Teaching Exponential Functions to Students of the Digital Era

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Teaching Exponential Functions to Students of the Digital Era

Marian R. Tufano

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Abstract

This curriculum is designed to teach the unit of exponential functions in Algebra 1 to students of the digital generation. The lessons are designed to engage students through technology rich lessons involving real world examples. The lessons provide visuals, multiple representations, and applications of exponential functions and are aligned to the New York State Common Core Learning Standards.
**Introduction**

Students in today’s generation can be known as the digital generation. They have grown up being surrounded by technology—televisions, computers, video games, and cell phones. All this technology affects the way they process and interact information (Jukes, Crockett, & McCain, 2010, p. 65). These students are different in the way they think and we as teachers need to teach in ways that engage them, just as the technology around them does.

Research states, “the digital generation wants their learning to be relevant and instantly useful, and more than anything else, they want to know what possible connections this has to them and their world” (Jukes, Crockett, & McCain, 2010, p. 69). With the Algebra 1 Common Core Learning Standards teachers are expected to teach students the connections to real world situations, especially when it comes to building functions. “Functions presented as expressions can model many important phenomena…A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions…” (New York State P-12 Common Core Learning Standards for Mathematics, n. d., P. 57). Teachers can use technology and real world situations to gain the interest and attention of their students.

**Purpose**

The purpose of this curriculum is to provide a series of lessons targeted towards students of the digital era. This bombardment of technology affects the ways students learn information and the ways teachers need to engage their students. The curriculum will be composed of technology rich lessons on exponential functions taught through real life situations. The lessons will be taught to students of the ninth grade taking the course of Algebra 1. Through the implementation of various technological tools (i.e. Google Chromebooks and TI-Nspires) and
encompassing of real world situations, teachers will be able to engage their students while teaching exponential functions.

**Literature Review**

**Students of The Digital Era**

Students in today’s generation can be known as the digital generation. They have grown up being surrounded by technology—televisions, computers, video games, and cell phones. All this technology affects the way they process and interact information (Jukes, Crockett, & McCain, 2010). These students are different in the way they think and we as teachers need to teach in ways that engage them, just as the technology around them does.

Research states, “the digital generation wants their learning to be relevant and instantly useful, and more than anything else, they want to know what possible connections this has to them and their world” (Jukes, Crockett, & McCain, 2010). With the Algebra 1 Common Core Learning Standards teachers are expected to teach students the connections to real world situations, especially when it comes to building functions. “Functions presented as expressions can model many important phenomena…A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions…” (New York State P-12 Common Core Learning Standards for Mathematics, n. d., P. 57). Teachers can use technology and real world situations to gain the interest and attention of their students.

**Multiple Representations Through Technology**

Students in the digital generation have been exposed to graphics and need to be taught with images as they are visually fluent (Jukes, Crockett, & McCain, 2010). Visuals are key to student learning. Research states that individuals are able to remember 65% more when a visual
is added to the new material at hand (Jukes, Crockett, & McCain, 2010). Algebraic concepts can be represented through different representations: tabular, graphical, and symbolic (DePeau and Kadler, 2010, p. 268). Along with visual fluency it is also important for students to have representational fluency to work between the different representations of the algebraic concept (DePeau and Kadler, 2010, p. 268). Technology tools such as the TI-Nspire can be used for student exploration of a function through these three representations (DePeau and Kadler, 2010).

Another tool for exploration is Desmos. It is a website that acts as an online graphing calculator. This tool is beneficial to the digital era students as it provides them with a large visual of the function they may be graphing. Orr (2017) discusses the two different tools within Desmos—the classroom activities and the activity builder (p. 549). The activity builder allows students to work through mathematics productively while receiving feedback and thinking deeply about the mathematics (Orr, 2017, p. 549). Orr (2017) states Desmos tools create a deep understanding between the algebraic representation of a function and the graphical representation (p. 550). Desmos can be used to support the representational fluency among students.

**Real Life Situations**

Jukes, Crockett, & McCain (2010) state how educators feel it is necessary to teach the memorization of the material, when what they really need is relevance of the material to the real world. Visuals and real life connections are key to student learning. Jukes, Crockett, & McCain (2010), state, “Illustrations, and applications of learning must have significance to the students if we want to maximize the chance that the learning will stick”. Many functions within the Algebra 1 curriculum can be taught through real world examples. Bush, Gibbons, Karp, and Dillon (2015) use the real life situation of an outbreak of a disease to teach the concept of exponential functions (p. 90). Students were able to create tables to represent how many people were affected
by the disease after a round (Bush, Gibbons, Karp, and Dillon, 2015). They used their graphing
calculators to input their data values and create an exponential regression line to create the
equation that models this situation (Bush, Gibbons, Karp, and Dillon, 2015, p. 95). Such an
activity ties in real life relevance, visual representations, and technology—all key parts to
learning for a student of the digital generation.

Relevance, representations, and technology are key to teaching the digital aged students.
Although the students of this age our surrounded by technology, it is not to say they can perform
the given classroom tasks without guidance. Thompson (2013, p. 23) found that these digital
natives are not completely self-sufficient when using technology. It is crucial as the teacher to
scaffold the use of technology for students (Thompson, 2013, p.23). Thompson (2013) studies
also showed that students no not need constant entertainment and technology (p. 21). So
although guided technology should be a part of the classroom practice, it does not need to
consume it. For example, Bryan (2012) provides an example of teaching through real world
scenarios with an activity where students use random numbers to simulate radioactive decay
(Bryan, 2012, p. 52). The students collect data and then use graphing calculators to chart the
decay of the nuclei, they are able to see through both the table and the graphical display how the
radioactive material is decaying at an exponential rate (Bryan, 2012, p. 54). Seeking to provide
relevance, representations, with integrated technology, this curriculum provides technology rich
lessons involving real life situations described in this provided literature review in order engage
and educate the digital aged students.
Methods

This chapter consists of lessons to teach the Algebra 1 topic of exponential functions. There are four lessons in the curriculum followed by a summative assessment. The lessons incorporate the students using technology tools such as their TI-Nspires along with many Desmos activities which would be completed on students’ chromebooks. The lessons are based on real life scenarios. The combination of the technology tools and real world applications within the curriculum are designed to engage students of the digital era. The curriculum goes as follows:

