Decreasing Math Anxiety Through Teaching Quadratic Equations

Kaitlyn Kaufman
kkauf2@u.brockport.edu

Follow this and additional works at: https://digitalcommons.brockport.edu/ehd_theses

Part of the Algebra Commons, and the Secondary Education Commons

To learn more about our programs visit: http://www.brockport.edu/ehd/

Repository Citation
https://digitalcommons.brockport.edu/ehd_theses/787

This Thesis is brought to you for free and open access by the Education and Human Development at Digital Commons @Brockport. It has been accepted for inclusion in Education and Human Development Master's Theses by an authorized administrator of Digital Commons @Brockport. For more information, please contact digitalcommons@brockport.edu.
Decreasing Math Anxiety

Through Teaching Quadratic Equations

Kaitlyn E. Kaufman

The College at Brockport, State University of New York (SUNY)
Math anxiety is known as having a feeling of fear that interferes with math performance. Many students today suffer from math anxiety as they push through each developmental stage in their schooling. A majority of students develop math anxiety through traditional classroom methods, such as drill and practice, assessments, memorizing, and textbooks. According to research, teachers can help decrease math anxiety in students by incorporating specific teaching styles, methods, and strategies, related to decrease math anxiety, into lessons. These teaching styles, methods, and strategies include, but not limited to, constructivist teaching, concrete-to-representation-to-abstract model, student-centered learning, and interactive lessons. Based on this research, a unit plan was created for teachers that focus on decreasing math anxiety in students by teaching quadratic equations. The unit plan aligns to the New York State (NYS) Common Core State Standards (CCSS) and was tailored for students who experience math anxiety in Algebra I.

*Keywords*: math anxiety, quadratics, CCSS, constructivist teaching
Chapter One: Introduction

Anxiety related to learning mathematics, often referred to as math anxiety, is highly prevalent in all levels of education (Chernoff & Stone, 2014; Finlayson, 2014; Park, Ramirez, & Beilock, 2014). Math anxiety is defined as a “feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations” (Finlayson, 2014, p. 100). The tensions that students may face could be a “mild tension to a strong fear of mathematics” (Finlayson, 2014, p. 100). These fears result in students having lower performance scores, lack of self-confidence, and avoidance of mathematics (Chernoff & Stone, 2014). Often students with math anxiety tend to experience anxiety no matter what type of instructional strategies are used in the classroom (Finlayson, 2014).

A unit plan designed around the best practices established through research in the field of math anxiety may support teacher’s efforts to decrease anxiety during the learning of high school mathematics. This unit plan offers teachers various teaching styles, strategies, and methods, which establishes fundamental mathematical skills, to decrease math anxiety while still preparing students for formal assessments. The goal of this thesis is to support teachers of Algebra I to learn ways to support students with math anxiety during the learning of mathematics.
Chapter Two: Literature Review

Theoretical Perspective

As students advance in their mathematical classes, through the learning of higher-cognitive concepts, students will be exposed to more complex tasks compared to simple tasks. According to Van Merriënboer et al. (2006), “complex tasks have many different solutions, are ecologically valid, cannot be mastered in a single session and pose a very high load on the learner’s cognitive system” (p. 343). Complex instruction also forces students to use their working memory in order to understand and learn new concepts (Pollock, Chandler, & Sweller, 2002). This type of instruction can overwhelm students and put pressure on their working memory, which can hinder students learning.

Cognitive load theory defines learning as a permanent change in long term memory, which indicates the ability to recall and apply content learned in future problem solving. The cognitive load that teachers can seek to manage to support students with math anxiety is extraneous cognitive load. This type of cognitive load instructs teachers to create instruction that will reduce unnecessary working memory loads (Pollock et al., 2002). Intrinsic cognitive load “is imposed by the intellectual complexity of information” and “if understanding is to occur, it will impose a heavy working memory load irrespective of instructional design considerations” (Pollock et al., 2002, p. 62). Thus, when learning complex mathematical content (and the complexity is dependent upon the current content knowledge of the learner) it is only extraneous cognitive load that teachers can seek to manage, and for students with math anxiety, this is crucial for learning.

In addition to Pollock et al. (2002) work, Van Merriënboer et al. (2006) examined two different ways germane-load, “a load that directly contributes to learning...to the
learner’s construction of cognitive structures and processes that improve performance,”
methods affect learning. One is low contextual interference (LCI) and the other is high
contextual interference (HCI) (p. 344). In LCI, “one version of a task is repeatedly practiced
before another version of the task is introduced” (Van Merriënboer et al., 2006, p. 344).
This type of interference is a common practice made by teachers, since teachers learned
this way when they were in school. Even though this is a common practice, LCI did not
produce higher performance during retention tests. In HCI, “all versions of the task are
mixed and practiced in a random order” (Van Merriënboer et al., 2006, p. 344). This
method showed less effective performance during practice, but has proven to produce
higher performance on test scores (Van Merriënboer et al., 2006, p. 344; Magill & Hall,
1990).

Many student look for feedback on formative and informative assessments, in order
to gain an understanding of their mathematical understanding. As students begin to solve
complex tasks, it is important that teachers provide constructive feedback to students.
Feedback is the “process by which an environment returns to individuals a portion of the
information in their response output necessary to compare their present strategy with a
representation of an ideal strategy” (Balzer et al., 1989, p. 412; Doherty & Balzer, 1988).
Through the use of feedback, students will gain a better understanding of what they
comprehend, as well as improving their mathematical knowledge and reducing incorrect
answers.

State Standards to Common Core Standards

In 2010 there were 42 states that adopted and began to implement the new
Common Core State Standards (CCSS), including New York State (NYS). The Common Core
Standards place emphasis on demonstrating understanding, balancing procedures, and solving nonroutine problems (Porter et al., 2011: Hinde, 2014). These standards also stress the importance of basic algebra, such as order of operations, exponents, and simple equations, as well as, geometric concepts, and measurements. The state standards placed more emphasis on memorization and performance procedures, as well as instructional technology, advanced algebra, and advanced geometry (Porter et al., 2011). Both the Common Core Standards and state content standards focus on conjecture as well as number sense and operations.

The Common Core curriculum illustrates eight standards for mathematical practices that provide students with a deeper understanding of mathematical concepts (Hinde, 2014). The eight standards for mathematical practices are: “(1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; (8) Look for and express regularity in repeated reason” (Hinde, 2014, p. 50). These standards allow students to move away from memorization and/or procedures, in order for students to reach mathematical goals.

Teachers are expected to incorporate the standards they are to teach, either the state standards, most often known as the National Council of Teacher Mathematics (NCTM) Standards or the Common Core standards into their lesson plans. These standards, or policies within school systems where teachers teach, often, require teachers to rely on pre-made mathematics curricula, which may not align with the students’ needs in any given classroom. It is important for teachers to remember that there are multiple ways teachers
can use curriculum vision and coherence to support their students learning. Such curriculum vision should align with their own teacher autonomy and views about how learning occurs in the classrooms where they teach (Cirillo et al., 2009).

According to Cirillo et al. (2009), curriculum vision is understanding the mathematics that students must learn, as well as creating coherence in learning mathematics. For teachers to have a well-defined curriculum vision, they must examine each mathematical standard and objective. Teachers can then create curriculums by building off of each standard/objective and their own mathematical beliefs. These factors will allow a teacher to create curriculum coherence. Making connections between the students and teacher with a focus on mathematical concepts will allow a teacher to create coherence (Cirillo et al., 2009).

There are many factors that influence a teacher’s practice and curriculum vision. These include: district standards, state standards, national standards, new textbooks, technology, and research in how students learn mathematics (Cirillo et al., 2009). Cirillo et al. (2009) believe that the key to teaching and learning mathematics is the “combination of knowledge, beliefs, resources, vision, and support necessary to assist teachers in providing coherent mathematics instruction” (p. 75).

