Alternative Assessments: Moving Learning Forward in a Geometry Classroom

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Alternative Assessments:
Moving Learning Forward in a Geometry Classroom
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Abstract

This project was created by a teacher for teachers, providing alternative assessments for a geometry classroom. The assessments emphasize collaboration and group work as part of the assessment task. Topics include geometric proofs, geometric constructions, Pythagorean Theorem, trigonometry, polygons and polyhedra, volume and surface area. Recommendations for how and when to use each alternative assessment have been provided by the author. Additionally scoring rubrics have been provided. The alternative assessments are aligned to the Common Core State Standards, specifically geometry standards throughout secondary grade levels.
Introduction

Assessment has been identified as the single most influential factor in student learning and is a complex activity (Williams, 2014, p. 566). Figure 1 presents a cartoon commenting on an error made in educational assessment; educators design assessments that are believed to be fair and universal however, assessments are not fair and universal when the type of learners in one classroom vary so drastically. Thus arises the question, how can teachers assess in a way that allows all students to show their mastery of skills? A shift in pedagogy to a more constructivist classroom, one that promotes collaboration and group work, requires assessment that is for learning instead of simply an assessment of learning. Methods of assessment should reflect the diversity in methods of instruction as well as the diversity in the learners (Williams, 2014). The idea of alternative assessments is in sharp contrast to the current traditional approaches. Alternative assessment is closely linked with the notion of authentic assessment and can include various ways for a teacher to gather feedback involving students more actively in collaborating and assessing.

Purpose

The purpose of this curriculum project is to provide alternative assessments and group work opportunities that allow for all levels and types of learners to show mastery in Geometry content. This curriculum project will give several examples of alternative assessments aligned to Common Core State Standards for Geometry. These ideas are presented by a teacher for a teacher and can be used to incorporate group work/activities within assessments. At this time
with lack of textbooks aligned to the common core, resources such as these are in even more of a critical need. The assessments presented in this project may include collaborative work, group projects, and group summative assessments as well as grading tools. This project will be helpful in moving the work of secondary mathematics classrooms forward by providing new and innovative ideas that incorporate as well as move beyond the material that already exists.

**Literature Review**

**Zone of Proximal Development and Small Group Problem Solving**

The study of metacognition has been widely acknowledged as key to understanding mathematical problem solving. However Goos, Galbraith, and Renshaw (2002) lamented that there is still a need for an adequate theoretical model to further explain the “mechanisms of self-monitoring and self-regulation” and other aspects of mathematical thinking (p. 193). In particular, the potential for small group work to develop students’ mathematical thinking and problem solving skills has remained largely unexplored. The teacher’s role in orchestrating student discussions and interactions is also lacking in research. The implementation of small group work, discussion based classes, and other characteristics of this framework for learning would require shifts in teaching methods and classroom environments. In order to justify these changes, Goos et al. (2002) decided to pursue further research specifically addressing the role of the students as they worked together and the role of the teacher in creating a classroom culture of inquiry.

The framework for the study was Vygotsky’s notion of the zone of proximal development (ZPD). Part of Vygotsky’s research included noting that when children played together they acted above their normal level of development (Goos et al., p. 195, 2002). Traditionally ZPD has been defined as “the distance between the actual developmental level as
determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). This would imply that there is potential for learning to occur in peer groups where students must collaborate and combine ideas in order to make progress. In other words, students possess some of the knowledge and skill but require their peers’ contribution in order to move forward with solving. Goos et al. (2002) decided to use the phrase “collaborative ZPD” to emphasize the distinction between expert-novice and equal status interactions. It is important to note that Goos et al. (2002) use the term “collaborative” to refer to when “students with similar levels of competence share their ideas in order to solve jointly a challenging problem” (p. 196). Thus it does not include peer tutoring or tasks that become divided among peers to be solved individually. Goos et al. (2002) used the term “mutuality” to make this distinction in peer collaboration.

A major theme that emerged from the research of Goos et al. (2002) was that challenge is a defining piece of the ZPD. Teaching and learning in the ZPD requires moving students past their current capabilities towards new forms of reasoning and problem solving. Within peer interactions the back and forth of transactive challenges and metacognitive decisions was significant in creating zones of proximal development that led to problem solving outcomes. Challenges that required clarification or justification stirred discussion and monitoring that led to errors being noticed or led to strategies being affirmed (Goos et al., 2002). Danish, Peppler, Phelps, and Washington (2011) argued that rather than asking students to engage with ideas they are
already familiar with, within the ZPD students are challenged to engage in new, more advanced ways of thinking, with support. As seen in Figure 2, Vygotsky believed that when a student is in the ZPD during a particular task, providing the appropriate assistance will give the student the boost they need in order to be successful in achieving the task (McLeod, 2010). If successful, learning in the ZPD “at the edge of one’s competence, can then result in the appropriation on the part of the child of new psychological tools” (Danish et al., 2011, p. 456). Conversely in situations of metacognitive failures, students failed to engage with each other’s ideas and thus were unable to create a ZPD due to the lack of challenges.

In conclusion, within the collaborative ZPDs the process of “articulating and justifying strategies for attacking a problem represent the social means through which students appropriated mathematical reasoning tools” (Goos et al., p. 218, 2002). There is significance in transactive discussion in contributing to productive metacognitive decisions by making a student’s thinking public and open to criticism. Up until Goos et al. (2002) study, research in the area of metacognition had treated monitoring and regulation as individual and internalized. Now, the view of metacognition has been extended to include collaborative conversations between peers of comparable skill level. Further research is needed on the role of the teacher in this collaboration process as well as other practical questions arising from this study.

**Justification for Collaborative Learning**

Author Jo Boaler explained her perspective on the link between knowledge, practice and identity in learners through three different studies. In the first study, Boaler (2002) studied students learning mathematics in two secondary schools over the course of three years (p. 42). Amber Hill used a traditional approach of teacher-led demonstration while Phoenix Park used an approach where students worked on teacher-designed open-ended projects. One finding of this
study was that the teaching practice led not only to the development of knowledge but the form of knowledge produced. Boaler (2002) recorded that Amber Hill students performed well in familiar situations, similar to the textbook but struggled with “open, applied, or discussion-based situations” (p. 43). On the other hand students from Phoenix Park developed flexible thinking that was useful in different situations including when asked questions unlike anything they had previously seen. Boaler (2002) argues that a main conclusion of this study was that “knowledge and practices are intricately related and that studies of learning need to go beyond knowledge to consider the practices in which students engage and in which they need to engage in the future” (p. 43). But Boaler’s representation of learning did not end with the relationship of knowledge and practice, there was a third element that she called identity.

In the second study Boaler discussed the contrast between traditional advanced placement calculus classes and group work, discussion-based calculus classes. The students in the more traditional classes had an identity of “received knowers” where they solely received knowledge from their teacher (Boaler, 2002, p. 44). This created a conflict of identity in learners who felt frustrated by the lack of opportunity for expression and interpretation. However, in the other calculus classes where students “regarded their role to be learning and understanding mathematical relationships” there was not a conflict of identity. Students who wanted to continue to do more than just receive knowledge were able to form plans for themselves to continue being mathematical learners (Boaler, 2002, p. 44).

