coordinates of the vertices for triangle $R'S'T'$.

4. Quadrilateral $KLMN$ has vertices $K(2, 0), L(4, 0), M(5, -2),$ and $N(1, -2)$. Graph the figure and its rotated image after a counterclockwise rotation of $90°$ about the origin. Then give the coordinates of the vertices for quadrilateral $K'L'M'N'$.
### Standards: NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

### MP Mathematical Practices

1. Make sense of problems and persevere in solving them.

---

### Lesson Title: Dilations

#### Essential Question:
How do you dilate figures on a coordinate grid?

#### Lesson Materials:
- Note Guide
- Pencil

#### Key Vocabulary:
1. Dilate
2. Scale Factor
3. Coordinate
4. Preserved

#### Classroom Activities:

**Teach:**

1. Students find a partner. The partners face each other so that they can see the pupil of the other partner. Teacher turns off the lights. Students discuss what they notice. As students are looking at each other’s eyes, teacher turns on the lights. Students discuss what they notice.
2. Teacher assists students with completing guided notes.
3. Students attempt the three example problems on page two of note guide.
4. Teacher and students have discussion regarding methods to finding scale factor and summarize information in note guide.
5. Students practice finding scale factor in partners.

**Student May Struggle With:**

1. Determining the image coordinates.
2. Plotting points on a coordinate grid.

---

**Ticket Out the Door:**

Students will create a dilations problem. They will then trade problems with a partner and complete their partner’s problem. The students will trade back to check solutions.

**Homework:**

Day Eight Homework Worksheet

**Assessments:**

Formative Assessments:
- Teacher Observations
- Ticket Out the Door

Summative Assessments:
- Unit Test
- Day Eight Homework Worksheet

---

**Resources Used:**

- [https://colesyteach.wordpress.com/](https://colesyteach.wordpress.com/)
- McGraw Hill Glencoe Textbook Series
Describing Transformations
Name: _______________________________________     Date: _____________
Math 8 – Class Period ________________________       Notes – Day Eight

Observe the dilation image of triangle ABC with the center of dilation at the origin and a scale factor of 2.

Notice how EVERY coordinate of the original triangle was multiplied the scale factor 2.

Example: Plot the points L(1, 1), A(2, 3), and B(4, 1). Then graph the L’A’B’ under a dilation with a scale factor of 3.

What happens to your pupil when you look into a bright light or you enter a dark room?

The same concept of "getting larger or smaller" applies to the word \textit{dilate} used in math.

Dilations

What happens to your pupil when you look into a bright light or you enter a dark room?

The same concept of "getting larger or smaller" applies to the word \textit{dilate} used in math.

Original | New
---|---
A(-2, 2) | A’(-4, 4)
B(1, -1) | B’(2, -2)
C(0, 2) | C’(0, 4)

Notice how EVERY coordinate of the original triangle was multiplied the scale factor 2.

What property is preserved under dilations? What property is \textbf{NOT} preserved under dilations?

__________________________      __________________________

Example: Plot the points L(1, 1), A(2, 3), and B(4, 1). Then graph the L’A’B’ under a dilation with a scale factor of 3.

Original | Work | New
---|---|---
L(1, 1) | | 
A(2, 3) | | 
B(4, 1) | | 
Describing Transformations

Example: Plot the parallelogram P(-2, -1), A(6, -1), R(0, 2) and K(6, 2).
   Dilate the image using a scale factor of 2.

<table>
<thead>
<tr>
<th>Original</th>
<th>Work</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(-2, 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A(6, -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R(0, 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K(6, 2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example: Plot the points T(2, 4), E(-4, -4), and A(6, -2).
   Dilate the image using a scale factor of \( \frac{1}{2} \).

<table>
<thead>
<tr>
<th>Original</th>
<th>Work</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(2, 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E(-4, -4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A(6, -2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example: Plot the points R(-8, -8), E(-4, 0), and D(0, 4).
   Dilate the image using a scale factor of \( \frac{1}{4} \).