**Math Anxiety**

Math anxiety, according to Richardson and Suinn, is “characterized by ‘fear about performing mathematics and is associated with delayed acquisition of core mathematics, number concepts, and poor math competence’” (Finlayson, 2014, p. 100). Research has shown that “math anxiety is frequently linked to the teaching style of mathematics in the classroom” (Finlayson, 2014, p. 100). Most mathematics teachers use traditional
instruction methods (Finlayson, 2014) which are most typically: (a) Drill and practice; (b) Assessment via testing/correct answers; (c) Textbooks and workbooks; (d) Memorizing formulas and applying rules; and (e) Students working individually (Chernoff & Stone, 2014, p. 30; Finlayson, 2014, p. 100). This may be because teachers must focus on covering the accepted curriculum, which is currently most likely the Common Core State Standards (CCSS). Also, such instruction uses textbooks, workbooks, and emphasizes computational skills (Finlayson, 2014). Additionally, students can become overwhelmed with the pace they are expected to learn content, which can also increase student anxiety when learning mathematics.

Mathematics is “often taught as if the students have the same ability, preferred leaning style, and pace of working” (Finlayson, 2014, p. 101). Teachers also tend to “focus on repetition and speed or ‘timed tests’ as important tools for improving mathematical prowess and skills” (Finlayson, 2014, p. 101). When teachers are focus on repetition and speed, it forces students to drill and memorize mathematical concepts. This can increase math anxiety as well as cause students to develop negative outlooks on learning mathematics, especially if students are not successful on assessments when the focus is speed and accuracy (Finlayson, 2014).

Students can also develop a fear of failure, which can increase math anxiety (Finlayson, 2014). When students are presented with a problem, they feel the pressure that there is only one right answer, or that there is only one way to answer that question (Finlayson, 2014, p. 108). Students “felt a lot of stress when they thought that they had the wrong answer; they didn’t know how to do the mathematics to get the right answer; or to whom they could turn to get help” (Finlayson, 2014, p. 108). This type of behavior can
result in ‘global avoidance tendency’ (Chernoff & Stone, 2014, p. 30). According to Ashcraft (year), global avoidance tendency is when students avoid studying for mathematics, which then may limit future education as well as impair their ability to survive and excel in modern society (Chernoff & Stone, 2014). This can limit students in two ways: (1) students most often rush through mathematics in an attempt to limit the amount of required; and (2) students often seek to reduce, via course selection, the amount of time spent studying mathematics (Chernoff & Stone, 2014). When students avoid studying for a mathematics exam this often creates more anxiety and a stronger fear of failure. Students also develop test anxiety if: (a) all classes were not attended; (b) all assignments were not completed; (c) they crammed for the exams; (d) they had not kept up on their work; or (e) had not completed assignments without fully understanding the process of doing the mathematics (Finlayson, 2014). As a result, students often become unsure how to problem solve (Finlayson, 2014).

**Math Anxiety and Performance**

Algebra I provides students with the foundational mathematics that will support their learning in higher level mathematical classes. These students may not conceptually understand number order, computation, or how to problem solve, which can cause stress and anxiety when learning mathematics. Students who fear learning math “show a strong tendency to avoid learning mathematics, hold negative attitudes towards mathematics, and have weak self-confidence in doing mathematics” (Finlayson, 2014, p. 100). This can cause students “avoiding courses and careers that involve mathematics” (Park, Ramirez, & Beilock, 2014, p.103). Many students, who experience math anxiety, may also develop stomachaches, headaches, increased heart rates, and sweating according to Chernoff and
Stone (2014, p. 29). These symptoms align with students feeling anxious and nervous because they are overwhelmed with mathematical concepts.

There have been numerous studies that have examined the link between math anxiety and performance in mathematics which reveal a significant relationship between the two (Chernoff & Stone, 2014; Park, Ramirez, & Beilock, 2014). Students with a high degree of math anxiety receive lower grades in mathematics courses across k-12 and college courses when compared to students without math anxiety (Park, Ramirez, & Beilock, 2014). Research has shown that high math-anxious individuals (HMAs) underperform relative to low math-anxious individuals (LMAs) (or to students with no math anxiety) on basic numerical tasks (Park et al., 2014). Another factor that causes low performance in mathematics, according to Park et al. is that HMAs tend to worry and have intrusive thoughts regarding mathematical situations, which robs students of their cognitive resource in working memory (p.103). Students working memory holds intermediate steps in their mind and computes solutions to difficult problems on mathematics tests (Park, et al.). “When working memory is compromised, an individual’s ability to perform at a high level can suffer” (Park et al.p.103).

**Concrete-To-Representation-To-Abstract Model**

Through the years, the expectations of mathematics to be learned in high school have become more rigorous, especially in Algebra, which, in most states, is the first mathematics Carnegie unit required for graduation. Many states are now pushing middle school students into learning pre-algebra content, as well as enrollment in Algebra I courses (Witzel, 2005). As teacher’s present curriculum, hands-on instruction can benefit students in order to “build problem-solving and higher order thinking” (Witzel, 2005, p. 49-
Witzel (2005) compared the CRA (concrete-to-representational-to abstract) model to traditional abstract instruction with middle school students.

The CRA model is a process where students use concrete objects, picture representations, and abstract numbers (Witzel, 2005). CRA has three learning stages where “students learn through physical manipulation of concrete objects, followed by learning through pictorial representations of the concrete manipulations, and ending with solving problems using abstract notation” (Witzel, 2005, p. 50). Students who experience math difficulties can benefit from the CRA model. The purpose of the CRA model is to allow students to use multiple strategies by using “visual and auditory interactions with content...kinesthetic and tactile, through the use of hands-on manipulations of objects and matching of pictorial drawings” (Witzel, 2005, p.51).

“An effective curriculum is one that can help students acquire the new symbol system as well as teach the proper steps to solving algebraic problems” (Witzel, 2005, p. 50-51; Stacey & MacGregor). The CRA model allows students to use multiple representations to solve a mathematical concept as well as reinforcing prior knowledge. This model stresses the importance of reaching out to all types of students, for example students who struggle in math and students who have higher math achievement, in order to teach effective math skills and connect to their conceptual understanding.

**Prevention Strategies**

To reduce a student’s math anxiety, teachers can use the Constructivist teaching approach (Finlayson, 2014). Constructivist teaching is a process in which “learning occurs as learners are actively involved in a process of meaning and knowledge construction as opposed to passively receiving information” (Finlayson, 2014, p. 102). This type of
classroom instruction practices: (1) Beginning with the whole and then expending into parts; (2) Pursuing students questions/interests; (3) Learning is interactive and builds on what students already know; (4) Instructor interacts/negotiates with students; (5) Assessments are based on student work, observations, point of views, and tests; and (6) Students work in groups (Finlayson, 2014).

A Constructivist classroom engages students through activities that are interactive, interesting, and student-centered (Finlayson, 2014). “The teacher facilitates the learning process, encouraging students to take risks, to be responsible and to be critical and independent thinkers” (Finlayson, 2014, p. 102). Finlayson notes that Constructivist teaching focuses on the process and understanding of mathematical concepts by guiding students through problems so that they are less worried about right or wrong answers (2014, p. 102). The assessments for this type of classroom include “student work and projects, observations, conferences, anecdotal notes, checklists, diagnostic interviews, and tests” (Finlayson, 2014, p. 102). Constructivist teaching provides students with a classroom environment, were they feel comfortable and confident in completing mathematic problems through interactive tasks.

Implementing contextual problems can also reduce stress and anxiety in students through the learning of mathematics. Professionals are now calling for the use of activities that incorporate authentic real world problems, in order to engage and motivate students. Teachers find that they can integrate student’s interests into context problems, to gain student attention. Beswick (2011) examined the effectiveness of context problems in order to enhance student achievement, participation, and engagement.
Beswick (2011) provided five reasons why teachers should implement contextual problems in the mathematic curriculums. First, mathematic classes should prepare students to meet the economic needs of society. It has been reported that in 1982, adults were unable to “apply the mathematics that they had learned at school in vocational and other everyday contexts” (Beswick, 2011, p. 369). Secondly, teachers should use mathematics to “teach students about issues deemed important, as well as providing a context in which mathematics could be applied” (Beswick, 2011, p. 370). According to Beswick (2011), contextual problems can improve students’ understanding of mathematical concepts, as well as enhancing their appreciation of mathematics. Teachers can use “context problems to make the experience of learning mathematics mirror more closely the activities of mathematicians” (Beswick, 2011, p. 370). Lastly, context problems can improve student affects in relation to mathematics. Middle school students become disengaged in mathematics, thus context problems can improve a student’s attitude and viewpoint in mathematics (Beswick, 2011, p. 371).