As part of the third study, Boaler (2002) wrote that a common misconception of when students are given more agency and authority is that the students are not learning enough and are left to wander in unproductive directions (p. 45). Boaler (2002) discussed the notion of a “dance of agency” in which there are times for creating and then there are times for discipline and
standard procedures (p. 46). This “dance” is what allows for the development of knowledge, problem solving skills, and the ability to have a productive relationship with mathematics. Students who are able to investigate and question in a teacher-guided setting, are able to transfer mathematics to new situations and persevere to solve the problem.

While traditional teaching style allows for positive results on textbook questions, the goal should not merely be for students to pass standardized tests. In addition, students in this traditional setting are more likely to become docile learners who give up easily when faced with difficult problems. Boaler’s work closely aligns with the mathematical practices outlined by Common Core and the main goal of helping students be college and career ready. Through collaboration with their peers and classmates, students are able to not only acquire the necessary knowledge to perform on tests but also develop solving strategies and perseverance. These problem solving skills transfer not only to new mathematical questions but also to other situations outside of the classroom. Students are active participants in their learning via teacher-designed investigation and discussion of concepts and are prepared for new and “real world” situations when they leave.

Making Sense

Mathematics as a subject is often met with disdain by individuals. Donovan and Bransford (2005) argued that the negative feelings people have towards mathematics as a subject is due to the fact that mathematics is rarely taught in a way that helps people make sense and meaning out of the content. Donovan and Bransford (2005) highlight three key principles that should be used to prevent this consequence. Instead of “connecting with, building on, and refining the mathematical understandings, intuitions, and resourcefulness” that students bring with them to the classroom (Principle 1), instruction often overrides students’ reasoning and
replaces it with procedures and algorithms that cause disconnect from meaning making (Donovan & Bransford, 2005, p. 217). Instead of organizing the skills required to do mathematics fluently around a set of core mathematical ideas (Principle 2), the skills and competencies often become the central focus. Due to the fact that procedural knowledge often becomes the focus disconnected from meaning making, students do not use metacognitive strategies when they engage in solving problems (Principle 3). If instruction was delivered with these principles in mind, then perhaps there would be a different attitude towards mathematics.

In order to apply Principle 1, Donovan and Bransford (2005) claim teachers must engage students’ preconceptions. Both children and adults engage in informal mathematical problem solving using untrained strategies in their day-to-day lives. However when asked to complete the same types of problems in a formal setting, the same child or adult is unable to solve the problem. For example, California housewives were able to solve mathematical problems when comparison shopping but they could not solve problems presented abstractly in a classroom that required the same mathematical skills (Donovan & Bransford, 2005). These informal strategies and mathematical reasoning skills can serve as a foundation for learning more abstract mathematics in the classroom. Donovan and Bransford (2005) emphasized that this link from informal to formal is not automatic.

In addition to engaging students by building on existing knowledge, teachers must address and engage students’ preconceptions about mathematics. Unfortunately, students often have several counterproductive, early-formed, preconceptions that interfere with their learning. Some of these preconceptions might include the conclusion that mathematics is just “not for them” or that mathematics is all about following steps to get the right answer or that some people just can’t “do math” and some people can (Donovan & Bransford, 2005). Teachers must break
down these preconceptions and replace them with mindsets and problem solving skills that lead to sense making. This also requires the student to take on a significant role in their learning. As Donovan and Bransford (2005) pointed out, in our society is has become socially acceptable and often desirable not to put forth effort in learning mathematics. In other countries, success is attributed to how much effort a student puts into learning and not their ability. Also, the idea of struggling is valued by teachers in other countries. It is the combination of building a foundation of skills, linked to existing knowledge, which leads to sense making, allowing for dynamic problem solving in an ever evolving mathematical field. At the root of this is a teacher who provides students with challenging problems and the time to struggle through them with support.

These preconceptions are all linked together. If students only memorize rules and procedures, when faced with a new abstract problem they will have no problem solving strategies to pull from in order to make sense of the question presented. When they are unsuccessful, this then leads to the notion that they are “bad at math” or unable to “do math” and slowly they build a fixed mindset about the subject. Donovan and Bransford (2005) suggest “math talk” as a method to engage students. When students work in collaborative small groups, they are able to clarify their strategies with peers and compare approaches, working together to solve challenging problems. “Math talk” and student-centered discussion allows for informal problem solving but also guides thinking toward more advanced understandings.

Donovan and Bransford (2005) discussed allowing for multiple strategies. The idea that there is one algorithm or one rule for solving problems is part of the traditional instructional approach. As previously mentioned, during collaborative small group time students are able to compare and share their strategies with one another. Then as a large group students have an opportunity to display their work on the board and/or explain their methods for solving a
particular problem. For a teacher this can be unsettling. This type of classroom community can only function productively if the teacher is open to the idea that they might not fully understand a student’s method immediately. But as Donovan and Bransford (2005) argue, this is okay! This provides a perfect opportunity to model the math talk language, and sense making through asking clarifying questions that students are asked to do.

In order to apply Principle 2 Donovan and Bransford (2005) urge teachers to remember that this level of understanding and problem solving can only be achieved through balancing conceptual understanding and procedural fluency. While mathematical procedures should not be the central focus, a student must have a network of knowledge building as they move up through their schooling. Students must pull from their prior knowledge of concepts and procedures, use these to support their new understandings, and connect prior knowledge to new knowledge in order to organize new concepts and competencies. A teacher must find the balance between knowledge-centered learning and learner-centered learning in order to effectively help students make these connections.

Donovan and Bransford (2005) explained that students often self-fulfill their own prophecy that they will never be able to learn mathematics. Part of combating this is to help students be metacognitive thinkers which enables student self-monitoring. This is Donovan and Bransford’s (2005) Principle 3. Not only do students need to use various problem solving strategies to answer abstract mathematical problems, students must make sense of their answers. Instruction to support metacognition includes having students make their thinking visible. This could be done through the previously discussed strategy of “math talk” and through their own self-assessment of learning during group work time with their peers. Donovan and Bransford (2005) explained that in order to for visible thinking to occur, students must feel comfortable in
their classroom environments to become vulnerable with their work. This also requires students to have confidence in not only presenting their thinking when they are confident it is correct but also to have the confidence and self-awareness to ask for help when they are stuck or when they cannot find their error.

**Shifting Classroom Environment and Pedagogy**

Educational reform has been a topic for many years. Windschitl (2002) said the latest calls for reform in pedagogy is based on constructivism which he explained as the vision of children “constructing their own knowledge”. Implementing constructivist instruction presents a wide variety of challenges. Windschitl (2002) used four frames of reference to describe some of the dilemmas faced by teachers wanting to use a constructivist approach. The first frame is conceptual dilemmas which describe a teacher’s attempt to understand the philosophical, psychological, and epistemological ideas behind constructivism. The second frame is pedagogical dilemmas which come from designing curriculum and learning experiences. The third frame is cultural dilemmas which arise due to the reorientation of classroom roles. The fourth frame is political dilemmas which come from resistance to the progressive reform.