<table>
<thead>
<tr>
<th>Original</th>
<th>Work</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(-8, -8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E(-4, 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D(0, 4)</td>
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</tbody>
</table>
Describing Transformations

Finding the Scale Factor from the Original Figure and the Image

** Image Coordinate **
** New Coordinate **

** Do this for each point to be sure they are the same! **

Example: Under a dilation, triangle A(0,0), B(0,4), C(6,0) becomes triangle A'(0,0), B'(0,10), C(15,0). What is the scale factor for this dilation?

Example: Under a dilation, quadrilateral A(-8,8), B(-8, 4), C(0,8), D(0,4) becomes triangle A'(-2,2), B’ (-2,1), C’(0,2), D'(0,1). What is the scale factor for this dilation?
Describing Transformations

Name: _____________________________      Date: ________
Math 8 – Class Period _______       Day Eight Homework

Find the coordinates of the vertices of each figure after a dilation with the given scale factor $k$. Then graph the original image and the dilation.

1. $S(–2, 1), U(0, 1), N(–1, –1); k = 4$

2. $M(–3, 1), A(1, 3), T(2, –2), H(–4, –2); k = \frac{1}{2}$

3. $F(–2, 1), U(–1, 2), N(3, 1); k = 2$

4. $P(–4, 2), L(2, 4), A(2, –4), Y(–4, –2); k = \frac{1}{4}$

5. Rachel and her cousin, Lena, live in different cities that are about 100 miles apart. On a map, the two cities measure 5 inches apart. What is the scale factor used for the map?

6. A square has vertices $J(–1, 4), U(5, 4), M(5, –2), P(–1, –2)$. After a dilation, square $JUMP$ has vertices $J(–0.5, 2), U(2.5, 2), M(2.5, –1), P(–0.5, –1)$. What is the scale factor of the dilation?

7. A landscape designer has a drawing of a flower bed that measures 6 inches by 9 inches. The owner wants the actual flower bed to be 5 feet by 7.5 feet. What is the scale factor the designer must use to install the new flower bed?
### Standards: NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

### MP Mathematical Practices

1. Make sense of problems and persevere in solving them.

### Lesson Title: Combining Transformations

### Essential Question:

How do you apply multiple transformations at once?

### Lesson Materials:
- Note Packet
- Pencil

### Key Vocabulary:

1. Translations
2. Rotations
3. Reflections
4. Dilations

### Classroom Activities:

#### Teach:

1. Students will work together in groups to complete the transformation review section of their note packet.
2. Teacher will complete whole group instruction on Multiple Transformations. Teacher will demonstrate example one and example two.
3. Students will work together to complete the remaining exercises in the note packet.

#### Students Could Struggle With:

1. Determining whether the figure slid, flipped, turned, or changed size.
2. Writing specific rules for the transformations.
3. Determining the number of degrees the figure rotated.

### Ticket Out the Door:

Teacher will post directions on the board which require students to transform a triangle using multiple transformations. Students will complete their work on a half sheet of paper and turn it in.

### Homework:

None

### Assessments:

Formative Assessments:
- Teacher Observations
- Ticket Out the Door

Summative Assessments:
- Unit Test

### Resources Used:

McGraw Hill Glencoe Textbook Series
Describing Transformations

Transformations Review

Translations (____________________)

Notations:
1) _________________________________________________________________
2) _________________________________________________________________
3) _________________________________________________________________

Example: Translate four units left and three units down.

Reflections (________________________)

Notations: __________________________, ____________________________, __________________________

Example: Reflect the following shape over the line y=x.

Example: Reflect the following shape over the x-axis.
Describing Transformations

**Rotations**

Notations: _________________________

Example: Rotate the figure 90° clockwise.

**Dilations**

Example: Dilate the figure when $k = 2$.