Beswick (2011) stated, “enhanced student achievement in mathematics should be based on understanding of important ideas and not simply on the performance of meaningless procedures” (p. 382). Students should learn mathematics through the process of engagement instead of learning procedures and applying them (Beswick, 2011, p. 382). Through the use of engagement, teachers are creating meaningful and memorable activities that students can connect to. Ultimately, engagement can motivate students to participate in mathematical lessons and reach academic goals.

Context problems are “intended to enhance the accessibility of the mathematics, reveal students’ mathematical thinking and understanding the suggest strategies for
solving the mathematical problem” (Beswick, 2011, p. 377). These problems increase student engagement, participation, and achievement in a mathematical classroom. Overall, “context problems have the potential to improve students’ mathematical understanding but...are complex, idiosyncratic and...dependent upon contexts” (Beswick, 2011, p. 379).

Peer collaboration in the classroom can also improve student’s math anxiety by allowing students to work together to enhance their learning and complete a common goal. “Collaboration has been the basis for the development of a community that fosters children’s learning where the idea is that both the children and the adults take ‘varying but coordinated responsibilities to foster children’s learning’” (Fawcett & Garton, 2005, p. 158; Rogoff, Turkanis, & Bartlett, 2001). Fawcett and Garton (2005) extended the study by Garton and Pratt (2001) to examine the effect of peer collaboration on 7-year-old students problem solving ability. This work provides an insight of how peer collaboration can benefit, not only 7-year-old students, but also all high school students including students taking algebra.

The relationship between peer collaboration and cognitive development is based on the Piaget or Vygotsky’s theories (Fawcett & Garton, 2005; Tudge, 1992). Piaget believed that a “child’s cognitive development depended on manipulation of, and active interaction with, the environment,” i.e., Piaget viewed social interactions as important in intellectual development (Fawcett & Garton, 2005, p. 158; Piaget, 1959). Vygotsky argued, “cognitive development is most likely to occur when two participants, who differ in terms of their initial level of competence, work collaboratively on a task to arrive at a shared understanding” (Fawcett & Garton, 2005, p. 158; Garton, 1992; Johnson & Johnson, 1994). Thus, when teachers pair students together, they should do so based on student’s
mathematical ability. Teachers should match students, such as students with lower
cognitive ability with students who have a higher cognitive ability, in order to develop their
understanding. Through the combination of Piaget’s and Vygotsky’s theories, it is suggested
“the benefits of peer collaboration arise from active participation in interaction and verbal
communication with a partner who has a different perspective, either due to more
knowledge, or a different viewpoint” (Fawcett & Garton, 2005, p. 160; Kruger, 1993).

Collaboration with students as they work through tasks, shows great benefit for all
types of students. Students are exposed to higher level thinking as well as verbal
interaction between peers, which can help decrease math anxiety. As teachers incorporate
peer collaboration, the tasks must be appropriate and structured in order for students to
work together and complete a common goal. Through this approach teachers are
“providing information within children’s zone of proximal development,” which coincides
with Vygotsky’s theory, and also “create[s] the socio-cognitive conflict necessary from a
Piagetian perspective” (Fawcett & Garton, 2005, p. 166).

**Research Question**

Through the use of a constructivist teaching approach, what strategies and methods
can teachers implement to students, who have math anxiety, so that they can achieve
academic success?
Chapter 3: Methods

This unit plan will align with the Algebra I curriculum as well as the NYS Common Core to teach quadratic equations. The lessons that are designed for the unit plan will use the Constructivist teaching approach in order to decrease student anxiety in mathematics. A Unit Overview and Unit Timeline are provided for teachers to better manage their time. This suggested 11 – 12 day timeline is based on a 45-minute class period. As the students are presented with each lesson, they will be given a creative and fun note sheet, which will help students comprehend and remember the material. Students will also conduct hands-on activities by themselves and in groups. This approach will push students into practicing their math comprehension and prepare them for the end of the unit test.

It is to be noted that the worksheets and activities for this curriculum do not follow APA format. This was intended so teachers can maximum the amount of information on each page, which will save paper. Answer keys to all materials may be found in the Appendix.
# UNIT PLAN: TABLE OF CONTENTS

## INTRODUCTION

Table of Contents .................................................................................................................................................. 17  
Unit Overview ..................................................................................................................................................... 18  
Unit Timeline ..................................................................................................................................................... 18  

## LESSON PLANS

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Equation Cover Page</td>
<td>19</td>
</tr>
<tr>
<td>Day 1: Solving Quadratic Equations by Graphing</td>
<td>20</td>
</tr>
<tr>
<td>Day 2: Solving Quadratic Equations by Graphing – Activity</td>
<td>24</td>
</tr>
<tr>
<td>Day 3: Solving Quadratic Equations by Factoring</td>
<td>32</td>
</tr>
<tr>
<td>Day 4: Solving Quadratic Equations by Factoring – Activity</td>
<td>36</td>
</tr>
<tr>
<td>Day 5: Solving Quadratic Equations by Completing the Square – Box Method</td>
<td>41</td>
</tr>
<tr>
<td>Day 6: Solving Quadratic Equations by Completing the Square – “It” Method</td>
<td>50</td>
</tr>
<tr>
<td>Day 7: Deriving the Quadratic Formula – Discriminants and Axis of Symmetry</td>
<td>55</td>
</tr>
<tr>
<td>Day 8: Solving Quadratic Equations by Using the Quadratic Formula</td>
<td>60</td>
</tr>
<tr>
<td>Day 9: Review Activity – Quadratic Equation Graphic Organizer</td>
<td>64</td>
</tr>
<tr>
<td>Day 10: Review Activity – Relay Review</td>
<td>69</td>
</tr>
<tr>
<td>Day 11 – 12: Assessment – Quadratic Equation Test</td>
<td>76</td>
</tr>
</tbody>
</table>
# UNIT PLAN: QUADRATIC EQUATIONS

## UNIT OVERVIEW

**Subject Area**
Ninth Grade Mathematics – Algebra 1

**Approximate Time Needed**
11 – 12 Days
(Based on 45 minute class periods)

**New York State Standards Addressed**
- **CCLS – Math: A.SSE.1** Interpret expressions that represent a quantity in terms of its context
- **CCLS – Math: A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see \( x^2 - y^2 \) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\)
- **CCLS – Math: A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression
- **CCLS – Math: A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include arising from linear and quadratic functions, and simple rational and exponential functions
- **CCLS – Math: A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
- **CCLS – Math: A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
- **CCLS – Math: A.REI.4** Solve quadratic equations in one variable
- **CCLS – Math: F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
- **CCLS – Math: F.IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima

## UNIT TIMELINE

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving Quadratics by Graphing</td>
<td>Solving Quadratics by Graphing - Activity</td>
<td>Solving Quadratics by Factoring</td>
<td>Solving Quadratics by Factoring – Activity</td>
<td>Solving Quadratics by Completing the Square – Box Method</td>
</tr>
<tr>
<td>Day 6</td>
<td>Day 7</td>
<td>Day 8</td>
<td>Day 9</td>
<td>Day 10</td>
</tr>
<tr>
<td>Solving Quadratics by Completing the Square – “It” Method</td>
<td>Deriving the Quadratic Formula and Determining Discriminants and Axis of Symmetry</td>
<td>Solving Quadratics by Using the Quadratic Formula</td>
<td>Review Activity – Graphic Organizer</td>
<td>Review Activity – Relay Activity</td>
</tr>
<tr>
<td>Day 11</td>
<td>Day 12</td>
<td>Day 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assessment – Quadratic Equation Test</td>
<td>Possible Assessment – Quadratic Equation Test</td>
<td>(Start of next Unit)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Factoring

\[ x^2 + 8x + 7 = 0 \]

\[ (x + 7)(x + 1) = 0 \]

\[ x + 7 = 0 \quad \text{or} \quad x + 1 = 0 \]

\[ x = -7 \quad \text{or} \quad x = -1 \]