- Lesson 1: Introduction to Exponential Functions Day 1
- Lesson 2: Introduction to Exponential Functions Day 2
- Lesson 3: Exponential Growth and Decay
- Lesson 4: Exponential Models and Percents
- Summative Assessment

You will find an outline to each lesson containing the overview, common core learning standards, objectives, the design, and assessment of each lesson. Answer keys to the lessons and homework can be found in the appendix.
Lesson 1: Introduction to Exponential Functions Day 1

A. Lesson Overview, Lesson Purpose and Rationale

Lesson Overview: In this lesson students will be completing a Desmos activity: https://teacher.desmos.com/activitybuilder/custom/56c7457e11c7724106e683b1. Students will complete the activity within their partner groups. Then we will close class with a discussion and exit ticket. The students’ homework will be on identifying exponential functions from other types of functions in Algebra 1, the problems contain regents based questions from jmap.org: http://www.jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI.

Lesson Purpose: To introduce the behavior of an exponential function to students. This introduction lesson will allow students to have a basic understanding of this specific type of function. This will drive the rest of the unit. The students will be able to view the behavior both graphically and through a table.

Lesson Rationale: In the previous unit students learned about linear functions. This lesson will allow students to make connections between the past unit and current unit. Students will be able to better understand the behavior of an exponential function when comparing it to the behavior of a linear function. The students will see purpose of the lesson through the real life example of a repair shop and be engaged through the technology tool of Desmos.

B. Specific NYS Common Core Learning Standards (CCLS)

CCSS.MATH.CONTENT.HSF.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

CCSS.MATH.CONTENT.HSF.IF.C.9
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

CCSS.MATH.CONTENT.HSF.LE.A.1
Distinguish between situations that can be modeled with linear functions and with exponential functions.

CCSS.MATH.CONTENT.HSF.LE.A.3
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

C. Lesson Objectives

At the conclusion of this lesson, students will be able to:

- Identify a linear function versus an exponential function.
- Explain the differences between linear and exponential functions.
- Come up with their own examples of exponential functions in the real world.

D. Lesson Design

- Class will begin with passing out the half sheet to all of the students. I will ask the class to focus on the left side of the paper and write down everything they remember about linear functions from the previous unit, they may discuss with their partner. We will then share out and discuss as a class (5 minutes).
- We will then complete the Desmos activity. Students will work on this and have their partner to assist and ask questions with. During this time the teacher will circulate the
room. The teacher will also pause the activity at times to stop and have class discussions and/or address any common misconceptions (45 minutes).

- Once the activity is complete, students will work on filling the other side of their half sheet of paper out about what they learned about exponential functions. We will then share out as a class and students will turn in their sheet and begin their homework (10 minutes).

**E. Assessment**

- Students will be recording responses within the Desmos program. The teacher will be able to view responses through their computer and display student responses to prompt discussion.
- Teacher will also observe students and their progress through circulation.
- Finally, students will turn in the half sheet of paper students completed at the beginning and end of the lesson to see if students reached the lesson objectives. The sheets will be returned to students in the next lesson.
### Lesson 1: Introduction to Exponential Functions Day 1

<table>
<thead>
<tr>
<th>Linear</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Homework:
One characteristic of all linear functions is that they change by

1) equal factors over equal intervals  
2) unequal factors over equal intervals

Which situation could be modeled by using a linear function?

1) a bank account balance that grows at a rate of 5% per year, compounded annually  
2) a population of bacteria that doubles every 4.5 hours  
3) the cost of cell phone service that charges a base amount plus 20 cents per minute  
4) the concentration of medicine in a person's body that decays by a factor of one-third every hour

Which scenario represents exponential growth?

1) A water tank is filled at a rate of 2 gallons/minute.  
2) A vine grows 6 inches every week.  
3) A species of fly doubles its population every month during the summer.  
4) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

Ian is saving up to buy a new baseball glove. Every month he puts $10 into a jar. Which type of function best models the total amount of money in the jar after a given number of months?

1) linear  
2) exponential  
3) quadratic  
4) square root
Which type of function is graphed to the right?

1) linear  
2) quadratic  
3) Exponential  
4) absolute value

A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

1)  
2)  
3)  
4)

11. On January 1, a share of a certain stock cost $180. Each month thereafter, the cost of a share of this stock decreased by one-third. If $x$ represents the time, in months, and $y$ represents the cost of the stock, in dollars, which graph best represents the cost of a share over the following 5 months?
12. Antwaan leaves a cup of hot chocolate on the counter in his kitchen. Which graph is the best representation of the change in temperature of his hot chocolate over time?

1) ![Graph 1](image1)
2) ![Graph 2](image2)
3) ![Graph 3](image3)
4) ![Graph 4](image4)

13. The tables below show the values of four different functions for given values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
<th>$x$</th>
<th>$h(x)$</th>
<th>$x$</th>
<th>$k(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>17</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>4</td>
<td>13</td>
<td>4</td>
<td>24</td>
<td>4</td>
<td>28</td>
</tr>
</tbody>
</table>

Which table represents a linear function?

1) $f(x)$
2) $g(x)$
3) $h(x)$
4) $k(x)$
The function, \( t(x) \), is shown in the table to the right. Determine whether \( t(x) \) is linear or exponential. Explain your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>t(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td>10</td>
</tr>
<tr>
<td>−1</td>
<td>7.5</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Consider the pattern of squares shown below:

Which type of model, linear or exponential, should be used to determine how many squares are in the \( n \)th pattern? Explain your answer.
Desmos activity:

Avi and Benita’s Repair Shop

Avi and Benita run a repair shop. They need some help, so they hire you.

They have different options for how much they’ll pay you each day. (You’ll learn more about those rules in a moment.)

Just remember this: each day, you get to choose whose rules you prefer.

Ready? Continue to the next screen.

Complete the table.

<table>
<thead>
<tr>
<th>Day</th>
<th>Avi’s Rule (Dollars)</th>
<th>Benita’s Rule (Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table shows how much you’ll earn with each rule.

Complete the table for Day 4.

Then predict: Who do you think pays more on Day 10?

Avi

Benita

Sample Responses

- On Day 4, Avi’s rule pays $0.08.
- On Day 4, Benita’s rule pays $400.