---

**Multiple Transformations**

1. Translate the figure 1 unit right and 5 units up. Then reflect the new triangle over the y-axis.

2. Perform the transformations in question one in the opposite order. Do you end up with the same figure?
Describing Transformations

3. Translate the triangle 4 units up. Reflect the triangle over the x-axis.

4. Translate the triangle right 1 unit, then 3 units up. Then rotate the new triangle 180 clockwise.

5. Translate the triangle 2 units left. Then rotate the new triangle 90 clockwise.
## Describing Transformations

### Day Ten

<table>
<thead>
<tr>
<th>Standards: NYSCCSS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.G</strong> Understand congruence and similarity using physical models, transparencies, or geometry software.</td>
</tr>
<tr>
<td><strong>2.</strong> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td>
</tr>
<tr>
<td><strong>4.</strong> Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</td>
</tr>
</tbody>
</table>

### Lesson Title:
Discovering Transformations

### Essential Question:
How do you determine which transformation has been applied?

### Lesson Materials:
- Note Guide
- Pencil

### Key Vocabulary:
1. Translation
2. Rotation
3. Reflection
4. Dilation

### Classroom Activities:
1. Students will work in groups to discover which transformation occurred. They will use the coordinates to prove their answer.
2. Teacher will walk around the room and facilitate learning.
3. Students will present results to the class as an answer key.
4. Teacher and students will have open discussions if students arrive at different answers.

### MP Mathematical Practices
1. Make sense of problems and persevere in solving them.

### Ticket Out the Door:
Students will be asked to write a paragraph describing their strategy for determining which transformation occurred.

### Homework:
Day Ten Homework Worksheet

### Assessments:
Formative Assessments:
- Teacher Observations
- Ticket Out the Door

Summative Assessments:
- Unit Test
- Day Ten Homework

### Resources Used:
Engage NY – 8th Grade Math – Module 2
Identifying Transformations

Goal: To identify a transformation or series of transformations that maps one figure onto another.

Transformation Options:
- Translate (Give specific directions – ex. \( T(-2,3) \))
- Rotate (State _______ or ____________ and the # of degrees)
- Reflect (Over _________, ____________, or _________)
- Dilate (State the ______ _______)

Example One
Identify a transformation that takes figure \( WXYZ \) to \( STUV \)

Example Two
Identify a transformation that takes figure \( GHI \)
Describing Transformations

**Example Three**
Identify a transformation that takes figure $ABCD$ to figure $FGHI$.

**Example Four**
Find a sequence of transformations that maps $RSTUV$ to $ABCDE$.

**Example Five**
Find a sequence of transformations that maps $GHIJKL$ to $RQPONM$. 
1. In Mr. Lunderwood’s class, Johnny was asked to describe as many transformations as he could. Johnny responded saying a slide, a flip and a turn. Correct Johnny’s answer using correct mathematic vocabulary.

2. Looking at the following pre-images and images, identify the possible transformation that occurred.

   a. 
   
   b. 
   
   c. 

3. A figure has vertices at A(-3, 2), B(-1, -1) and C(-4, -2). After a transformation has been performed, the image of the figure has vertices at A’(3, 2), B’(1, -1), and C’(4, -2). Graph both the pre-image and the image on the grid below. Then identify the transformation that occurred.

4. A figure has vertices at J(-2, 3), K(0, 3), L(0, 1) and M(-2, 1). The vertices of the image of the figure are J’(2, 1), K’(4, 1), L’(4, -1) and M’(2, -1). Graph both the pre-image and the image on the grid. Then identify the transformation that occurred.
### Standards: NYSCCSS

#### 8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   - Lines are taken to lines, and line segments to line segments of the same length.
   - Angles are taken to angles of the same measure.
   - Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

### MP Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
8. Look for and express regularity in repeated reasoning.

### Lesson Title: Review Day

### Essential Question:
What information do I still struggle with regarding this unit?

### Lesson Materials:
- Note Guide
- Pencil

### Key Vocabulary:
1. Translation
2. Rotation
3. Dilation
4. Reflection

### Classroom Activities:

#### Teach:

1. Large paper will be placed around the room to represent different sections of the graphic organizer. Students will work with a partner to complete their section of the graphic organizer.
2. Students will receive time to move around the room and add to posters if they feel that information may be missing.
3. Students will rotate around the room and fill in their entire complete graphic organizer.
4. Students will begin working on homework worksheet if there is extra time.