Solution Set \{ -7, -1 \}

Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Unit: Solving Quadratic Equations

Graphing

Completing the Square

\[ x^2 - 9 = 4x \]

\[ x^2 - 4x + \_ = 9 + \_ \]

\[ x^2 - 4x + 4 = 9 + 4 \]

\[ \sqrt{(x-2)^2} = \sqrt{13} \]

\[ x-2 = \pm \sqrt{13} \]

\[ x = 2 \pm \sqrt{13} \]

\[ x \approx 2 - \sqrt{13} \quad 2 + \sqrt{13} \]
Day 1
Solving Quadratic Equations by Graphing

NYS Learning Standards:
- **CCLS – Math: F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
- **CCLS – Math: F.IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima

Goal(s) and Objective(s):

Today's Goal: (Communicated to students on the front board)
**Graphing Quadratic Equations**

Learning Objectives:
1. Students will be able to define quadratic equation.
2. Given a quadratic equation, students will rewrite the equation in standard form and vertex form.
3. Students will be able to graph a quadratic equation.
4. Students will be able to interpret key features of a graph and table, such as domain, range, y-intercept, x-intercepts, vertex, and maximum or minimum.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators

Instructional Plan:
1. **Notes:**
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board.
2. **Group Practice:**
   Students will be placed in groups, where they will complete the remaining problems in the notes.
3. **Closure:**
   Before the end of the period, gather all students together in order to go over the group problems. This is a way to see if students met the goal and objectives for the day, as well as wrapping up the lesson.
Solving Quadratic Equations By Graphing

First, let's define Quadratic Equation

_____________________________________________________________________
_____________________________________________________________________

Standard Form: __________________________________________________________________

Vertex Form: __________________________________________________________________

How do we Graph a Quadratic Equation?

Steps:
1. Create a table for $x$ and $y$ values
2. Plot the points and connect the dots
3. Label line with the given equation
4. Find and label the vertex, $y$-intercept $(x = 0)$, and $x$-intercepts $(y = 0)$
5. State if equation is a maximum or minimum
6. State the domain and range of equation

Class Example 1:

$$y = x^2 + x - 6$$

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain: ___________________________
Range: ___________________________
y - intercept:____________________
x - intercepts: ___________________
Vertex: ___________________________
Vertex Form: _______________________
Minimum or Maximum: ___________________
Class Example 2:

\[ y = 2x^2 - 6x - 4 \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain: ___________________________

Range: ___________________________

y - intercept: ______________

x - intercepts: __________________

Vertex: __________________________

Vertex Form: ______________________

Minimum or Maximum: __________________

---

Group Example 1:

\[ y = x^2 + 4 \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain: ___________________________

Range: ___________________________

y - intercept: ______________

x - intercepts: __________________

Vertex: __________________________

Vertex Form: ______________________

Minimum or Maximum: __________________
Group Example 2:

\[ y = x^2 - 5x + 6 \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain: ___________________________

Range: ___________________________

y - intercept: _________________

x - intercepts: _____________________

Vertex: _____________________________

Vertex Form: ___________________________

Minimum or Maximum: ______________________
Day 2
Solving Quadratic Equations by Graphing - Activity

NYS Learning Standards:
- **CCLS – Math: F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
- **CCLS – Math: F.IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima

Goal(s) and Objective(s):
- **Today's Goal:** (Communicated to students on the front board)
  Graphing Quadratic Equations
  
  **Learning Objectives:**
  1. Students will be able to graph a quadratic equation.
  2. Given a quadratic equation, students will rewrite the equation in standard form and vertex form.
  3. Students will be able to interpret key features of a graph and table, such as domain, range, y-intercept, x-intercepts, vertex, and maximum or minimum.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculator
- Scissors
- Tape

Instructional Plan:
1. **Individual Work:**
   Students will start working on the review problems on their own, in order to gain an understanding of what each student knows.
2. **Partner Work:**
   Each student will pick a partner and finish/check the review problems together. After students have completed the problems, go over and provided students with the correct answers.
3. **Group Work:**
   Students will be placed in groups of 3-4 and will work on completing the group activity.
4. **Closure:**
   Before the end of the period, students will hand the activity in for a grade. This is a way to see if students met the goal and objectives for the lesson.
Solving Quadratic Equations By Graphing

Review Problem 1:

\[ y = x^2 + 2x - 3 \]

\[ \begin{array}{c|c}
X & Y \\
\hline
& \\
& \\
& \\
& \\
& \\
& \\
\end{array} \]

Domain: _____________________________
Range: _____________________________
y - intercept: ________________
x - intercepts: _______________________
Vertex: ______________________________
Vertex Form: _________________________
Minimum or Maximum: __________________

Review Problem 2:

\[ y = -x^2 + 5x \]

\[ \begin{array}{c|c}
X & Y \\
\hline
& \\
& \\
& \\
& \\
& \\
& \\
\end{array} \]

Domain: _____________________________
Range: _____________________________
y - intercept: ________________
x - intercepts: _______________________
Vertex: ______________________________
Vertex Form: _________________________
Minimum or Maximum: __________________
Group Activity

Directions:
1. Cut out each puzzle piece.
2. Work in your group to complete 6 puzzles. Each puzzle will have a piece with a graph, an equation in standard form, an equation in vertex form, and x-/y-intercept(s).
3. Once you complete all of the puzzles, all group members will raise their hand to get their puzzles checked by the teacher.
4. If all of the puzzles are correct, your group will record each puzzle in a box containing all of the pieces.
   a. Remember: Use the example on the board to see how each box should be filled in.
5. If a puzzle is incorrect, your group will go through and correct the mistake(s) until correct.
6. After all the boxes are filled in, hand in your work sheet with all your group members’ names for a class grade.

Equation: __________________________________________
Vertex Form: _______________________________________

x – intercept(s): ______________________________________
y – intercept: ________________________________________

Equation: __________________________________________
Vertex Form: _______________________________________

x – intercept(s): ______________________________________
y – intercept: ________________________________________

Equation: __________________________________________
Vertex Form: _______________________________________

x – intercept(s): ______________________________________
y – intercept: ________________________________________
Equation: ________________________________

Vertex Form: ________________________________

x-intercept(s): ________________________________

y-intercept: ________________________________
\[ y = x^2 \quad 5x + 6 \]

\[ y = 3(x - 0)^2 + 3 \]

\[ y = (x+1)^2 + 4 \]

**x-intercepts:**
(-1, 0)  
(2, 0)

**x-intercepts:**
(-1, 0)  
(1, 0)

**y-intercept:**
(0, -2)

**y-intercept:**
(0, 3)
$y = 2x^2 + 2x + 4$

- $x$-intercepts: $(-1, 0)$, $(2, 0)$
- $y$-intercept: $(0, -6)$

$y = 3x^2 - 3x - 6$

- $x$-intercepts: $(-3, 0)$, $(1, 0)$
- $y$-intercept: $(0, 3)$
\[ y = 3x^2 + 3 \]

\[ y = 3(x - \frac{1}{2})^2 + \frac{27}{4} \]

- **x-intercepts:** 
  - \((-1, 0)\)
  - \((2, 0)\)

- **y-intercept:** 
  - \((0, 4)\)

\[ y = 3(x - \frac{1}{2})^2 - \frac{27}{4} \]

\[ y = \frac{3}{2}(x - \frac{3}{2})^2 - \frac{27}{4} \]

\[ y = x^2 + 2x + 3 \]

- **x-intercepts:** 
  - \((2, 0)\)
  - \((3, 0)\)

- **y-intercept:** 
  - \((0, 6)\)
Day 3
Solving Quadratic Equations by Factoring

**NYS Learning Standards:**

- **CCLS – Math: A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
- **CCLS – Math: A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x^2 - y^2)$, thus recognizing it as a difference squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$
- **CCLS – Math: A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines
- **CCLS – Math: A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include arising from linear and quadratic functions, and simple rational and exponential functions
- **CCLS – Math: A.REI.4.b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$

**Goal(s) and Objective(s):**

*Today's Goal:* (Communicated to students on the front board)
Factoring Quadratic Equations

**Learning Objectives:**

1. Students will be able to define the quadratic factor form and the zero product property.
2. Given a quadratic equation, students will be able to evaluate the equation by using the box method to determine the x-intercepts.