Extend the table.

<table>
<thead>
<tr>
<th>Day</th>
<th>Avi’s Rule (Dollars)</th>
<th>Benita’s Rule (Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table shows the actual Day 4 values.

Let’s settle the Day 10 question by extending the table.

Enter values for Days 5-10.

When you’re done, continue to the next screen.
TEACHING EXPONENTIAL FUNCTIONS TO STUDENTS OF THE DIGITAL ERA

STUDENT SCREEN PREVIEW

Visualize the rules.

Avi's rule is shown in RED.
Benita's rule is shown in BLUE.

Describe what you notice about the graphs.

In particular, how are they similar? How are they different?

Teacher Moves

Highlight unique answers for the class.

• On Day 4, Avi's rule pays $0.08.
• On Day 4, Benita's rule pays $2.00

STUDENT SCREEN PREVIEW

Day 20

Which rule do you think will pay more on Day 20?

Use math to support your answer.

Use the sketch tool on the graph if that helps to illustrate your thinking.

Avi's Rule

Benita's Rule

Teacher Moves

STUDENT SCREEN PREVIEW

Visualize Days 11-20.

Complete the table, then press "Try It."

<table>
<thead>
<tr>
<th>Day</th>
<th>Avi's Rule (Dollars)</th>
<th>Benita's Rule (Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Teacher Moves

The work here, especially for Avi's rule, gets pretty tedious. Consider showing students how they can use expressions (in the table!) to calculate values.

For example, students can write $5.12 \cdot 2$ for Day 11 in Avi's rule and Desmos will automatically calculate the result.

Students can push this approach even further by writing either $10.24 \div 2$ or (better yet!) $5.12 \cdot 2 \cdot 2$ or (better still) $5.12 \cdot 2^2$ for Day 12.
Now what do you notice about the graphs?

In particular, is there anything you're surprised by?

"For what days should I get paid by Avi's rule? And for what days should I get paid by Benita's rule?"

What advice would you give to Cal?

Cal should choose Benita's rule for Days 1-18, and Avi's rule from Day 19 onward.

Instead of letting her decide each day which rule she'd like to get paid by, they're making Dee pick one rule from the beginning (and she has to stick with it).

What advice would you give to Dee? Is there anything you need to know before you can give sound and helpful advice?
Lesson 2: Introduction to Exponential Functions Day 2

A. Lesson Overview, Lesson Purpose and Rationale

Lesson Overview: In this lesson students will complete an activity based off the NCTM article "Epidemics, Exponential Functions, and Modeling" (Bush, Gibbons, Karp, and Dillon, 2015). They will view a clip from the movie "Contagion," (Warner Brothers Studio, 2011) http://www.youtube.com/watch?v=VZGHGVIedzA. This will set up the activity where students will model an outbreak through tables, graphs, and symbolically.

Lesson Purpose: Students now have an understanding of how exponential functions behave. In this lesson they will now learn how to express an exponential function through an equation, table, and graph and see the connection between the different representations, and increase their representational fluency.

Lesson Rationale: In the previous lesson students learned about the behavior of an exponential function in comparison to a linear function. Now that they have a basic understanding of the function they will work on building exponential functions through tables, graphs, and equations. They will do this through the real life example of epidemics.

B. Specific NYS Common Core Learning Standards (CCLS)

CCSS.MATH.CONTENT.HS.F.BF.A.1
Write a function that describes a relationship between two quantities.

CCSS.MATH.CONTENT.HS.F.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

C. Lesson Objectives

At the conclusion of this lesson, students will be able to:

- Students will be able to construct a table, graph, and equation based off a real life situation.
- Students will be able to see how manipulating different variables of a situation affect the table, graph, and equation.
- Students will make connections between the multiple representations.
- Students will use their technology (TI-Nspires) to help them construct the different representations.
- Students will have a solidified understanding of the exponential functions and their behavior.

D. Lesson Design

- Class will begin with asking the class what they know about epidemics and how they spread (5 minutes).
- Then the youtube clipped will be played for the students to get them further thinking about epidemics (2 minutes).
- Students will then complete round 1 of the lesson, completing a table, graph, and the equation based off the situation in the movie. Students will then stop after round 1 and then discuss as a class what they came up with. The goal will be to make sure students know how to properly construct the table, graph, and equation based off the real life situation of an epidemic. The students will resume the lesson activity and complete
round 2 and 3, then complete the follow up questions. Once the class has completed the follow up questions, the students will once again share out the results (48 minutes).

- The students for homework will think of other real life situations that can be modeled by exponential functions.

**E. Assessment**

- Teacher will walk around throughout the majority of this lesson. Through circulation the teacher will be able to assist students and observe the student work. If there are any common misconceptions the teacher will address these when the lesson is paused for class discussion.
- Through the class discussions, the teacher will also be able to observe if students are reaching the lesson objectives.
Lesson 2: Introduction to Exponential Functions Day 2

Name:                                                                                                                   Date:

Round 1: Model the epidemic from the movie *Contagion* in the table, graph, and in an equation. Think back to yesterday’s lesson how an exponential function behave!

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>Days past since the start of outbreak</th>
<th>f(x)</th>
<th>Number of people affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph:

Equation:

*Equation of an Exponential Function*

\[ f(x) = a(b)^x \]

- \(a\) \(\rightarrow\) initial amount
- \(b\) \(\rightarrow\) growth factor

Now you try and create the equation for this example!

\[ f(x) = \]

Now graph it! Does your graph and table on our calculator match the table above?
Round 2: This time let’s say that 5 people started with the infection instead of 2. Create the table, graph, and equation for this situation. Use your technology to assist you.

**Table:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days past since the start of outbreak</td>
<td>Number of people affected</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

**Equation:**

*Equation of an Exponential Function*

\[ f(x) = a(b)^x \]

\[ f(x) = \]
Round 3: This round let us start with 2 people being infected, but the infection triples with each day that passes.

**Table:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>Days past since the start of outbreak</th>
<th>$f(x)$</th>
<th>Number of people affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
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<td>6</td>
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<td>7</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Graph:**

**Equation:**

Equation of an Exponential Function

$$f(x) = a(b)^x$$

Follow-up questions:

1. What do you notice as we change the initial amount of infected people?

2. What do you notice as we change the growth rate?
3. What do you notice about the y-intercept in each problem (make connections between the equation, table, graph)?