### Students May Struggle With:

1. Remembering the rules for each transformation.

### Ticket Out the Door:

Students will write one thing that they learned throughout the class period. Students will also make a detailed plan of how they will prepare for their test.

### Homework:

Day Eleven Homework Worksheet

### Assessments:

#### Formative Assessments:
- Teacher Observations
- Ticket Out the Door

#### Summative Assessments:
- Unit Test
- Day Eleven Homework
## Describing Transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Picture</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection in x-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection in y-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dilation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Size</th>
<th>Shape</th>
<th>Location in Plane</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation 90 clockwise (270 counterclockwise)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation 180</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation 270 clockwise (90 counterclockwise)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Properties Preserved under Transformations

There are four properties of any figure: size, shape, location in plane and orientation.

Location in plane means that the figure stays in the same place.

Orientation is preserved when the letters that you label the figure with stay in the same place (they are not flipped around).

Put a check under the properties that are preserved (stay the same) for each of the transformations.
1. Plot the points R(-5, 6), A(-5, 1), F(-1, 6) and T(-1, 1). Reflect the shape over the y-axis.

<table>
<thead>
<tr>
<th>Original</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(-5, 6)</td>
<td></td>
</tr>
<tr>
<td>A(-5, 1)</td>
<td></td>
</tr>
<tr>
<td>F(-1, 6)</td>
<td></td>
</tr>
<tr>
<td>T(-1, 1)</td>
<td></td>
</tr>
</tbody>
</table>

2. What are the coordinates of T' if T(-6, -5) is reflected over the x-axis?
   a) (-6, 5)  b) (5, 6)  c) (6, -5)  d) (-5, -6)

3. Plot the points P(3, 5), A(6, 5), and N(6, 9). Rotate the image 90˚ about the origin.

<table>
<thead>
<tr>
<th>Original</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(3, 5)</td>
<td></td>
</tr>
<tr>
<td>A(6, 5)</td>
<td></td>
</tr>
<tr>
<td>N(6, 9)</td>
<td></td>
</tr>
</tbody>
</table>

4. What are the coordinates of B' when B(-4, 5) is rotated 180˚ about the origin?
   a) (-4, -5)  b) (4, 5)  c) (4, -5)  d) (5, -4)
Describing Transformations

5. Plot the points \( F(-5, -4) \), \( U(-7, -6) \), and \( N(-2, -5) \). Plot the image of \( \text{FUN} \) under the transformation \( T_{(8, 3)} \).

<table>
<thead>
<tr>
<th>Original</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(-5, -4) )</td>
<td></td>
</tr>
<tr>
<td>( U(-7, -6) )</td>
<td></td>
</tr>
<tr>
<td>( N(-2, -5) )</td>
<td></td>
</tr>
</tbody>
</table>

6. Given the ordered pair \( F(3, -5) \), find \( F' \) under \( T_{(-4, 2)} \).

   a) \( (7, 7) \)  
   b) \( (1, 3) \)  
   c) \( (-1, -3) \)  
   d) \( (-7, 3) \)

7. Plot the points \( S(2, -4) \), \( H(4, -6) \), \( O(6, -4) \), and \( W(8, -6) \). Plot \( S'H'O'W' \) under \( D_{\frac{1}{2}} \).