**Materials:**

- Smart Board
- Worksheets (see attached)
- Graphing Calculators

**Instructional Plan:**

1. **Notes:**
   
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board.

2. **Group/Partner Practice:**
   
   Students will create a partnership or group of 3 to complete the remaining group problems in the notes.

3. **Closure:**
   
   The class as a whole will gather back together to check their work. Groups will be able to present their work to the class, in order to share their knowledge of the concepts. This process allows for a deeper understanding of which students comprehend the material and which students have misconceptions.
Solving Quadratic Equations By Factoring

Let's Review:

Quadratic Standard Form: ______________________________________________________________

So, what's New??

Quadratic Factor Form: ______________________________________________________________

Zero Product Property: ____________________________________________________________________________________

How do we Factor a Quadratic Equation using the Box Method?

Steps:
1. Place all of the terms on one side of the equal side and set equal to 0.
2. Draw a two-by-two square.
3. Place the first term of the quadratic equation in the upper-left corner and the third term of the quadratic equation in the lower-right corner.
4. If $a = 1$, then list all of the factors of the third term.
   If $a > 1$, then multiply the first term and the third term together, and then list all of the factors of the new term.
5. Looking at the factors, determine which set of factors add up to equal the second term.
6. Place the selected factors into the remaining squares with the given variable.
7. Outside of each row, place the greatest common factor (GCF) of the two squares.
   Row 1 – GCF of upper-left corner and upper-right corner
   Row 2 – GCF of lower-left corner and lower-right corner
8. Outside of each column, place the greatest common factor (GCF) of the two squares.
   Column 1 – GCF of upper-left corner and the lower-left corner
   Column 2 – GCF of upper-right corner and lower-right corner
9. Rewrite the terms on the side and top to create the factors to the quadratic equation.
10. Use the zero product property to find the x-intercepts.
Class Example 1:

\[0 = x^2 + 10x + 21\]

Class Example 2:

\[x^2 = 9\]

Class Example 3:

\[3t^2 - 13t = 4\]
Group Example 1:

\[ 0 = x^2 + 8x + 7 \]

Group Example 2:

\[ m^2 + m = 90 \]

Group Example 3:

\[ 2a^2 = 7a \]
Day 4
Solving Quadratic Equations by Factoring - Activity

NYS Learning Standards:

- **CCLS – Math: A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
- **CCLS – Math: A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see \( x^2 - y^2 \) as \((x^2) - (y^2)\), thus recognizing it as a difference squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\)
- **CCLS – Math: A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines
- **CCLS – Math: A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include arising from linear and quadratic functions, and simple rational and exponential functions
- **CCLS – Math: A.REI.4.b** Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \)

Goal(s) and Objective(s):

Today's Goal: (Communicated to students on the front board)
Factoring Quadratic Equations

Learning Objectives:
1. Students will be able to evaluate quadratic equations by using the box method to determine the x-intercepts.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators
- Lined Paper
- Scissors
- Tape

Instructional Plan:
1. **Individual Work:**
   Students will start working on the review problems on their own, in order to gain an understanding of what each student knows.
2. **Partner Work:**
   Each student will pick a partner and finish/check the review problems together. After students have completed the problems, go over and provided students with the correct answers.
3. **Group Work:**
   Students will be placed in groups of 2 – 3 and complete the matching activity. Each student will use a separate sheet of paper to show all of his or her work.
4. **Closure:**
Before the end of the period, students will hand in their separate sheet of paper with all of their work for a grade. Having student’s hand in their work shows which students understood the learning objectives for the lesson.
Solving Quadratic Equations By Factoring

Review Example 1:

\[ k^2 - 13k = -40 \]

Review Example 2:

\[ y = x^2 - 36 \]
**Matching Factors to Quadratic Equations**

**Directions:** Cut out all of the factors and match to the corresponding quadratic equation. Then determine the x values for each equation. Make sure you show all your work on a separate sheet of paper.

<table>
<thead>
<tr>
<th>$y = 3x^2 + 10x + 7$</th>
<th>$y = x^2 + 7x + 8$</th>
<th>$y = 3x^2 + 6x + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$y = 2x^2 + 9x + 10$</th>
<th>$y = 7x^2 + 2x + 5$</th>
<th>$y = x^2 + 2x + 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$y = 6x^2 + 12x + 18$</th>
<th>$y = x^2 + 12x + 32$</th>
<th>$y = x^2 + 81$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$y = x^2 + 11x + 24$</th>
<th>$y = x^2 + 3x + 2$</th>
<th>$y = x^2 + 8x + 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$y = x^2 + 4$</th>
<th>$y = 3x^2 + 10x + 3$</th>
<th>$y = x^2 + 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Factor Cut Outs

\[(x - 8) \quad (2x - 5) \quad (x - 5) \quad (x + 8)\]

\[(x + 1) \quad (3x + 3) \quad (x + 1) \quad (7x - 5)\]

\[(x + 7) \quad (x + 1) \quad (3x + 7) \quad (x + 4)\]

\[(6x - 18) \quad (x - 2) \quad (x - 9) \quad (x - 3)\]

\[(x - 5) \quad (x + 10) \quad (x + 2) \quad (x + 1)\]

\[(3x - 1) \quad (x + 2) \quad (x - 3) \quad (x - 10)\]

\[(x - 2) \quad (x + 1) \quad (x + 9) \quad (x - 8)\]

\[(x + 1) \quad (x - 3)\]
Day 5
Solving Quadratics by Completing the Square – Box Method

NYS Learning Standards:
- CCLS – Math: A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients
- CCLS – Math: A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 +r)n as the product of P and a factor not depending on P
- CCLS – Math: A.SSE.3.a Factor a quadratic expression to reveal the zeros of the function it defines
- CCLS – Math: A.SSE.3.b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines
- CCLS – Math: A.REI.4.a Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form
- CCLS – Math: A.REI.4.b Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b
- CCLS – Math: A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include arising from linear and quadratic functions, and simple rational and exponential functions

Goal(s) and Objective(s):

Today’s Goal: (Communicated to students on the front board)
Solving Quadratic Equations By Completing The Square

Learning Objectives:
1. Students will conceptually understand what completing the square means.
2. Given a quadratic equation, students will be able to evaluate an equation by using the box method, in order to complete the square to find the x-intercepts.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators
- Scissors (optional)
- Colored Pencils

Instructional Plan:
1. Notes:
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board. Students will also use the squares and rectangles as a representation tool to conceptual understand how to complete the square. The students will receive the squares and rectangles already cut up or on sheets of paper for the students to cut out.
2. Group Work:
Students will be placed in groups of 3 and work on the group examples in their notes. As the students work together they will use the squares and rectangles to complete the square, just as they did in the notes.

3. **Closure:**
Before the end of the class period, students will gather together to go over the examples. Groups can go up to the board and present their work by explaining their process and knowledge. Having students present provides a clearer idea of which students understood the goal and objectives for the day.
Solving Quadratic Equations By Completing The Square

Completing the Square?!? What does that even mean?

Let’s look at the equation $x^2 + 4x + 4 = 0$. If we were to factor this, using the box method, what would we get?

So, what does this really look like?