As Windschitl (2002) argued, without a firm working understanding, teachers cannot be expected to link constructivist objectives for learning with the appropriate types of instruction and assessment for their classroom contexts. He explained, “this is not only because constructivist is a theory of learning rather than of teaching, but also because the implied precepts for instruction break radically from the traditional educational model in which teachers themselves were schooled” (Windschitl, 2002, p. 138). This makes it especially difficult for teachers to picture constructivist pedagogy. To understand constructivism is difficult enough,
and transforming a classroom practice in meaningful ways also requires thinking like a constructivist.

The traditional approach of a teacher-centered classroom needs to mold into a student-centered classroom. Windschitl (2002) listed the following as characteristics of a teacher in the classroom holding a constructivist view:

- A prior awareness of ideas that children bring to the learning situation
- Clearly defined conceptual goals for learners and an understanding of how learners might progress toward these
- Use of teaching strategies which involve challenge to the initial ideas of learners and ways of making new ideas accessible to them
- Provision of opportunities for the learners to utilize new ideas in a range of contexts
- Provision of a classroom atmosphere which encourages children to put forth and discuss ideas (p. 140)

These are clearly different from that of a traditional classroom and teaching style. From this list of characteristics it follows that there needs to be a shift of focus. Typically teachers think about instruction as dispensing content but instead teachers should be placing the focus on students’ efforts to understand (Windschitl, 2002). What has been found to be most difficult about this notion is that this creates a more complex and unpredictable relationship between teacher and student. This requires that a teacher be able to use ideas presented by students that the teacher may not necessarily be prepared for ahead of time.
A common misconception is that this type of teacher to student relationship or this type of classroom environment is not structured. A constructivist teacher must develop a classroom that models the type of problem solving and perseverance required of a constructivist thinker. Part of this student centered learning can occur in group work settings where students are required to collaborate with peers and the teacher acts as more of a facilitator. This allows for students to “witness and participate in each other’s intellectual activity” which can be extremely beneficial (Windschitl, 2002, p. 146). However, Windschitl (2002) also described possible negative aspects of group work to include peers that are uninterested in helping their group members, bickering, exclusion, and academic free-loading (p. 146). It is up to the teacher to teach how to be a productive group member and to create a classroom environment where “opposing views become alternatives to be explored rather than competitors to be eliminated” (Windschitl, 2002, p. 147).

Windschitl (2002) went on to discuss that this type of classroom environment where students encounter new problems and are asked to generate original solutions can feel ambiguous and high-risk to students. Despite this, teachers must remember to hold their students accountable for the quality of their solutions. This presents a dilemma about the balance between teachers’ obligation to content and obligation to the learner. On one hand teachers are to encourage and teach students to make their own meaning, but then are also responsible for students’ complete understanding of the material. It may mean that not all students grasp the same level of understanding. This is again in stark contrast to the traditional approach of teaching. In the past it was expected that teachers “control the intellectual activity to ensure uniform ‘exposure’ to the curriculum” but over time this led to passive learners rather than active and engaged participants (Windschitl, 2002, p. 151). By challenging students in their thinking,
they become better mathematicians and better active thinkers. During group work and class discussions questions such as “why do you know that is the answer?” or “what do you mean by that?” help students formulate an understanding.

Windschitl (2002) also discussed the political dilemmas and push back from administration, community members, parents, and students. Particularly, many state policies discourage educators from spending time on activities such as inquiry into their own teaching or adapting instruction. At the same time, states urge educators to teach using methods that promote deeper thinking. Part of this frustration comes from issues surrounded by the “standards movement” and standardized testing. Success on these tests might be an argument for keeping to more traditional methods. However students are often assessed using problems they have not encountered in class. Thus students must possess a deep level of understanding, problem solving skills, and perseverance.

Windschitl (2002) discussed the necessity of allowing for new teachers to observe and be mentored by “model” constructivist teachers. As Windschitl (2002) said, “to transform practice that can sustain progressive educational change, researchers, reformers, and practitioners must jointly fashion a vision of constructivism that involves more than theories of learning or instruction” because it must also include the whole picture involving conflicts and tensions like the ones presented here (p. 165). The teachers are the central figures in classrooms and it is up to them to examine and share how this pedagogy will help our school’s flourish when put into practice correctly.

**Alternative Assessments**
This curriculum project is not presenting curriculum but rather assessment ideas that will align with content standards and curriculum being taught in the Geometry classroom. This chapter consists of the alternative assessments. For each assessment idea presented, recommendations for where the assessment will fit within the curriculum across the spectrum of 7-12th grade learners will be provided. In addition, an explanation of the assessment type, sample response, and grading tools will be included. These assessments provide a way to give all learners the opportunity to demonstrate mastery. The assessments are presented as follows:

- Alternative Assessment 1: Collaborative Trigonometry Assignment
- Alternative Assessment 2: Construction Justification
- Alternative Assessment 3: Building Kaleidoscopes
- Alternative Assessment 4: Proof Collaboration

All images, solutions, and answer keys to the assessments will be provided in the appendix.
Alternative Assessment 1: Collaborative Trigonometry Assignment

This assessment is best for a high school geometry classroom.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Trigonometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>Designed for a unit on Trigonometry including but not limited to the following unit objectives:</td>
</tr>
<tr>
<td></td>
<td>1. Define the three trigonometric functions sine, cosine, and tangent, in terms of the right triangle definitions.</td>
</tr>
<tr>
<td></td>
<td>2. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems to solve for an unknown side.</td>
</tr>
<tr>
<td></td>
<td>3. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems to solve for an unknown angle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Common Core State Standards</th>
<th>Define trigonometric ratios and solve problems involving right triangles.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometry ratios for acute triangles</td>
</tr>
<tr>
<td></td>
<td>G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
</tr>
</tbody>
</table>

Collaborative Assignment

The following assessment should be given as a collaborative assignment to groups of students. Groups of three to four students should be chosen by the teacher ahead of time.

Students should collaborate with their group members to complete the problem to the best of their ability. Collaborative assignments given during class serve different purposes:

- Teachers seek to encourage and develop essential collaborative skills and to provide opportunities for students to further their mathematical understandings. Problems are thoughtfully selected to promote collaboration and further development of mathematical ideas.

- A collaborative assignment connects current content to other mathematical concepts and/or extends understanding of current content to a deeper level. Challenging problems are selected that require collaboration as well as individual preparation prior to the assessment.
A picnic table in the shape of a regular octagon is shown in the accompanying diagram. If the length of AE is 6 feet, find the length of one side of the table to the nearest tenth of a foot, and find the area of the table’s surface to the nearest tenth of a square foot.
ALTERNATIVE ASSESSMENTS IN GEOMETRY

Student Scoring

A general rubric has been provided to assess students. (A sample answer to the problem is provided in the appendix.) The rubric has been modified to be appropriate for the particular assessment, but is focused on skills necessary for success with any mathematical problem (Exemplars, 2016). Individual students will be given two separate grades for the collaborative assignment. Each student will be given a rubric based grade and a grade based on the quality and accuracy of the mathematical work he/she completes. These two grades may then be combined to create one overall grade for each student for the collaborative assignment.