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( S(2, -4) )</td>
<td></td>
</tr>
<tr>
<td>( H(4, -6) )</td>
<td></td>
</tr>
<tr>
<td>( O(6, -4) )</td>
<td></td>
</tr>
<tr>
<td>( W(8, -6) )</td>
<td></td>
</tr>
</tbody>
</table>

8. Given the point \( K(4, -8) \), what are the coordinates of \( K' \) under \( D_2 \)?

   a) \( (2, -4) \)  
   b) \( (-8, 4) \)  
   c) \( (-4, 8) \)  
   d) \( (8, -16) \)
### Standards: NYSCCSS

8.G Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

### MP Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### Lesson Materials:

- Unit Test

### Key Vocabulary:

All Vocabulary from Unit

### Essential Question:

What is a transformation?
How do you perform a transformation on a figure?
How do you determine which transformation has been applied?

### Classroom Activities:

#### Teach:

1. Teacher will ask students if they have any questions from their homework or the unit in general.
2. Teacher will answer student questions.
3. Student will complete the Unit Assessment.

### Ticket Out the Door:

None

### Homework:

None

### Assessments:

Summative Assessment: Unit Test

### Resources Used:

Engage NY – 8th Grade Math – Module 2 & 3
http://www.regentsprep.org/regents/math/geometry/gt3/idilate2.htm
McGraw Hill Glencoe Textbook Series
1.) If $ABCD$ is reflected over the x-axis and translated 5 units to the right, which is the resulting image of point $B$?

a) (-1, -2)  

b) (-11, 2)  

c) (-1, 2)  

d) (11,2)

2.) A line segment has endpoints Q (-5, -6) and P (-5, 1). Which of the following figures is the image after a dilation?

a) Q' (-5,6), P' (-5, -1)  

b) Q' (5,-6), P' (5, 1)  

c) Q' (-10,-12), P'(-10,2)  

d) Q' (-6,-5, ), P' (-1, -5)

3.) The following graph shows the image of $\triangle GJH$ under what sequence of transformations?

a) Reflection over x axis followed by a dilation  

b) Rotation 90 clockwise followed by a reflection  

c) Reflection over the y axis followed by a translation 3 units up  

d) Rotation 180 followed by a reflection over the x axis

4.) Anna drew the two figures shown on the coordinate grid. Which transformation did Anna apply to Figure A to get to Figure B?

a) rotated 90º  

b) Dilated by 6  

c) Reflected in the y-axis  

d) Translated 6 units to the left
Describing Transformations

5. What is T’ when point T (4,5) is translated 3 units to the right and 2 units down?
   a) (7,7) b) (1,3) c) (7,3) d) (-7, -3)

6. Carrie rotated her puzzle piece 180° clockwise to see if she could use it.
   Which image represents the position of the puzzle piece after 180° clockwise rotation?
   a) b) c) d)

7. Which drawing best represents a reflection over the vertical line segment in the center of the rectangle?
   a) b) c) d)

8. The graph shows segment M’N’ is a dilation of segment MN. What is the scale factor of the dilation?
   a) 4 b) \( \frac{1}{2} \) c) 2 d) \( \frac{1}{4} \)

9. Which letter does not have line symmetry?
   a) R b) I c) V d) B
10. The grid below shows a triangle located in the first quadrant. If the triangle was reflected over the x-axis, what would it look like and what quadrant would it lie in?

11. ΔA'B'C' is the image of ΔABC after a transformation. What was the transformation?
   - a) A reflection in the y-axis
   - b) A 90° rotation
   - c) Translation
   - d) A dilation with scale factor 2

12. The area of triangle RST is 36 square inches. Under which transformation, could the area of image, triangle R'S'T', be greater than 36 square inches?
   - a) dilation
   - b) reflection
   - c) translation
   - d) rotation

13. What property is not preserved under a dilation?
   - a) angle measures
   - b) shape
   - c) size
   - d) orientation

14. What is W' when point W (7,5) is translated 4 units to the left and 3 units up?
   - a) (3,8)
   - b) (8,3)
   - c) (11,8)
   - d) (11, 2)
15. A rectangle is plotted on the graph below. Which image shows a 90° clockwise rotation about the origin?