4. Using the equation from Round 1, calculate how many people would be infected after 30 days?

5. Using the equation from Round 1, calculate how many people would be infected after 60 days? Is this a reasonable amount?

6. Calculate the average rate of change from day 0 to day 5 for all three rounds of the infection. Which scenario has the greatest rate of change?

**Homework:** Think of another example of a real life situation that can be modeled by an exponential function.
Lesson 3: Exponential Growth and Decay

A. Lesson Overview, Lesson Purpose and Rationale

Lesson Overview: This lesson and homework has been built from emathinstruction curriculum (Unit 6 – Exponents, Exponents, Exponents and More Exponents, n.d.). Students will examine the difference between exponential growth and exponential decay. After completing the notes, the class will then complete another Desmos activity: https://teacher.desmos.com/activitybuilder/custom/5b11a0b985f3a05de9cf6ee5a. In this particular Desmos activity students will create two truths a lie using the knowledge they gained from the first part of the lesson. The students create their own graph of a growth or decay function and then three statements to go along with it. The students will then complete their classmates 2 truths and a lie to further solidify the objectives of this lesson.

Lesson Purpose: Students will gain an understanding of growth and decay exponential functions through two real life examples. Through the examples they will further learn the structure of an exponential function equation to see what makes a growth versus decay equation.

Lesson Rationale: Students now have a basic understanding of exponential functions. They will learn the properties of an exponential growth function versus exponential decay function through real life examples. The Desmos activity will engage the students in practicing this newly learned information through a game like activity.

B. Specific NYS Common Core Learning Standards (CCLS)

- **CCSS.MATH.CONTENT.HSF.BF.A.1**
  Write a function that describes a relationship between two quantities.

- **CCSS.MATH.CONTENT.HSF.IF.B.4**
  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **CCSS.MATH.CONTENT.HSF.IF.A.2**
  Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **CCSS.MATH.CONTENT.HSF.LE.A.1.C**
  Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **CCSS.MATH.CONTENT.HSF.LE.B.5**
  Interpret the parameters in a linear or exponential function in terms of a context.

C. Lesson Objectives

At the conclusion of this lesson, students will be able to:

- Understand an exponential growth and exponential decay functions.
- Understand how to write an equation for an exponential growth and decay function.
- Further understand the graphs of exponential functions.

D. Lesson Design

- Class will begin with students sharing out their homework prompt of coming up with other real life examples of exponential functions. The teacher will then ask the students to think of an example where instead of the amount of something doubling, it is halved.
If students cannot think of an example, prompt them into thinking about science class and half-lives (5 minutes).

- Then as a class the teacher and students will complete exercise 1 and 2 together in which the students will see an example of exponential growth and then exponential decay (25 minutes).
- The students will then transition into the Desmos activity. As a class complete the example together. Then the teacher will demonstrate how to make their own “two truths and a lie”. The teacher will assign half the class to create a growth function and the other half a decay function. The students will then complete their classmates examples to further solidify the lesson objectives (20 minutes).
- Class will end with students starting their homework as the teacher circulates the room (10 minutes).

### E. Assessment

- The beginning of class the teacher will listen to the students’ homework responses where they had to come up with their own examples of exponential functions, this will allow the teacher to see if they have mastered the skills taught in the first two introduction lessons.
- Then as the students and teacher complete exercise 1 and 2, the class discussions will allow the teacher to see if the students are understanding the difference between growth and decay functions.
- The Desmos activity will further allow for teacher observation of student success of identifying features of growth and decay functions.
- The homework that the students will start in class will be used as a formative assessment to assess the lesson objectives in lesson 3.
Lesson 3: Exponential Growth and Decay

**Exercise #1:** The number of people who have heard a rumor often grows exponentially. Consider a rumor that starts with 3 people. Every hour, the number of people who know the rumor doubles.

(a) Fill in the table below for the total number of people, \( N \), who knew the rumor after it has spread a certain number of hours, \( h \).

<table>
<thead>
<tr>
<th>( h )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(h) )</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What operation is happening to the total number of people who knew the rumor as the hours go by?

(c) Graph this situation. Label the y-intercept with its coordinates.

(d) Why are we only graphing in the first quadrant?

(e) Write a formula giving the number of people who know the rumor, \( N(h) \), if you know the number of hours, \( h \), it has been spreading.

(f) What significance do the two values used in (e) have in context of the problem?

(g) Use this to predict how many students will know the rumor after 40 hours.

Is this reasonable?
**Exercise #2:** Jean (from Finland) is heading towards a windmill in a very peculiar way. He starts 160 feet from the windmill. On his first trip he walks half the distance to the windmill. On his next trip he walks half of what is left. On each consecutive trip he walks half of the distance he has left. We are going to model the distance, $D$, that Jean has remaining to the windmill after $n$-trips.

(a) Fill in the table below for the amount of distance that Jean has left after $n$-trips.

<table>
<thead>
<tr>
<th>$n$ (trips)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ (ft)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What do you notice is happening to the distance?

Write this as multiplication instead:

(c) Graph this situation. Label the y-intercept with its coordinates.

(d) Write a function giving the distance Jean has left, $D(n)$, if you know the number of trips, $n$, he has taken.

(e) What significance do the two values you used in (d) have in context of the problem?

(f) When will Jean finally reach the windmill?
Homework:

Match each situation below to its graph.
1. Mary had a bag of 100 apples. Each day, half are taken away. _________
2. Two mice decide they want to start a family. Each day, the family doubles. _________
3. Jenna is addicted to her phone. On Monday, she sent 10 snapchats. Every day, she sends triple the amount of snap chats than the day before. _________
4. At the start of hunting season, the population of deer in one area was 400. Each week, the population decreased by one-third. _________
5. A group of students decided to pay it forward and do good deeds for the community. The function $D = 20(3)^n$ represents the number of student deeds, $D$, for the number of days, $n$. Explain what 20 and 3 represent in the context of this problem.

6. A piece of paper is 0.01 centimeters (cm) thick. When you fold it once, it becomes 0.02 centimeters thick. If you fold it again, it doubles again to 0.04 centimeters thick.