<table>
<thead>
<tr>
<th>Problem Solving</th>
<th>Reasoning and Proof</th>
<th>Communication</th>
<th>Connections</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Novice</strong></td>
<td>No strategy is chosen, or a strategy is chosen that will not lead to a solution.</td>
<td>Arguments are made with no mathematical basis.</td>
<td>No awareness of audience or purpose is communicated.</td>
<td>No connections are made or connections are mathematically or contextually irrelevant.</td>
</tr>
<tr>
<td></td>
<td>Little or no evidence of engagement in the task is present.</td>
<td>No correct reasoning nor justification for reasoning is present.</td>
<td>No formal mathematical terms or symbolic notations are evident.</td>
<td></td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td>A correct strategy is chosen based on the mathematical situation in the task.</td>
<td>Arguments are constructed with adequate mathematical basis.</td>
<td>A sense of audience or purpose is communicated.</td>
<td>A mathematical connection is made. Proper context are identified that link both the mathematics and the situation in the task.</td>
</tr>
<tr>
<td></td>
<td>Planning or monitoring of strategy is evident.</td>
<td>A systematic approach and/or justification of correct reasoning is present.</td>
<td>Communication of an approach is evident through a methodical, organized, coherent, sequenced and labeled response.</td>
<td>Some examples may include one or more of the following:</td>
</tr>
<tr>
<td></td>
<td>Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</td>
<td></td>
<td>Formal math language is used to share and clarify ideas. At least two formal math terms or symbolic notations are evident, in any combination.</td>
<td></td>
</tr>
<tr>
<td><strong>Exemplary</strong></td>
<td>An efficient strategy is chosen and progress towards a solution is evaluated.</td>
<td>Deductive arguments are used to justify decisions and may result in formal proofs.</td>
<td>A sense of audience and purpose is communicated.</td>
<td>Mathematical connections are used to extend the solution to other mathematics or to a deeper understanding of the mathematics in the task.</td>
</tr>
<tr>
<td></td>
<td>Adjustments in strategy if necessary, are made along the way, and/or alternative strategies are considered.</td>
<td>Evidence is used to justify and support decisions made and conclusions reached.</td>
<td>Communication at the Target level is achieved, and communication of argument is supported by mathematical properties.</td>
<td>Some examples may include one or more of the following:</td>
</tr>
<tr>
<td></td>
<td>Evidence of analyzing the situation in mathematical terms and extending prior knowledge is present.</td>
<td>Formal math language and symbolic notation is used to consolidate math thinking and to communicate ideas.</td>
<td>Formal math language and symbolic notation is used to consolidate math thinking and to communicate ideas.</td>
<td></td>
</tr>
</tbody>
</table>

20
When considering the problem solving and connections categories of the rubric, please note this particular assessment requires that students draw upon previously learned content of the Pythagorean Theorem, knowledge of regular polygons and their properties, area of polygons, and to connect to their knowledge of trigonometry.
**Alternative Assessment 2: Construction Justification**

This assessment is best for an advanced level 8th grade class with students who are able to exceed grade level expectations or otherwise is best for a high school geometry class. This is due to the fact that students must complete a set of constructions.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Patterns in Shape</th>
</tr>
</thead>
</table>
| Objectives | Designed for a unit on patterns in shape including but not limited to the following **unit objectives**:  
1. Justify the relationship \( a^2 + b^2 = c^2 \).  
2. Recognize when and how to apply the Pythagorean Theorem.  
3. Know how to construct a perpendicular line.  
4. Know how to construct a parallel line. |

| Common Core State Standards | **Understand and apply the Pythagorean Theorem.**  
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.  

**Make geometric constructions.**  
G-CO.12. Make a formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. |

**Pythagorean Theorem Construction**

Students should be in groups of 3 to 4 students in order to be given the opportunity to collaborate and discuss their ideas with their peers. While students are working on their constructions and application problems, the teacher should be moving throughout the room, checking in with groups, asking guiding questions when necessary to help students stay on track. Students must be provided with a compass, straightedge, tape, and scissors.
Follow the below instructions with your group members:

1. Given triangle ABC below, correctly label the sides of the triangle formed. From each of these sides label the areas of $a^2$, $b^2$, and $c^2$ that extend out from the sides of the triangle.

2. Find the midpoint of the square off of the longer leg. Label this point $z$. How did you accurately find the midpoint?

3. Using your compass and straight edge, construct a line parallel to the hypotenuse of the triangle that passes through point $z$. Label this line $p$.

4. If you did not already do so in the process of step 3, using your compass and straight edge, construct a line perpendicular to the hypotenuse of the triangle. Label this line $q$.

5. Extend lines $p$ and $q$ and identify the 4 defined areas in the large square.

6. Cut out these areas generated from the constructed parallel and perpendicular lines. Also cut out the square that extends from the shorter leg. Use these areas to tile over the square extending from the hypotenuse.

Discussion: What have you proven by construction?

Application: Use what you have proven!
1. Bridging, shown in the diagram below provides stability between adjacent floor joists. It is generally used when floor spans are greater than 8 feet. If the floor joists are set approximately 20 inches apart, to what length should the bridging be cut?

![Diagram of bridging]

2. The slide at the playground has a height of 8 feet. The base of the slide measured from the bottom of the ladder to the bottom of the slide on the ground is 10 feet. What is the length of the slide?

3. You can make a tent by throwing a piece of cloth over a clothesline, then securing the edge to the ground with stakes so that an isosceles triangle is formed. How long would the cloth have to be so that the opening of the tent was 8 feet wide and 3 feet high?
4. A baseball “diamond” is actually a square with sides of 90 feet. A runner tries to steal second base! How far must the catcher throw the ball to second base?

Using this information, should runners try to steal second base or third base? Justify your response.

5. When building a house, floor space must be planned for the staircases. If the vertical distance between the first and second floor is 1.8 meters, and a contractor is using the standard step pattern of 28 cm wide and 18 cm tall, how many stairs are needed? What is the linear distance needed for the stair case? What will the length of the bannister need to be to the nearest hundredth of a meter?
Student Scoring

The same rubric provided for alternative assessment 1 can be used. Student work should be assessed for accuracy not only on the constructions but also the applied practice problems. The teacher should provide individualized feedback on student work. Additionally, assignments such as this one provide opportunities for students to self-assess. The following general rubric is provided as a way for students to reflect on their own participation and learning experience (Exemplars, 2009). The student self-assessment rubric is closely aligned with the teacher rubric, which also allows for students and teachers to consider and reflect upon feedback from both parties.