What about $x^2 + 8x + 16 = 0$?

What does this look like?
When we “completed the square,” we were able to determine the factors for each of the equations by looking at the number of rectangles and small squares. By using our squares and rectangles, as well as the box method, we can determine factors that are not easy to find.

To complete the square, we are going to use the form below and solve for the square value.

\[ x^2 + bx + c = c + \square \]

Now, how exactly do we complete the square?

Using the quadratic equation form, we will:

1. Divide all terms by \( a \) (the coefficient of \( x^2 \)) to get a coefficient of 1 for \( x^2 \)
2. Move the \( c \) term to the right side of the equation
3. Place the first term, \( x^2 \) into the upper left corner of the box
4. Determine two factors that add up to equal \( b \) and place one of the numbers in the top right corner and the other in the bottom left corner
5. Place each number and variable on the outside of the box
6. Find the bottom right corner by “completing the square” by multiplying the outside numbers
7. Place the bottom left corner value into the “squares” in the equation
8. Simplify the left hand and right hand side of the equation
9. Take the square root on both sides of the equation
   Remember: Place a plus and minus sign in front of the square root term
10. Solve for \( x \)

Class Example 1:

\[ x^2 + 6x + 8 = 0 \]

Picture:  
Box Method:  
Solution:
Class Example 2:

\[ x^2 - 12x + 23 = 0 \]

Picture:                                 Box Method:                                Solution:

Group Example 1:

\[ x^2 + 2x - 3 = 0 \]

Picture:                                 Box Method:                                Solution:
Group Example 2:

\[ x^2 + 6x + 3 = 0 \]

Picture:  

\[ \begin{array}{ccc}
\hline
\hline
& & \\
\hline
\hline
\end{array} \]

Box Method:  

Solution:

Group Example 3:

\[ x^2 - 4x + 8 = 0 \]

Picture:  

\[ \begin{array}{ccc}
\hline
\hline
& & \\
\hline
\hline
\end{array} \]

Box Method:  

Solution:
Completing the Square Cut Outs

$x^2$

$x^2$

$x^2$

$x^2$

$x^2$

$x^2$

$x^2$

$x^2$

$x^2$
Day 6
Solving Quadratics by Completing the Square – “It” Method

NYS Learning Standards:

- **CCLS – Math: A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients
- **CCLS – Math: A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 +r)n as the product of P and a factor not depending on P
- **CCLS – Math: A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines
- **CCLS – Math: A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines
- **CCLS – Math: A.REI.4.a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form
- **CCLS – Math: A.REI.4.b** Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b
- **CCLS – Math: A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include arising from linear and quadratic functions, and simple rational and exponential functions

Goal(s) and Objective(s):

**Today's Goal: (Communicated to students on the front board)**
Solving Quadratic Equations By Completing The Square

**Learning Objectives:**
1. Students will conceptually understand what completing the square means.
2. Given a quadratic equation, students will be able to evaluate an equation by using the “It” method, in order to complete the square to find the x-intercepts.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators

Instructional Plan:
1. **Individual Work:**
   Students will start working on the review problem on their own, in order to gain an understanding of what each student knows.
2. **Partner Work:**
   Each student will pick a partner and finish/check the review problem together. After students have completed the problem, go over and provided students with the correct answer.
3. **Notes:**
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board.
4. **Partner Work:**
Students will choose their partner and work together on completing the group examples in their notes.

5. **Closure:**
Students will be able to present their work to the class before the period ends. Allowing students to present their work provides an insight to which students may have misconceptions of the material from the day.
Solving Quadratic Equations By Completing The Square

Recall: What is the completing square form?

______________________________________________________________________________________________

Review Question:

Complete the square to find the \( x \) value(s) for \( 2x^2 - 8x + 6 = 0 \)

Box Method: Solution:

Based on what we just did and looking at the completing square form, what can we conclude about the square value?

The “square” value = ________________________________

**The “It” Method**

“It” represents _________________

Steps for the “It” Method:

1. _________________ It
2. _________________ It
3. _________________ It

Remember: When we _______________ It, we have to do it to ____________ sides!
Class Example 1:
\[ x^2 + 2x - 4 = 0 \]

Class Example 2:
\[ 3x^2 - 12x - 6 = 0 \]

Class Example 3:
\[ x^2 - 4x + 10 = 0 \]
Group Example 1:

\[ x^2 + 8x - 1 = 0 \]

Group Example 2:

\[ x^2 - 6x - 3 = 0 \]

Group Example 3:

\[ x^2 - 6x + 34 = 0 \]
Day 7
Deriving the Quadratic Formula and Determining Discriminants and Axis of Symmetry

NYS Learning Standards:
- CCLS – Math: A.REI.4 Solve quadratic equations in one variable
- CCLS – Math: A.REI.4.a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form
- CCLS – Math: A.REI.4.b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$

Goal(s) and Objective(s):

Today’s Goal: (Communicated to students on front board)
Deriving the Quadratic Formula and Applying the Discriminant and Axis of Symmetry

Learning Objectives:
1. Students will be able to derive the quadratic formula from the quadratic equation standard form.
2. Students will be able to define and develop the formulas for the discriminant and axis of symmetry.
3. Given a quadratic equation, students will be able to determine the number and nature of the roots, as well as the axis of symmetry to the equation.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators
- Scissors (optional)
- Highlighters

Instructional Plan:
1. Notes:
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board. Students will use the cut out equations in order to derive the quadratic formula. Each student will receive the rectangles containing the equations already cut up or on sheets of paper for the students to cut out.
2. Group Practice:
   Students will select a partner, where they will complete the remaining problems in their notes.
3. Closure:
   Before the end of the period, gather all students together in order to go over the group problems. This is a way to see if students met the goal and objectives for the day, as well as wrapping up the lesson.
Solving Quadratic Equations By The Quadratic Formula

What is the standard form for a quadratic equation?

_______________________________________________________________________________

Now using the standard form let's solve for x.

Quadratic formula: ________________________________
The quadratic formula allows us to find x-intercepts of quadratic equations that are not easily factorable. 
Before we dig in to using the quadratic formula, let’s break the formula down to understand how powerful the quadratic formula really is!

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

So, what is that thing called a discriminant?

A discriminant tells us _____________________________________________________________________________________
________________________________________________________________________________________________________________

If \(b^2 - 4ac \leq 0\), then we have ______________________________
If \(b^2 - 4ac > 0\), then we have ______________________________
If \(b^2 - 4ac < 0\), then we have ______________________________

What about the axis of symmetry?

The axis of symmetry tells us _____________________________________________________________________
________________________________________________________________________________________________________________

Now that we have a better understanding of the quadratic formula, let’s practice by finding the discriminant, the number of roots, and the axis of symmetry. (Use the graphing calculator to check your answers.)

Class Example 1:

\[ r^2 + 5r + 2 = 0 \]
Class Example 2:

\[ x^2 - 9 = 6x \]

Class Example 3:

\[ 9n^2 - 3n - 8 = -10 \]

Group Example 1:

\[ 9m^2 + 6m + 6 = 5 \]

Group Example 2:

\[ 7n^2 + 16n = 8n \]

Group Example 3:

\[ 4r^2 - 4r = 6 \]
Standard Form to Quadratic Formula

\[ (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2} \]

\[ x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = c + \frac{b^2}{4a^2} \]

\[ x^2 + \frac{b}{a}x = c \]

\[ x = \frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \]

\[ (x + \frac{b}{2a})^2 = \frac{c + \frac{b^2}{4a^2}}{a} \]

\[ (x + \frac{b}{2a})^2 = \frac{4ac + b^2}{4a^2} \]

\[ x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \]
Solving Quadratics by Using the Quadratic Formula

NYS Learning Standards:
- **CCLS – Math: A.REI.4** Solve quadratic equations in one variable
- **CCLS – Math: A.REI.4.a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form
- **CCLS – Math: A.REI.4.b** Solve quadratic equations by inspection (e.g., for \(x^2 = 49\), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \(a \pm bi\) for real numbers \(a\) and \(b\)

Goal(s) and Objective(s):

Today’s Goal: (Communicated to students on the front board)
Solving Quadratic Equations By Using the Quadratic Formula

Learning Objectives:
1. Students will be able to evaluate quadratic equations by using the quadratic formula to determine the x-intercepts.