<table>
<thead>
<tr>
<th>$f$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(f)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Graph this situation.

(b) State the coordinates of the $y$-intercept:

(c) Why are we only graphing in the first quadrant?

(d) What do you notice is happening to the thickness?

(e) Write a function giving the thickness of the paper, $T(f)$, if you know the number of folds, $f$.

(f) What significance do 0.01 and 2 have relating back to the original problem?

(g) Using your function, determine how thick the paper would be after 12 folds. Use proper units. Verify on the table on your calculator.
Desmos activity:

**Two Truths and a Lie: Exponentials**

Students will practice their understanding of the features and vocabulary of exponential graphs by creating an exponential curve, writing two true and one false statement about it, and inviting their peers to separate truth from lies.

**Where’s the Lie?**

Here are three statements that Tiana wrote about this exponential.

- A. For very large values of \( x \), the exponential has very large values of \( y \).
- B. The \( y \)-intercept of the exponential is \((0,1)\).
- C. The point \((−2,4)\) is on the exponential.

**Teacher Moves**

The goal of this screen is to familiarize students with the types of questions they’ll create in the upcoming Challenge Creator.

Highlight unique answers for the class. Ask students to justify their responses and critique each others’ reasoning.

Sample Answer: For very large values of \( x \), the exponential has very SMALL values of \( y \).

**Class Gallery**

**Make My Challenge**

Here students will write two truths and a lie about their own exponential, and examine the truth claims made by their classmates.

Encourage students to complete each other’s challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students’ understanding of the mathematics. Highlight those for students and also ask them what they learned from the experience.

We intend for this to be a social and creative experience for students and encourage you to emphasize those virtues whenever you see them in your class.
Lesson 4: Exponential Models and Percents

A. Lesson Overview, Lesson Purpose and Rationale

**Lesson Overview:** In this lesson students will learn about exponential growth and decay problems involving percents. The students will reflect on how they have seen percents when they go shopping, then transition into a real life example of an exponential growth problem, then an exponential decay problem. Students will then complete another Desmos activity, [https://teacher.desmos.com/activitybuilder/custom/5c0dd1a5dbdefd757cc93765](https://teacher.desmos.com/activitybuilder/custom/5c0dd1a5dbdefd757cc93765). Students’ homework contains regents based questions from [jmap.org](http://www.jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI).

**Lesson Purpose:** The purpose of this lesson is for students to be able to build exponential growth and decay functions involving percents.

**Lesson Rationale:** In the previous lesson students learned about growth and decay functions. The students will further their knowledge about growth and decay as they learn how to build functions involving percents. The Desmos activity will be used to engage the students in real world problems involving percents.

B. Specific NYS Common Core Learning Standards (CCLS)

- **CCSS.MATH.CONTENT.HSF.BF.A.1** Write a function that describes a relationship between two quantities.
- **CCSS.MATH.CONTENT.HSF.IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **CCSS.MATH.CONTENT.HSF.LE.A.1.C** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **CCSS.MATH.CONTENT.HSF.LE.A.1.C** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

C. Lesson Objectives

At the conclusion of this lesson, students will be able to:
- Build an exponential growth or decay function involving percents.
- Understand the graph of an exponential growth or decay function involving percents.
- Make predictions and comparisons given more than one exponential function.

D. Lesson Design

- Students will start the lesson by reviewing the structure of exponential growth and decay, then share out their results from exercise 1. From there a class discussion will begin on percents and how they are seen in everyday life, using shopping as an example and then complete exercise 2 as a class (7 minutes).
- The teacher will lead the students through exercise 3 as the student learn how to build the equation of an exponential growth and decay situation involving a percent (7 minutes).
- The students will then work through the Desmos activity. The activity will walk the students through an example involving two prices of cars and their depreciation rates, saving accounts, and population changes. The students will use their knowledge to
make predictions and comparisons in these situations. They will examine the situations through their equation, tables, and graphs (40 minutes).

- The lesson will end with pulling up some of the students’ work from the Desmos activity and discussing the students’ findings (6 minutes).

E. **Assessment**

- The teacher will assess students’ understanding from the previous lesson with exercise 1.
- The teacher will be in circulation while students are completing the Desmos activity and will also be able to view students’ work on the Desmos program. If the teacher views or sees common misconceptions the teacher will pause the class for discussion.
- The homework for this lesson will serve as a formative assessment as well.
- The next class a summative assessment will be given on the unit.
Lesson 4: Exponential Models and Percents

<table>
<thead>
<tr>
<th>Name:</th>
<th>Date:</th>
</tr>
</thead>
</table>

**Exercise #1:** Fill in the table for each of the functions below:

<table>
<thead>
<tr>
<th>Function</th>
<th>Growth or Decay?</th>
<th>“GROWTH” Factor</th>
<th>“Starting Point” (or y-intercept)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 2^x$</td>
<td></td>
<td></td>
<td>(0, 1)</td>
</tr>
<tr>
<td>$y = \left(\frac{1}{3}\right)^x$</td>
<td></td>
<td></td>
<td>(0, 1)</td>
</tr>
<tr>
<td>$y = 4 \left(\frac{3}{4}\right)^x$</td>
<td></td>
<td></td>
<td>(0, 4)</td>
</tr>
<tr>
<td>$y = 3 \left(\frac{3}{2}\right)^x$</td>
<td></td>
<td></td>
<td>(0, 3)</td>
</tr>
<tr>
<td>$y = 4.6 \left(\frac{3}{2}\right)^x$</td>
<td></td>
<td></td>
<td>(0, 4.6)</td>
</tr>
<tr>
<td>$y = 50 \left(\frac{1}{2}\right)^x$</td>
<td></td>
<td></td>
<td>(0, 50)</td>
</tr>
</tbody>
</table>

**Exercise #2:** You are going shopping and you find new pair of shoes that you have to buy. What percent are you actually paying for the shoes if these things happen to the price?
- Decrease by 50%
- Increase by 50%
- Decrease by 20%
- Decrease by 25%
- Decrease by 10%
- Percent growth of 5%
Exercise #3: Examine an example of an exponential growth and then an exponential decay function involving percents.