<table>
<thead>
<tr>
<th></th>
<th>Novice</th>
<th>Target</th>
<th>Exemplary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understanding</strong></td>
<td>I did not understand the problem or parts of the problem.</td>
<td>I got it. I understood the problem and have an appropriate solution. All parts of the problem are addressed.</td>
<td>I got it! I did it in new ways and showed you how it worked. I can tell you what math concepts are used.</td>
</tr>
</tbody>
</table>
| **Strategies, Reasoning, Procedures** | I could not get started. I do not know how to begin.  
or  
I am stuck. I have part of the solution, but now I do not know what to do. | I have a correct solution. I used a plan to solve the problem. | My solution is effective and inventive. I used big math ideas to solve the problem. I addressed the important details. I showed you some other ways I can solve this problem. I checked to make sure my answer was right. |
| **Communication** | I did not explain how I solved the problem. I did not use pictures, tables or graphs to show you how I solved the problem. | I clearly explained how I solved the problem. I used math language and pictures, tables, graphs, and numbers to explain how I did the problem. | I clearly detailed how I solved the problem. I included all the steps so you don’t have to guess what I did. I used words, numbers, pictures, graphs and/or models. |
Alternative Assessment 3: Building Kaleidoscopes

This assessment is best suited for an advanced level 7th grade class, 8th grade class or a high school Geometry class.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Patterns in Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>Designed for a unit on patterns in shape including but not limited to the following unit objectives:</td>
</tr>
<tr>
<td></td>
<td>1. Identify common 3D shapes of cylinders, cones, prisms, and pyramids.</td>
</tr>
<tr>
<td></td>
<td>2. Use volume formulas for cylinders and other 3D shapes.</td>
</tr>
<tr>
<td></td>
<td>3. Use surface area formulas for cylinders and other 3D shapes.</td>
</tr>
<tr>
<td></td>
<td>4. Use volume and surface area in a real world context.</td>
</tr>
<tr>
<td>Common Core State Standards</td>
<td>Solve real-life and mathematical problems involving angle measure, area,</td>
</tr>
<tr>
<td></td>
<td>surface area, and volume.</td>
</tr>
<tr>
<td></td>
<td>7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and rea of a circle.</td>
</tr>
<tr>
<td></td>
<td>7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two-and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.</td>
</tr>
<tr>
<td></td>
<td>Solve real world-mathematical problems involving volume of cylinders, cones, and spheres.</td>
</tr>
<tr>
<td></td>
<td>8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world mathematical problems.</td>
</tr>
<tr>
<td></td>
<td>Explain volume formulas and use them to solve problems.</td>
</tr>
<tr>
<td></td>
<td>G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</td>
</tr>
<tr>
<td></td>
<td>Apply geometric concepts in modeling situations.</td>
</tr>
<tr>
<td></td>
<td>G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
</tr>
<tr>
<td>Connected to:</td>
<td>Though outside of the 7-12th grade scope, this project is also connected to the following Common Core State Standards:</td>
</tr>
<tr>
<td></td>
<td>Solve real-world and mathematical problems involving area, surface area, and volume.</td>
</tr>
<tr>
<td></td>
<td>6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and</td>
</tr>
</tbody>
</table>
show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

**Kaleidoscope Building**

Students should be, at minimum, in pairs. This provides students with the opportunity to collaborate and share ideas as they are working on the assembly of the kaleidoscope. Students must be provided with a piece of Mylar, a cardboard toilet paper tube, plastic wrap, colored plastic beads, a small “medicine dosage” cup, tape, scissors, and measurement tools. Step by step images of the assembly process have been provided in the appendix.
Kaleidoscope Project  

Name: ______________________

Directions for Assembly:

1. Measure the piece of Mylar you were given. Let the length be the longer side and the width be the shorter side. Write the dimensions of your Mylar piece below:

   Length:_______________  
   Width:________________

2. Divide the width by three. *Carefully* measure out three congruent sections of Mylar and on your cardboard sheet. Cut the Mylar and the cardboard into thirds.

3. Carefully tape the Mylar together to form a prism. Carefully tape the cardboard to the outside of the Mylar to create sturdiness. Now you have a 3D shape! If the shape was closed, what shape would the base be? What type of prism would it be? Draw the base of the prism below. How could you calculate the height of the base?

4. Find the lateral area of the “prism”.  
   Lateral Area:________________

5. Wrap your plastic wrap around the ends of the “prism” to create bases. Find the total surface area of the prism.  
   Surface Area:________________
6. Find the volume of the prism. Volume: ____________________

7. Now move to the cardboard toilet paper roll. What shape is this? Draw a picture diagram below.

8. Find the lateral area of the cardboard roll. Lateral Area:______________

9. If the roll was closed, by bases, what shape would the base be? What would the total surface area of the shape be? Surface Area:_______________

10. Find the volume of the roll. Volume:_______________
11. Compare the volume of the prism to the roll. Which is largest? When the prism is placed inside of the roll, how much space will be left between the prism and roll?

12. Place the prism inside of the roll. Place plastic wrap over the ends of the roll to create bases.

13. Now take 5 to 7 plastic beads, and place them in your medicine dosage cup. Attach this to one of the bases, using plastic wrap and tape.

14. You have created a kaleidoscope! How did we use geometry to create this kaleidoscope?
**Student Scoring**

This assessment is a project that incorporates key concepts of volume, surface area, prisms, and cylinders. A general rubric has been created that is aligned with this project as well as the standards for mathematical practice, similar to that of assessment 1 and assessment 2.

<table>
<thead>
<tr>
<th></th>
<th>Problem Solving</th>
<th>Communication</th>
<th>Connections</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Novice</strong></td>
<td>No strategy is chosen, or a strategy is chosen that will not lead to a solution.</td>
<td>No awareness of audience or purpose is communicated.</td>
<td>No connections are made or connections are mathematically or contextually irrelevant.</td>
<td>No attempt is made to construct a mathematical representation.</td>
</tr>
<tr>
<td></td>
<td>Little or no evidence of engagement in the task is present.</td>
<td>No formal mathematical terms or symbolic notations are evident.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td>A correct strategy is chosen based on the mathematical situation in the task.</td>
<td>A sense of audience or purpose is communicated.</td>
<td>A mathematical connection is made. Proper context are identified that link the mathematics and the situation in the task.</td>
<td>An appropriate and accurate mathematical representation(s) is constructed and refined to solve problems or portray solutions.</td>
</tr>
<tr>
<td></td>
<td>Planning or monitoring of strategy is evident.</td>
<td>Communication of an approach is evident through a methodical, organized, coherent, sequenced and labeled response.</td>
<td>Some examples may include one or more of the following:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Evidence of solidifying prior knowledge and applying it to the problem-solving situation is present.</td>
<td>Formal math language is used to share and clarify ideas. At least two formal math terms or symbolic notations are evident, in any combination</td>
<td>• Clarification of the mathematical context of the task.</td>
<td></td>
</tr>
<tr>
<td><strong>Exemplary</strong></td>
<td>An efficient strategy is chosen and progress towards a solution is evaluated.</td>
<td>A sense of audience and purpose is communicated.</td>
<td>A mathematical connection is used to extend the solution to other mathematics or to a deeper understanding of the mathematics in the task.</td>
<td>An appropriate mathematical representation(s) is constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.</td>
</tr>
<tr>
<td></td>
<td>Adjustments in strategy if necessary, are made along the way, and/or alternative strategies are considered.</td>
<td>Communication at the Target level is achieved, and communication of argument is supported by mathematical properties.</td>
<td>Some examples may include one or more of the following:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Evidence of analyzing the situation in mathematical terms and extending prior knowledge is present.</td>
<td>Formal math language and symbolic notation is used to consolidate math thinking and to communicate ideas.</td>
<td>• Testing and accepting or rejecting of a hypothesis or conjecture</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Explanation of phenomenon</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Generalizing and extending the solution to other cases</td>
<td></td>
</tr>
</tbody>
</table>
(Exemplars, 2016). The Reasoning and Proof category has been removed as it was not deemed applicable to this particular project.