Materials:
- Smart Board
- Worksheets (see attached)
- Graphing Calculators
- Lined Paper
- Scissors
- Tape

Instructional Plan:
1. Notes:
   Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board.
2. Group Practice:
   Students will be placed in groups, where they will complete the matching activity. Each student will be required to show all his or her work on a separate sheet of paper.
3. Closure:
   At the end of the period, students will hand in their separate sheet of paper with all of their work for a class grade. This process will provide an insight to which students comprehend the material and which students don’t.
Solving Quadratic Equations By The Quadratic Formula

Recall from yesterday, what is the quadratic formula?

__________________________________________________________________________________

Using the quadratic formula, lets find the $x$ value(s) to the following equations.

Example 1:

$$x^2 - 5x - 14 = 0$$

Example 2:

$$x^2 + 4x + 5 = 0$$
Matching Quadratic Equation To Solutions

Directions: Cut out each equation square and solve for x using the quadratic formula. Be sure to show all your work on a separate sheet of paper. Then, record the solutions for the given equation on each square. Finally, match each equation square to the corresponding box in the table.

<table>
<thead>
<tr>
<th>One Solution</th>
<th>Two Rational Solutions</th>
<th>Two Complex Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


\[ 3x^2 + 5x + 1 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ x^2 + 5x + 5 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 5x^2 + 2x + 1 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 5x^2 + 50x + 125 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 4x^2 + 11x + 20 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ x^2 + 5x + 24 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ x^2 + 3x + 10 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 4x^2 - 12x + 9 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 3x^2 + 6x + 3 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]

\[ 4x^2 + 12x + 9 = 0 \]
\[ x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}} \]
Day 9
Review Activity – Quadratic Equation Graphic Organizer

NYS Learning Standards:
• Through this learning activity, all the NYS standards will be incorporated.

Goal(s) and Objective(s):

Today’s Goal: (Communicated to students on front board)
Create a Quadratic Equation Organizer

Learning Objectives:
1. Students will be able to define and use each method, graphing, factoring, completing the square, and the quadratic formula, in order to solve a quadratic equation.

Materials:
• Smart Board
• Worksheets (see attached)
• Completed Graphic Organizer (teacher example)
• Large Sheet of White Paper (17 x 22)
• Scissors
• Tape
• Highlighters or Colored Pencils

Instructional Plan:
1. Notes:
Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Smart Board. By the end of the class period, students will have their own graphic organizer composed of definitions, steps, and examples. Students can use the graphic organizer as reference tool as they complete the Relay Activity, the next day, and to study from for their assessment.
Graphing

Steps:
1. Create table for $x$ and $y$ values
2. Plot and connect points
3. Label line
4. Find:
   - vertex $(h, k)$
   - vertex form $y = a(x - h)^2 + k$
   - $y$-intercept ($x = 0$)
   - $x$-intercepts ($y = 0$)
5. State min. or max.
6. State range and domain

Factoring

$ax^2 + bx + c = 0$

Box Method:

<table>
<thead>
<tr>
<th>$ax^2$</th>
<th>$\frac{1}{2}bx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}bx$</td>
<td>$c$</td>
</tr>
</tbody>
</table>
Completing the Square

\[ x^2 + bx + \square = c + \square \]

Two Methods:
1. **Box Method**

\[
\begin{array}{|c|c|}
\hline
x^2 & \frac{1}{2}bx \\
\hline
\frac{1}{2}bx & \square \\
\hline
\end{array}
\]

2. **“It” Method** ("It" = b)
   a. Half “It”
   b. Square “It”
   c. Add “It”

**Quadratic Formula:**

\[ y = ax^2 + bx + c \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**Discriminant:**

If \( b^2 - 4ac > 0 \) → 2 real roots
If \( b^2 - 4ac < 0 \) → 2 complex roots
If \( b^2 - 4ac = 0 \) → 1 real root
Example:

\[ y = x^2 - x - 2 \]

- vertex: ______________________
- vertex form: _____________________________
- y-intercept: _____________________________
- x-intercepts: _____________________________
- min. or max.: _____________________________
- domain: __________________________________
- range: __________________________________

Example:

\[ y = x^2 + x - 12 \]
Example:

\[ y = x^2 + 2x \quad 35 \]

Example:

\[ y = 2x^2 \quad 5x \quad 3 \]
Day 10
Review Activity – Relay Review

**NYS Learning Standards:**
- Through this learning activity, all the NYS standards will be incorporated.

**Goal(s) and Objective(s):**

**Today’s Goal:** (Communicated to students on front board)
Review Quadratic Equations

**Learning Objectives:**
1. Students will be able to apply all of the methods, graphing, factoring, completing the square, and the quadratic formula, to solve quadratic equations.

**Materials:**
- Smart Board
- Worksheets (see attached)
- Questions Cut Out
- Graphing Calculator
- Graphing Paper
- Lined Paper

**Instructional Plan:**

1. **Group Work:**
   Students will be placed in groups of 3-4 and work on completing the Relay Review activity. One person from each group will select a question from the board and work as a group to find the answer. After every student has their own work on their paper, the student who selected the problem will show the teacher to check for correctness. If correct, another student will select a question and repeat the same process. If the question is incorrect, the student will go back to their group and work through the problem again until correct. Each group will answer as many questions as they can before the class period is over.

2. **Closure:**
   Before the end of the class period, students will hand in their Relay Review packet for a class grade. Students will also have the opportunity to ask questions and review the all of the material before the assessment.
Relay Review

Directions:
1. One person in your group gets a question from the board.
2. Answer the question as a group (everyone must do their own work on their own worksheet).
3. When completed, the student who choose the question must show the teacher to check for correctness.
4. If correct, a different student goes to the board and picks up another question.
5. If incorrect, the student must return back to their group and redo their question until correct.
6. Continue answering as many questions as possible and then hand in when the class period is over.

Use the following boxes to complete your problems. MAKE SURE TO PUT THE CORRECT PROBLEM IN THEIR BOX.

1. 
2. 
3. 
4.
1. Complete the Square:

\[ x^2 - 12x + 26 = 0 \]

2. Factor:

\[ y = x^2 + 7x + 8 \]

3. Quadratic Formula:

\[ 3x^2 - 16x + 5 = 0 \]

4. State if the equation has 1 real, 2 real, or 2 complex roots by using the discriminant:

\[ y = x^2 + x + 30 \]

5. Quadratic Formula:

\[ y = 2x^2 + 3x + 5 \]

6. Complete the Square:

\[ x^2 - 2x + 15 = 0 \]
7. Determine if the equation has 1 real, 2 real, or 2 complex roots by using the discriminant:

\[ y = x^2 \quad 2x + 1 \]

8. Factor:

\[ y = 5x^2 \quad 30x + 40 \]

9. Quadratic Formula:

\[ y = 9x^2 + 4x \quad 16 \]

10. Find the x- and y-intercepts by graphing:

\[ y = x^2 \quad 5x \quad 6 \]

11. Factor:

\[ 2x^2 + 6x \quad 108 = 0 \]

12. Write in vertex form and state the vertex by graphing:

\[ y = 3x^2 + 12x + 11 \]
13. Complete the Square:

\[ y = 10x^2 + 40x + 20 \]

14. Quadratic Formula:

\[ 4x^2 + 8x + 3 = 0 \]

15. Complete the Square:

\[ y = x^2 + 4x - 59 \]

16. Factor:

\[ 2x^2 + 17x + 21 = 0 \]
Day 11 – 12
Assessment – Quadratic Equation Test

**NYS Learning Standards:**
- This assessment aligns with all of the NYS standards in this curriculum.

**Goal(s) and Objective(s):**

Today’s Goal: (Communicated to students on front board)
Show me what you know about Quadratic Equations

**Materials:**
- Worksheets (see attached)
- Graphing Calculator
- Graphing Paper
- Lined Paper

**Instructional Plan:**

1. **Preparation for Assessment:**
   Have students write on a piece of paper something that is on their mind or bothering them. After every student has something written, have the students crinkle up the paper and throw it underneath their seat. This process will help students clear their head and get any type of distraction off their mind before taking the assessment.