<table>
<thead>
<tr>
<th>Growth: $f(t) = C (1 + r)^t$</th>
<th>Decay: $f(t) = C (1 - r)^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>You found a beanie baby from your childhood from the year 2000. The beanie baby was bought for $10. The value of the beanie baby since then increased 1% per year.</td>
<td>You purchase a brand new Jeep for $50,000. It is known that Jeeps depreciate at about 10% per year.</td>
</tr>
<tr>
<td>Write an equation that can be used to model the value of the beanie baby, $b(t)$, over $t$ years.</td>
<td>Write an equation to model the price of the Jeep, $P(t)$, over $t$ years.</td>
</tr>
<tr>
<td>According to your equation, how much is your beanie baby worth now in 2019?</td>
<td>If you decide to sell your Jeep in 7 years, how much will it be worth?</td>
</tr>
</tbody>
</table>
Homework:
The current population of a town is 10,000. If the population, $P$, increases by 20% each year, which equation could be used to find the population after $t$ years?

1) $P = 10,000(0.2)^t$

2) $P = 10,000(0.8)^t$

3) $P = 10,000(1.2)^t$

4) $P = 10,000(1.8)^t$

2. The country of Benin in West Africa has a population of 9.05 million people. The population is growing at a rate of 3.1% each year. Which function can be used to find the population 7 years from now?

1) $f(t) = (9.05 \times 10^6)(1 - 0.31)^t$

2) $f(t) = (9.05 \times 10^6)(1 + 0.31)^t$

3) $f(t) = (9.05 \times 10^6)(1 + 0.031)^t$

4) $f(t) = (9.05 \times 10^6)(1 - 0.031)^t$

3. Anne invested $1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) $A = 1000(1 - 0.013)^2$

2) $A = 1000(1 + 0.013)^2$

3) $A = 1000(1 - 1.3)^2$

4) $A = 1000(1 + 1.3)^2$

4. A student invests $500 for 3 years in a savings account that earns 4% interest per year. No further deposits or withdrawals are made during this time. Which statement does not yield the correct balance in the account at the end of 3 years?

1) $500(1.04)^3$

2) $500(1 - .04)^3$

3) $500(1 + .04)(1 + .04)(1 + .04)$

4) $500 + 500(.04) + 520(.04) + 540.8(.04)$
5. Kathy plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. Which equation represents the value, $v$, of the car after 3 years?

1) $v = 21,000(0.14)^3$
2) $v = 21,000(0.86)^3$
3) $v = 21,000(1.14)^3$
4) $v = 21,000(0.86)(3)$

6. The New York Volleyball Association invited 64 teams to compete in a tournament. After each round, half of the teams were eliminated. Which equation represents the number of teams, $t$, that remained in the tournament after $r$ rounds?

1) $t = 64(r)^{0.5}$
2) $t = 64(-0.5)^r$
3) $t = 64(1.5)^r$
4) $t = 64(0.5)^r$

7. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, $y$, of one account that was left inactive for a period of $x$ years. What is the $y$ intercept of this equation and what does it represent?

1) 0.98, the percent of money in the account initially
2) 0.98, the percent of money in the account after $x$ years
3) 5000, the amount of money in the account initially
4) 5000, the amount of money in the account after $x$ years

8. Rhonda deposited $3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find $B$, her account balance after $t$ years.
10. The number of carbon atoms in a fossil is given by the function \( y = 5100(0.95)^x \), where \( x \) represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

11. The value, \( v(t) \), of a car depreciates according to the function \( v(t) = P(0.85)^t \), where \( P \) is the purchase price of the car and \( t \) is the time, in years, since the car was purchased. State the percent that the value of the car decreases by each year. Justify your answer.
Desmos activity:

Exponential Functions and Percent Increase and Decrease
by Wes Overton

Cars lose their value every year that you own it.
Some brands of cars hold their value better than others.
Which car would you choose to buy?

Option A
$60,000 Lexus
16% Depreciation per year

Option B
$45,000 BMW
11% Depreciation per year

Estimate how many years it will be until the value of the BMW is worth more than the value of the Lexus.

<table>
<thead>
<tr>
<th>Type of Guess</th>
<th>Number of Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>No way it will occur this fast</td>
<td></td>
</tr>
<tr>
<td>No way it will take this long</td>
<td></td>
</tr>
<tr>
<td>Actual Estimate</td>
<td></td>
</tr>
</tbody>
</table>
Here is a boxplot of the classes’ estimate.

Which expression would be the most efficient for calculating the value of the $60,000 Lexus after one year?

- 60,000(0.16)
- 60,000 - 60,000(0.16)
- 60,000(0.84)
- 60,000 - 60,000(0.84)

Which expression would be the most efficient for calculating the value of the $45,000 Lexus after one year?

- 45,000(0.11)
- 45,000 - 45,000(0.11)
- 45,000(0.89)
- 45,000 - 45,000(0.89)
Use your expressions from the previous slides to complete the values in the table.

(There is a calculator built in the table)

<table>
<thead>
<tr>
<th>Years of Ownership</th>
<th>Value of the Lexus</th>
<th>Value of the BMW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60,000</td>
<td>45,000</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This screen is still locked

Estimate: Which account will have more money in 10 years?

Account A
Initial Deposit: $5,000
1% Growth each year

Account B
Initial Deposit: $3,000
5% Growth each year
Which expression would be the most efficient for calculating the value of Account A after one year?

- 5,000(0.01)
- 5,000 + 5,000(0.01)
- 5,000(1.01)
- 5,000(1 + 0.01)

Graph the value of both accounts after $x$ years.

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>Account A</th>
<th>Account B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$y$ intercept of function

Given the graph and equation of the BMW and the saving account,
give me one thing you notice about the equations and graphs and one thing you wonder.

Notice:

Wonder:

Share with Class
A city has done some research and found out that their population in 2005 started with 2,000 people and has grown by 2.5% each year since then. Given that this growth percentage remains constant, how many people could we predict would be in the town in 2015?

<table>
<thead>
<tr>
<th>Year</th>
<th>City Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>x years after 2005</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td></td>
</tr>
</tbody>
</table>

1. The study at the Alligator Swamp showed that the number of mosquitoes has tripled each year. In the beginning of the study there were 1,000 mosquitoes. Write an equation that models this growth and predict how many mosquitoes will be there in 5 years if the growth continues?