Alternative Assessment 4: Proof Collaboration

This assessment is best for a high school geometry class.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Patterns in Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>Designed for a unit on proofs including but not limited to the following unit objectives:</td>
</tr>
<tr>
<td></td>
<td>1. Use the triangle congruence theorems to prove statements about geometric figures.</td>
</tr>
<tr>
<td></td>
<td>2. Use properties of polygons in geometric proofs.</td>
</tr>
<tr>
<td></td>
<td>3. Use theorems about angle relationships formed by intersecting lines and angle relationships formed when a transversal intersects a pair of parallel lines.</td>
</tr>
</tbody>
</table>
| Common Core State Standards | Prove geometric theorems. G-CO.9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternative interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints. G-CO.10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G-CO.11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Prove theorems involving similarity. G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Students will be in pairs for this assessment. In order to pair students, teachers must prepare cardstock cutouts of the triangles involved in each proof ahead of time. For each proof, there is a total of two cutouts. Upon entry to the classroom, each student will be handed a single cardstock cut out. Around the room, poster sized paper should be hanging with each “Prove:” statement and diagram. Students find their assessment partner by finding the proof that matches their cardstock triangle, and confirming with their believed partner whether the pair of their triangles can fit together to match the diagram on the poster. Once a partnership has been confirmed, the pair should head back to a table to plan and write their formal proof. Once completed, the students will record their final proof onto the poster. Once all partners are finished, students will take a gallery walk to view the other proofs around the classroom. The following are recommended proofs for this activity, however the teacher can modify or add to this set to be appropriate for their classroom and students.
Given: \( \overline{HI} \) is the angle bisector of \( \angle H \)
\[
\overline{FH} \cong \overline{HJ}
\]
Prove: \( \angle F \cong \angle J \)

Diagram:

---

Given: \( \angle Q \) and \( \angle S \) are right angles.
\[
\overline{QR} \cong \overline{SR}
\]
Prove: \( \triangle QRT \cong \triangle SRT \)

Diagram:
Given: $O$ is the midpoint of $UT$

$O$ is the midpoint of $SP$

Prove: $\angle USO \cong \angle TPO$

---

Given: $PQRS$ is a rectangle

Prove: $\triangle QSP \cong \triangle QSR$

---

Given: $STUV$ is a rectangle

Prove: $\triangle STV \cong \triangle UVT$
Given: \( TP \) bisects \( \angle OTQ \)

\[ \overline{OT} \cong \overline{TQ} \]

Prove: \( \angle O \cong \angle Q \)

Student Scoring
A general rubric has been provided for this activity (Exemplars, 2016). This activity lends itself to an opportunity for peer assessment as well as teacher assessment. While students are taking a gallery walk, student pairs can assess each written proof and include feedback for their classmates. The ability to analyze and interpret their classmates’ work is an essential skill.

<table>
<thead>
<tr>
<th>Reasoning and Proof</th>
<th>Communication</th>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Novice</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arguments are made with no mathematical basis.</td>
<td>No awareness of audience or purpose is communicated.</td>
<td>No connections are made or connections are mathematically or contextually irrelevant.</td>
</tr>
<tr>
<td>No correct reasoning nor justification for reasoning is present.</td>
<td>No formal mathematical terms or symbolic notations are evident.</td>
<td></td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arguments are constructed with adequate mathematical basis.</td>
<td>A sense of audience or purpose is communicated.</td>
<td>A mathematical connection is made. Proper context are identified that link both the mathematics and the situation in the task.</td>
</tr>
<tr>
<td>A systematic approach and/or justification of correct reasoning is present.</td>
<td>Communication of an approach is evident through a methodical, organized, coherent, sequenced and labeled response.</td>
<td>Some examples may include one or more of the following:</td>
</tr>
<tr>
<td></td>
<td>Formal math language is used to share and clarify ideas. At least two formal math terms or symbolic notations are evident, in any combination.</td>
<td>• Clarification of the mathematical context of the task</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Exploration of mathematical phenomenon in the context of the broader topic in which the task is situated</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Noting patterns, structures and regularities</td>
</tr>
<tr>
<td><strong>Exemplary</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deductive arguments are used to justify decisions and may result in formal proofs.</td>
<td>A sense of audience and purpose is communicated.</td>
<td>Mathematical connections are used to extend the solution to other mathematics or to a deeper understanding of the mathematics in the task.</td>
</tr>
<tr>
<td>Evidence is used to justify and support decisions made and conclusions reached.</td>
<td>Communication at the Target level is achieved, and communication of argument is supported by mathematical properties.</td>
<td>Some examples may include one or more of the following:</td>
</tr>
<tr>
<td></td>
<td>Formal math language and symbolic notation is used to consolidate math thinking and to communicate ideas.</td>
<td>• Testing and accepting or rejecting of a hypothesis or conjecture</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Explanation of phenomenon</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Generalizing and extending the solution to other cases</td>
</tr>
</tbody>
</table>

Validity
These alternative assessments were either used in a high school geometry classroom or were reviewed by a veteran teacher for feedback. The veteran teacher had personal experience with using these alternative assessments in the classroom. The author’s recommendations for how best to implement these alternative assessments as well as the veteran teacher’s feedback is paraphrased as follows:

- **Collaborative Trigonometry Assignment**: The author recommends that teachers intentionally group students ahead of time. It is best if students are grouped so that low level students are with middle level students, and middle level students are with high level students. The gap between low level and high level students is too large and does not lend to productive discussion of ideas and mathematical reasoning.

- **Construction Justification**: The author recommends using this assessment only after students have been given several opportunities to practice constructions. This assignment is an excellent opportunity for students to use multiple constructions, together. Additionally, the author suggests that students use colored pencils or colored pens to help distinguish between the constructions throughout the process. Students had a tendency to get “lost” in all of the different lines and arcs drawn, so that when it came time to cut apart the square students cut along incorrect lines. The author suggests using this assessment as a way to review Pythagorean Theorem, since the author has found this to be a topic that students often do not remember well from prior classes.

- **Kaleidoscope Building**: The veteran teacher allowed students to create kaleidoscopes using different prisms in addition to the triangular prism. However, when students began to explore it turned into inquiry “for fun” instead of an exploration connected tightly to
geometry. Therefore, it is recommended that teachers extend thinking in other ways by posing additional questions. The veteran teacher suggested posing questions such as:

- When does it make sense to be thinking about surface area?
- Why do we need to find surface area?
- Which part of the kaleidoscope is this accounting for?

These questions can also be applied to thinking about volume. Additionally depending on the size of the bead, more or less beads should be used than the 5 to 7 suggestion in the directions.

- Proof Collaboration: The veteran teacher suggests that this assessment be used only after students have had exposure to all types of proofs (including but not limited to congruent triangles, relationships involving parallel lines, and proving properties of quadrilaterals). In other words, students should already have had time via homework assignments or classwork to struggle with writing formal proofs before completing this task.