2. **Module Assessment (Option 1):**
   Students will be given the entire class period to complete the assessment by their self.

3. **Module Assessment (Option 2):**
   Students will be given half of a class period or a whole class period to complete the assessment by their self. The second half of the class period or the next day, students will receive the same assessment and complete the problems with a group of 3-4 students. For this assessment, students score will be based, 75% of the students’ individual assessment and 25% on the group assessment.

**Note:** Teachers may choose which type of assessment to assign students based on the classroom environment and students math anxiety levels. If students are presented with the second option for the assessment, the teacher may choose to have students complete the assessment in 1 day or in 2 days.
Quadratic Equation Test

1. Using the following equation,
   \[ y = x^2 - 3x + 4 \]

   a) Graph and create a table for the equation.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a) State the equation vertex.

   b) Rewrite the equation in vertex form.

   c) State the x- and y-intercepts.
      \[ x\text{-intercepts: } \] _________________________
      \[ y\text{-intercepts: } \] _________________________

   d) Determine if the equation is a maximum or minimum.

   e) State the range and domain of the equation.
      \[ \text{range: } \] _________________________
      \[ \text{domain: } \] _________________________
2. Determine the $x$ value(s) for the equations by factoring:

   a) $x^2 + 3x - 10 = 0$

   b) $2x^2 + 9x - 5 = 0$

3. Determine the $x$ value(s) for the equations by completing the square:

   a) $x^2 - 10x + 20 = 0$
4. Determine the x value(s) for the equations by using the quadratic formula:

a) \(3x^2 + 5x - 8 = 0\)

b) \(x^2 - 5x + 5 = 0\)
5. Determine the \( x \) value(s) for the equation by graphing, factoring, completing the square or using the quadratic formula. State the method you used.

\[ y = x^2 + 12x + 36 \]

Method: ______________________________________________________

6. Using the discriminant, state if the equation has 1 real root, 2 real roots, or 2 complex roots:

a) \( y = 9x^2 - 12x + 4 \)

b) \( y = x^2 - 2x + 2 \)
Chapter 4: Validity

The validity of this curriculum project was assessed by a tenured, Algebra I mathematics teacher from an upstate NY public high school. The cooperating teacher was asked to critique the unit plan and answer the questionnaire based on his experience:

1. Based on your understanding of the Common Core Learning Standards, is this unit plan aligned to those standards?
2. Do you feel that this unit plan can help decrease math anxiety in Algebra 1 students?
3. What are the strengths of this unit plan?
4. What changes, if any, would you make to this unit plan?

After reviewing the curriculum, the cooperating teaching provided a detailed critique regarding the strengths and weaknesses of the unit plan. The cooperating teacher noted that there is a ton of detail in the curriculum and he enjoyed the puzzle-piece theme. Within the cooperating teacher’s critique, a question arised regarding student work: “Do you have a place for student groups to write up a summary at the end of the lesson?” The cooperating teacher noted, “A lot of pedagogical literature talks about the importance of students summarizing at the end of the lesson.” According to the cooperating teacher, the use of multiple representations are used quite often throughout the lessons. The cooperating teacher mentioned the idea of incorporating more multiple representations as students are introduced to new concepts. For example, the cooperating teacher noted, “you could go back to the graphs of the quadratics the students just factored in order to demonstrate the connections between the function and its standard form and factored for, how the factors point to the zeros of the graph, etc.”
In order to provide the author with more feedback, the cooperating teacher also completed a questionnaire. The feedback from the cooperating teacher is as follows:

1. **Based on your understanding of the Common Core Learning Standards, is this unit plan aligned to those standards?**

   The unit plan is clearly aligned to the CCLS learning standards. The pertinent learning standards are clearly indicated in the unit, as a whole, and for each lesson. Both the conceptual and procedural work indicated in the lessons represents work required for students to address the goals of the standards.

2. **Do you feel that this unit plan can help decrease math anxiety in Algebra I students?**

   Given the wide range of activities that allow access to the content from different angles, and for different learning styles, as well as multiple opportunities for students to collaborate in small groups, it would appear the unit plan provides opportunities to defuse and/or reduce student math anxiety.

3. **What are the strengths of this unit plan?**

   There are a number of strengths that characterize this learning plan: First, it is thorough in that addresses CCLS learning standards from both conceptual and procedural points of view. Second, it provides students with multiple opportunities to explore the pertinent mathematical content using different learning styles, in small and large group settings. Third, the unit plan provides lessons in which students present their critical thinking to the rest of the class for comment and critique. Finally, the unit provides the instructor and students multiple opportunities for assessing learning in formal and informal ways.

4. **What changes, if any, would you make to this unit plan?**
As I suggested in my first critique, the only weakness I found in my initial review related to the need for a re-test cycle should students demonstrate weaknesses in learning in any particular point – concept or procedure. Should that occur, the unit should provide the teacher with suggestions regarding re-teaching and re-assessment strategies.
Chapter 5: Summary

This curriculum project was designed to support teachers to teach quadratic equations to Algebra I students, who suffer from math anxiety. Through this unit plan, teachers can implement multiple tools, strategies, and methods to support student learning. Within the unit plan, teachers will be able to apply hands-on activities and multiple representations to engage students, develop conceptual understanding, procedural fluency, and mathematical reasoning, as well as introducing methods to help decrease math anxiety in students.

The curriculum was reviewed and critiqued by a tenured, Algebra I mathematic teacher. Overall, the feedback given was positive and the cooperative teacher provided a few changes to improve the curriculum project. The cooperative teacher noted that the curriculum aligned to the CCSS and provides support to teachers to decrease math anxiety in students. In addition, the curriculum incorporates multiple hands-on activities that engage students in the lessons and provides opportunities for student-centered learning. The cooperative teacher also provided many critiques that could help strengthen the curriculum, which can be applied to future research.

Future research is still needed to be done to support teachers in decreasing math anxiety in students. The cooperative teacher recommended applying applications, multiple representations, end of lesson summaries, and assessment data. The author's next steps will be to create a two-day lesson revolved around applications using quadratic equations. Introducing real-world problems would provide educators examples that students could relate to. These examples give teachers more opportunities to expose students to real life math problems and overall help decrease math anxiety. The author will also create more
representations, such as graphic organizers to hang in the classroom as references, applying more graphs, using the Frayer diagram, etc. Applying multiple representations to the curriculum provides teachers multiple methods and strategies to reinforce quadratic equations and help students conceptual understand each concept. By provided more representations, teachers can increase a students understanding, in order to decrease math anxiety. After each lesson, the author will also develop end of the lesson summaries. The cooperating teacher noted the importance of summarizing a lesson before the class is over. With that being said, the author will incorporate lesson summaries at the end of each lesson. This will allow teachers to gather data in regards to student understanding of concepts and skills, as well as noting which students are developing an increase or decrease in math anxiety. Lastly, the author will implement the DDI (data-driven instruction) model in order to show that teachers are responsive to the potential that particular students may not understand the concepts or skills. With further research, teachers can use student results from review, pre-assessments, post-assessments, and re-assessments to check for student understanding and the level of math anxiety a particular student may have.

Finally, the author plans to implement the curriculum into her own classroom. By teaching the curriculum, the author will have a better understanding of how the unit plan assist students with math anxiety. Through the teaching of the curriculum, the author can apply new research, resources, and methods/strategies to decrease math anxiety based on student performance. Implementing the suggestions from the cooperating teacher, and the author’s own ideas and knowledge will expose teachers to further resources to improve
and assist students in decrease math anxiety and increase students comprehension of the concepts.
References


in Mathematics, 62, 81-99.