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>Mosquito Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>5 years</td>
<td></td>
</tr>
</tbody>
</table>
Summative Assessment

Name:

Multiple Choice #1-7

1. Which function is shown in the graph to the right?

   (1) $f(x) = 2(3)^x$
   (2) $f(x) = 3(2)^x$
   (3) $f(x) = 2^x$
   (4) $f(x) = 3^x$

2. Milton had his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where $x$ is the number of years since he made his investment.

   Which statement best describes the rate of change of the value of his portfolio?

   (1) It decreases 78% per year.
   (2) It decreases 22% per year.
   (3) It increases 78% per year.
   (4) It increases 22% per year.

3. Which function below has a y-intercept at $(0, 8)$?

   (1) $y = 8(1.5)^x$
   (2) $y = 5^x + 8$
   (3) $y = 5(8)^x$
   (4) $y = 8^x - 1$

4. Which table of values represents an exponential relationship?

   (1) | $x$ | $f(x)$ |
     |-----|--------|
     | -1  | -15    |
     | 0   | -12    |
     | 1   | -9     |
     | 2   | -6     |
     | 3   | -3     |

   (2) | $x$ | $f(x)$ |
     |-----|--------|
     | -1  | 0      |
     | 0   | 10     |
     | 1   | 20     |
     | 2   | 30     |
     | 3   | 40     |

   (3) | $x$ | $f(x)$ |
     |-----|--------|
     | -1  | 1      |
     | 0   | 0      |
     | 1   | 1      |
     | 2   | 4      |
     | 3   | 9      |

   (4) | $x$ | $f(x)$ |
     |-----|--------|
     | -1  | 0.5    |
     | 0   | 1      |
     | 1   | 2      |
     | 2   | 4      |
     | 3   | 8      |
5. A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

![Graphs of population growth]

6. The table below represents the function F.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(x)</td>
<td>9</td>
<td>17</td>
<td>65</td>
<td>129</td>
<td>257</td>
</tr>
</tbody>
</table>

The equation that represents this function is:

(1) \( F(x) = 3^x \)  
(2) \( F(x) = 3x \)  
(3) \( F(x) = 2^x + 1 \)  
(4) \( F(x) = 3^x - 9 \)

7. Krystal was given $3000 when she turned 2 years old. Her parents invested it at a 2% interest rate compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Krystal had in the account when she turned 18?

(1) \( 3000(1 + 0.02)^{16} \)  
(2) \( 3000(1 - 0.02)^{16} \)  
(3) \( 3000(1 + 0.02)^{18} \)  
(4) \( 3000(1 - 0.02)^{18} \)
Short Response #8-11: Be sure to show all work.

8. Determine which exponential function has a greater average rate of change over the interval \(-1 \leq x \leq 2\).

<table>
<thead>
<tr>
<th>x</th>
<th>a(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
</tr>
</tbody>
</table>

9. The breakdown of a sample of a chemical compound is represented by the function \(p(t) = 300(0.5)^t\), where \(p(t)\) represents the number of milligrams of the substance and \(t\) represents the time, in years. In the function \(p(t)\), explain what 0.5 and 300 represent.

10. Linda deposited $3000 in an account in the Merrick National Bank, earning 3.5% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find \(B\), her account balance after \(t\) years.
11. An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Downloads</td>
<td>120</td>
<td>180</td>
<td>270</td>
<td>405</td>
</tr>
</tbody>
</table>

Write an exponential equation that models this data.

Use the model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download.

Would it be reasonable to use this model to predict the number of downloads past one year? Explain your reasoning.
Feedback

This curriculum was reviewed by a tenured mathematics teacher. This mathematics teacher has many years of experience teaching Algebra 1 and exponential functions. Their comments have been paraphrased below:

- **Lesson 1:** The students needed some time in the beginning to get used to the Desmos activity. The warm-up was a necessary step in helping students be successful with activity in learning about the difference between linear and exponential functions. I think the students really enjoyed using the Desmos activity. It provided them with good visuals and provided a break from direction instruction.

- **Lesson 2:** I liked the movie clip and class discussion to start class. This activity made me think about using examples from the show *The Walking Dead* as a way to show exponential growth. The lesson activity where the students completed the three rounds went well. I walked around during this time and assisted students with their TI-Nspires. We did the follow-up questions as a class. Students needed reminders of using proper function notation in questions #4 and #5 and also needed reminders about average rate of change seen in question #6.

- **Lesson 3:** The beginning problems did a good job at showing the students an example of exponential growth. The students enjoyed the Desmos activity part of the lesson. I needed to give repeated directions on this however since this Desmos activity was not as structured. An idea to help with this would be to make students come up with their problem ahead of time and turn it in for a homework assignment. This way you could review the two truths and a lie and make sure they are all accurate.

- **Lesson 4:** I liked the warm-up, it allowed students to make connections between the third lesson and this one. The discussion about percents the students see while shopping as well as the examples done together in exercise #3 helped the students be successful with the Desmos activity. I could tell the students were more comfortable with Desmos during this lesson. I liked this
Desmos activity because it provided a good visual for the students. I think it helped them make a good connection between the equation and the graph.

The teacher that reviewed these really enjoyed using the Desmos activity. They believed it was a great tool for providing visuals for the students in addition to engaging them in the lesson. One suggestion was to find a different technology tool for lesson 3. This Desmos activity was less structured, so giving the mentioned homework assignment to set students up for being successful would be helpful. These slight complications could be resolved by redesigning this lesson with more emphasis on the research of Thompson (2013). Sometimes lessons involving technology need to be scaffold to the students or the technology is not always necessary piece in engaging students (Thompson, 2013). The teacher also liked how all of the lessons, aside from the homework, all involved real life examples. Overall the lessons went well and did a good job at engaging the students.

**Conclusion**

This curriculum was made to assist in teaching the unit of exponential functions in Algebra 1 to secondary students. The author made the curriculum to help engage the students in today’s digital world by creating lessons that focus on using technology and real life applications.