    Additionally, due to the fact that students are randomly “assigned” a partner when they are given a triangle cut out, it is possible to have a partnership that is not best for the task. To combat this, the veteran teacher suggests intentionally giving students triangles so that partners are intentionally assigned, unbeknownst to the students.

These are suggestions and recommendations, however teachers should modify the alternative assessments provided to best meet the needs of their classrooms and students.

**Conclusion**

Geometry as a subject is unique in that the content lends itself to many hands on learning experiences. It is the author’s hope that the same hands on, collaborative classroom, with
diversity in content delivery will also allow for students to show mastery in hands on, collaborative, diverse ways. This project is meant to show how alternative assessments can be incorporated into the classroom, in a way that still accounts for the pressures that teachers face including meeting the Common Core State Standards while combating a lack of time needed. Although using alternative assessments can sometimes be more of a challenge and requires more effort from the teacher than using traditional pen and paper summative assessments, it is in the best interest of the learner to incorporate alternatives for showing mastery of content. The pursuit of meeting the needs of diverse learners is a worthy one. Teachers should encourage problem solving, creativity, and collaboration in students who will be entering a world in which these skills will be essential. Since these are skills teachers want students to utilize and refine, then opportunities to do so, that assess these skills, should be a part of the learning experience.
References:


Appendix

Alternative Assessment 1: Collaborative Trigonometry Assignment

Name: ________________________________

Directions: Answer the following with your group. Show all mathematical ideas and work.

A picnic table in the shape of a regular octagon is shown in the accompanying diagram. If the length of AE is 6 feet, find the length of one side of the table to the nearest tenth of a foot, and find the area of the table’s surface to the nearest tenth of a square foot.

**Length of one side:**

\[
\frac{360}{8} = 45 \degree
\]

\[
\frac{45}{2} = 22.5
\]

\[
\sin 22.5 = \frac{x}{3}
\]

\[
3 \cdot (\sin 22.5) = x
\]

\[
1.148050297 = x
\]

\[
AH = 2.3 \text{ ft}
\]

**Area of Table’s Surface:**

**Area of one section:**

\[
\text{Area} = \frac{1}{2} \cdot bh
\]

\[
= \frac{1}{2} \cdot (2.3) \cdot (2.7708)
\]

\[
= 3.18642 \text{ ft}^2
\]

\[
1.15^2 + h^2 = 3^2
\]

\[
1.3225 + h^2 = 9
\]

\[
h^2 = 7.6775
\]

\[
h = 2.7708 \text{ ft}
\]

**Area of picnic table:**

\[
3.18642(8) = 25.49136
\]

**AREA = 25.5 \text{ ft}^2**
ALTERNATIVE ASSESSMENTS IN GEOMETRY

Alternative Assessment 2: Construction Justification
Application: Use what you have proven!

1. Bridging, shown in the diagram below provides stability between adjacent floor joists. It is generally used when floor spans are greater than 8 feet. If the floor joists are set approximately 20 inches apart, to what length should the bridging be cut?

   ![Diagram of bridging](image)
   The bridging should be cut to 25 inches.

   \[15^2 + 20^2 = c^2\]
   \[225 + 400 = c^2\]
   \[\sqrt{625} = c\]

2. The slide at the playground has a height of 8 feet. The base of the slide measured from the bottom of the ladder to the bottom of the slide on the ground is 10 feet. What is the length of the slide?

   ![Diagram of slide](image)
   The slide is about 12.8 ft long.

   \[8^2 + 10^2 = c^2\]
   \[64 + 100 = c^2\]
   \[\sqrt{164} = c\]

3. You can make a tent by throwing a piece of cloth over a clothesline, then securing the edge to the ground with stakes so that an isosceles triangle is formed. How long would the cloth have to be so that the opening of the tent was 8 feet wide and 3 feet high?

   ![Diagram of tent](image)
   The cloth would need to be 10 feet long.

   \[3^2 + 4^2 = c^2\]
   \[9 + 16 = c^2\]
   \[\sqrt{25} = c\]
   \[5 = c\]
4. A baseball ‘diamond’ is actually a square with sides of 90 feet. A runner tries to steal second base! How far must the catcher throw the ball to second base?

The catcher must throw the ball about 127.3 feet.

\[ 90^2 + 90^2 = c^2 \]
\[ \frac{16200}{c} = c \]

Using this information, should runners try to steal second base or third base? Justify your response.

The distance to second base is greater than the distance (of 90 ft) to third base. So, the catcher must throw farther to second base.

5. When building a house, floor space must be planned for the staircases. If the vertical distance between the first and second floor is 1.8 meters, and a contractor is using the standard step pattern of 28 cm wide and 18 cm tall, how many stairs are needed? What is the linear distance needed for the stair case? What will the length of the bannister need to be to the nearest hundredth of a meter?

\[ 1.8 \, \text{m} = 180 \, \text{cm} \]
\[ \frac{180}{18} = 10 \quad \text{need 10 stairs up} \]

\[ 10(28) = 280 \quad \text{need a linear distance of 280 cm or 2.8 m} \]

\[ 1.8^2 + 2.8^2 = c^2 \]
\[ \sqrt{11.08} = c \quad \text{the bannister should be about 3.3 m} \]
Alternative Assessment 3: Kaleidoscope Building

Kaleidoscope Project

Name: _______________________

Directions for Assembly:

1. Measure the piece of Mylar you were given. Let the length be the longer side and the width be the shorter side. Write the dimensions of your Mylar piece below:

   Length: 10 cm  
   Width: 7.5 cm  

2. Divide the width by three. Carefully measure out three congruent sections of Mylar and on your cardboard sheet. Cut the Mylar and the cardboard into thirds.

3. Carefully tape the Mylar together to form a prism. Carefully tape the cardboard to the outside of the Mylar to create sturdiness. Now you have a 3D shape! If the shape was closed, what shape would the base be? What type of prism would it be? Draw the base of the prism below. How could you calculate the height of the base?

   Base Shape: triangle  \rightarrow Triangular Prism

   \[
   \frac{10}{3} (\frac{10}{3})^2 = \frac{(10)}{3} + h^2
   \]

   \[
   h = \frac{\sqrt{25}}{3} \approx 8.33 \text{ cm}
   \]

4. Find the lateral area of the "prism".  
   Lateral Area: 75 cm²

   \[10(7.5) = 75 \text{ cm}^2\]

5. Wrap your plastic wrap around the ends of the "prism" to create bases. Find the total surface area of the prism.  
   Surface Area: 102.78 cm²

   Area of Base = \(\frac{1}{2}(\frac{10}{3}) \sqrt{25} = 13.8 \text{ cm}^2\)

   Area of prism = \((13.8)(2) + 75 \approx 102.78 \text{ cm}^2\)

6. Find the volume of the prism.  
   Volume: 104.16 cm³

   \[V = \left(\frac{1}{2}bh\right)h_{\text{prism}}\]

   \[V = (13.8 \times 7.5) = 104.16 \text{ cm}^3\]
7. Now move to the cardboard toilet paper roll. What shape is this? Draw a picture diagram below.

```
   cylinder
```

8. Find the lateral area of the cardboard roll. 
   Lateral Area: 105 cm²

```
7.5 cm
14 cm
```

\[ 7.5(14) = 105 \text{ cm}^2 \]

9. If the roll was closed, by bases, what shape would the base be? What would the total surface area of the shape be?
   Surface Area: 136.19 cm²

Base shape: Circle

Area of base = \( \pi r^2 = \pi \left( \frac{14}{\pi} \cdot \frac{1}{2} \right)^2 \)

\[ = 15.697184423 \text{ cm}^2 \]

Area of cylinder = 136.194368846 cm²

10. Find the volume of the roll.
   Volume: 116.98 cm³

\[ V = \pi r^2 h \]
\[ V = \pi \left( \frac{14}{\pi} \cdot \frac{1}{2} \right)^2 (7.5) \]
\[ V = 116.978883173 \text{ cm}^3 \]

11. Compare the volume of the prism to the roll. Which is largest? When the prism is placed inside of the roll, how much space will be left between the prism and roll?