- A
- B
- C
- D
Part 2: Short Answer
16) Graph \( XYZ \) with vertices \( X(-1, 2), Y(7, 2), \) and \( Z(3, 6). \)

**Part A**: Translate \( XYZ \) 3 units left and 4 units down. Identify the coordinates of each new vertex.

<table>
<thead>
<tr>
<th>Original Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X(-1, 2) )</td>
<td></td>
</tr>
<tr>
<td>( Y(7, 2) )</td>
<td></td>
</tr>
<tr>
<td>( Z(3, 6) )</td>
<td></td>
</tr>
</tbody>
</table>

**Part B** Find the vertices of \( X' \) \( Y' \) \( Z' \) after a dilation with a scale factor of \( \frac{1}{2} \). Then graph the dilation.

<table>
<thead>
<tr>
<th>Old Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

17) Figure A has been transformed to Figure B.

a. Can Figure A be mapped onto Figure B using only translation? Explain. Use drawings, as needed, in your explanation.

b. Can Figure A be mapped onto Figure B using only reflection? Explain. Use drawings, as needed, in your explanation.
Describing Transformations

18) Rotate Δ XYZ around the origin, clockwise 90°. Label the image of the triangle X', Y', and Z'.

19) a. One triangle in the diagram below can be mapped onto the other using two transformations. Identify the transformations used.

b. Can you map one triangle onto the other using just one transformation? If so, identify the transformation used.
20) Reflect $\triangle ABC$ over the line $y = x$. Graph the image and state the new coordinates.

<table>
<thead>
<tr>
<th>Old Coordinates</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(-5,8)</td>
<td></td>
</tr>
<tr>
<td>B(-2,5)</td>
<td></td>
</tr>
<tr>
<td>C(-6,2)</td>
<td></td>
</tr>
</tbody>
</table>
Describing Transformations

Validity and Conclusion

Any curriculum developed transitions through three phases: the intended curriculum, the implemented curriculum, and the attained curriculum. These three phases occur in a cycle. The first phase to occur is the intended curriculum: the actual content and material that one would want the students to learn. In this unit plan, the intended curriculum relates to the mathematical content of Transformations and is aligned to the NYSCCSS. The curriculum outlined above is intended to be used as a resource for any educator working with the NYSCCSS. However, the curriculum will only remain valid if educators implement the curriculum with fidelity. Thus, it is essential that any educator who chooses to implement this curriculum has bought-in to the ideas expressed in the unit plan. If the unit plan is implemented as intended, students will be provided the opportunity to learn the rules of transformations to translate, reflect, rotate, and dilate objects both on a coordinate grid and abstractly across a plane. Students can also develop their skills within two of the Standards for Mathematical Practices: use appropriate tools strategically (mathematical practice five) and reason abstractly and quantitatively (mathematical practice two).

The unit plan provided can be an additional resource when teaching students about transformations of objects: translations, rotations, reflections and dilations. The curriculum is developed to provide students the opportunity to develop a deeper understanding of the transformations rather than simply memorizing and reciting rules that were provided by the educator. The unit plan is aligned with the NYSCCSS, as well as, two standards for mathematical practice: (2) Reason abstractly and quantitatively and (5) use appropriate tools strategically (CCSSI, 2010).

As written, the unit plan is one where the intended curriculum and the attained curriculum should be the same. The success of the curriculum requires the educator to follow the unit plan with fidelity which would require it to be implemented by educators who have bought-in to the ideas that the unit plan is trying to express. However, an educator must also understand that every class is different and they may need to differentiate the
Describing Transformations
to better meet the needs of their students. The lessons in this unit plan, should be used as guide and
alternate resource to support teachers who work in states where CCSS has been adopted.
Describing Transformations

References


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www.pbs.org/makingschoolswork/sbs/csp/buyin.html


http://www.jstor.org.ezproxy2.drake.brockport.edu/stable/40388648