   The volume of the cylinder is larger.

\[ 116.98 \text{ cm}^3 - 104.17 \text{ cm}^3 = 12.81 \text{ cm}^3 \]

12. Place the prism inside of the roll. Place plastic wrap over the ends of the roll to create bases.
13. Now take 5 to 7 plastic beads, and place them in your medicine dosage cup. Attach this to one of the bases, using plastic wrap and tape.

14. You have created a kaleidoscope! How did we use geometry to create this kaleidoscope?

   We created a prism that when placed inside the cylinder, allowed for a reflection of the beads to be seen.

   We carefully measured and constructed an equilateral triangle. The angle created makes a specific number of reflections. If we were to change the shape of the prism’s base, the reflection number would also change. With the triangle, we created a hexagon. This is because the angles in our triangle are all 60°, which acts as the central angle in the regular hexagon (360°/6 = 60°).

***This part of the project might require students to do research about kaleidoscopes. Students could then present their findings to the class as partnerships.
Step 1: Gather supplies and cut Mylar pieces. (Mylar can be ordered at: https://www.enasco.com/p/Mirror-Pack%2BSB23588?searchText=flexible++mirrors)

Step 2: Create triangular prism.

Step 3: Use plastic wrap to create the bases.
Step 4: Place prism inside cylinder and create bases of cylinder using plastic wrap.

Step 5: Place beads inside dosage cup, and tape cup to the end of the cylinder.
Alternative Assessment 4: Proof Collaboration

Below is *one* set of possible solutions. For several of these proofs, there are alternative justifications.

### Alternative Assessment 4: Proof Collaboration

**Diagram:**

- **Given:** $HI$ is the angle bisector of $\angle H$
  - $FH \cong HJ$

- **Prove:** $\angle F \cong \angle J$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $HI$ is the angle bisector of $\angle H$ and $FH \cong HJ$</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $HI \equiv HI$</td>
<td>2. Reflexive property</td>
</tr>
<tr>
<td>3. $\angle 1 \equiv \angle 2$</td>
<td>3. Since $HI$ bisects $\angle H$</td>
</tr>
<tr>
<td>4. $\triangle FH \cong \triangle JH$</td>
<td>4. By SAS Triangle Congruence Theorem</td>
</tr>
<tr>
<td>5. $\angle F \equiv \angle J$</td>
<td>5. Congruent Parts of congruent triangles are congruent</td>
</tr>
</tbody>
</table>

**Diagram:**

- **Given:** $\angle Q$ and $\angle S$ are right angles.
  - $QR \cong SR$

- **Prove:** $\triangle QRT \cong \triangle SRT$

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<tbody>
<tr>
<td>1. $\angle Q$ and $\angle S$ are right angles and $QR \cong SR$</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $\angle Q \cong \angle S$</td>
<td>2. Since both right angles</td>
</tr>
<tr>
<td>3. $RT \equiv RT$</td>
<td>3. Reflexive property</td>
</tr>
<tr>
<td>4. $\triangle QRT \cong \triangle SRT$</td>
<td>4. By Hypotenuse-Leg Triangle Congruence Theorem</td>
</tr>
</tbody>
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**Diagram:**

- **Given:** $O$ is the midpoint of $UT$
  - $O$ is the midpoint of $SP$

- **Prove:** $\angle USO \cong \angle TPO$

<table>
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<tr>
<td>1. $O$ is the midpoint of $UT$ and $PS$</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $PO \equiv SO$ and $TO \equiv TO$</td>
<td>2. By definition of midpoint</td>
</tr>
<tr>
<td>3. $\angle 1 \equiv \angle 2$</td>
<td>3. Vertical Angles Theorem</td>
</tr>
<tr>
<td>4. $\triangle PTO \cong \triangle SVO$</td>
<td>4. By SAS Triangle Congruence Theorem</td>
</tr>
<tr>
<td>5. $\angle USO \equiv \angle TPO$</td>
<td>5. Congruent parts of congruent triangles are congruent</td>
</tr>
</tbody>
</table>
Given: \(PQRS\) is a rectangle

Prove: \(\triangle QSP \cong \triangle QSR\)

<table>
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<tr>
<th>Statement</th>
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<tbody>
<tr>
<td>① (PQRS) is a rectangle</td>
<td>① given</td>
</tr>
<tr>
<td>② (PS \cong RQ) and (PQ \cong RS)</td>
<td>② Property of a Rectangle</td>
</tr>
<tr>
<td>③ (QS \cong QS)</td>
<td>③ Reflexive Property</td>
</tr>
<tr>
<td>④ (\triangle QSP \cong \triangle QSR)</td>
<td>④ By SSS Triangle Congruence Theorem</td>
</tr>
</tbody>
</table>

Given: \(STUV\) is a rectangle

Prove: \(\triangle STV \cong \triangle UVT\)

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<tr>
<td>① (STUV) is a rectangle</td>
<td>① given</td>
</tr>
<tr>
<td>② (SV \cong UV) and (SV \cong UT)</td>
<td>② Property of a Rectangle</td>
</tr>
<tr>
<td>③ (\angle S) and (\angle U) are right angles</td>
<td>③ Property of a Rectangle</td>
</tr>
<tr>
<td>④ (\angle S \cong \angle U)</td>
<td>④ Since both are right angles</td>
</tr>
<tr>
<td>⑤ (\triangle STV \cong \triangle UVT)</td>
<td>⑤ By SAS Triangle Congruence Theorem</td>
</tr>
</tbody>
</table>

Given: \(TP\) bisects \(\angle LOTQ\)

Prove: \(\angle L \cong \angle Q\)

<table>
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</thead>
<tbody>
<tr>
<td>① (TP) bisects (\angle LOTQ) and (\overline{OT} \cong \overline{TQ})</td>
<td>① given</td>
</tr>
<tr>
<td>② (\angle 1 \cong \angle 2)</td>
<td>② Since (TP) bisects (\angle LOTQ)</td>
</tr>
<tr>
<td>③ (\triangle LOTQ) is isosceles</td>
<td>③ Since (\overline{OT} \cong \overline{TQ})</td>
</tr>
<tr>
<td>④ (\angle L \cong \angle Q)</td>
<td>④ Since base angles are congruent in isosceles triangles</td>
</tr>
</tbody>
</table>