All the lessons within this curriculum contain using technology either through students’ chromebooks or their TI-Nspire calculators. This technology is necessary for several reasons. It is the way the students in today’s classroom are used to being presented information and it has affected the way they learn and think (Jukes, Crockett, & McCain, 2010, p. 65). The technology used in the lessons also provide visuals of the content which increases students’ learning and retention of the material (Jukes, Crockett, & McCain, 2010). The students also worked between different representations of the exponential functions through using technology helping students
visual and representational fluency. The technology rich lessons all involved problems involving relevant information to the students’ lives which is key to engaging students of this generation (Jukes, Crockett, & McCain, 2010, p. 69). Real life applications is at the heart of Common Core Learning Standards (New York State P-12 Common Core Learning Standards for Mathematics, n. d.) and it is with illustrations and applications that we maximize the information we teach our students to stick (Jukes, Crockett, & McCain, 2010). The author hopes other educators find this curriculum useful in their classrooms filled with students of the digital generation.
References


**Lesson 1: Introduction to Exponential Functions Day 1**

<table>
<thead>
<tr>
<th>Name:</th>
<th>Date:</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Linear</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>- straight line</td>
<td>- curved line</td>
</tr>
<tr>
<td>- $y = mx + b$</td>
<td>- not constant</td>
</tr>
<tr>
<td>- constant rate of change</td>
<td>- multiplication</td>
</tr>
<tr>
<td>- addition</td>
<td></td>
</tr>
</tbody>
</table>
Homework:
One characteristic of all linear functions is that they change by

1) equal factors over equal intervals  
2) unequal factors over equal intervals  
3) equal differences over equal intervals  
4) unequal differences over equal intervals

Which situation could be modeled by using a linear function?

1) a bank account balance that grows at a rate of 5% per year, compounded annually  
3) the cost of cell phone service that charges a base amount plus 20 cents per minute  
2) a population of bacteria that doubles every 4.5 hours  
4) the concentration of medicine in a person’s body that decays by a factor of one-third every hour

Which scenario represents exponential growth?

1) A water tank is filled at a rate of 2 gallons/minute.  
3) A species of fly doubles its population every month during the summer.  
2) A vine grows 6 inches every week.  
4) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

Ian is saving up to buy a new baseball glove. Every month he puts $10 into a jar. Which type of function best models the total amount of money in the jar after a given number of months?

1) linear  
2) exponential  
3) quadratic  
4) square root
Which type of function is graphed to the right?

1) linear  
2) quadratic  
3) exponential  
4) absolute value

A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

1)  
2)  
3)  
4)

11. On January 1, a share of a certain stock cost $180. Each month thereafter, the cost of a share of this stock decreased by one-third. If $x$ represents the time, in months, and $y$ represents the cost of the stock, in dollars, which graph best represents the cost of a share over the following 5 months?

1)  
2)  
3)  
4)
12. Antwaan leaves a cup of hot chocolate on the counter in his kitchen. Which graph is the best representation of the change in temperature of his hot chocolate over time?

1)  

2)  

3)  

4)  

13. The tables below show the values of four different functions for given values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
<th>$x$</th>
<th>$h(x)$</th>
<th>$x$</th>
<th>$k(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>17</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>4</td>
<td>13</td>
<td>4</td>
<td>24</td>
<td>4</td>
<td>28</td>
</tr>
</tbody>
</table>

Which table represents a linear function?  

1) $f(x)$  

2) $g(x)$  

3) $h(x)$  

4) $k(x)$
The function, \( t(x) \), is shown in the table to the right. Determine whether \( t(x) \) is linear or exponential. Explain your answer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( t(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>10</td>
</tr>
<tr>
<td>-1</td>
<td>7.5</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Linear because there is a constant rate of change.

Consider the pattern of squares shown below:

Which type of model, linear or exponential, should be used to determine how many squares are in the \( n \)th pattern? Explain your answer.

Exponential because the number of squares is doubling.
Lesson 2: Introduction to Exponential Functions Day 2

Round 1: Model the epidemic from the movie Contagion in the table, graph, and in an equation. Think back to yesterday’s lesson how an exponential function behave!

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>128</td>
</tr>
<tr>
<td>7</td>
<td>256</td>
</tr>
<tr>
<td>8</td>
<td>512</td>
</tr>
</tbody>
</table>

Equation:

Equation of an Exponential Function

\[ f(x) = a(b)^x \]

a \to initial amount
b \to growth factor

Now you try and create the equation for this example!

\[ f(x) = 2(2)^x \]

Now graph it! Does your graph and table on our calculator match the table above?
Round 2: This time let’s say that 5 people started with the infection instead of 2. Create the table, graph, and equation for this situation. Use your technology to assist you.

Table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days past since the start of outbreak</td>
<td>Number of people affected</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
<tr>
<td>7</td>
<td>640</td>
</tr>
<tr>
<td>8</td>
<td>1280</td>
</tr>
</tbody>
</table>

Equation:

\[
f(x) = a(b)^x\]

\[
f(x) = 5(a)^x\]
Round 3: This round let us start with 2 people being infected, but the infection triples with each day that passes.

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>Days past since the start of outbreak</th>
<th>f(x) Number of people affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>162</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>486</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>1458</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>4374</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>13122</td>
</tr>
</tbody>
</table>

Graph:

Equation:

Equation of an Exponential Function

\[ f(x) = a(b)^x \]

\[ f(x) = 2(3)^x \]

Follow-up questions:

1. What do you notice as we change the initial amount of infected people?
   
   More people were infected than in the previous rounds.

2. What do you notice as we change the growth rate?
   
   This had the greatest impact on the number of people infected. The numbers grew quickly.
3. What do you notice about the y-intercept in each problem (make connections between the equation, table, graph)?

4. Using the equation from Round 1, calculate how many people would be infected after 30 days?

   \[ f(x) = 2(2)^x \]
   
   \[ f(30) = 2(2)^{30} \]
   
   \[ f(30) = 2147483648 \]

5. Using the equation from Round 1, calculate how many people would be infected after 60 days? Is this a reasonable amount?

   \[ f(60) = 2(2)^{60} \]

   \[ f(60) = 2305843009213693952 \]

   No, this number is far too high.

6. Calculate the average rate of change from day 0 to day 5 for all three rounds of the infection. Which scenario has the greatest rate of change?

   \[
   \begin{align*}
   \text{Round 1} & : \frac{2 - 64}{0 - 5} = \frac{-62}{-5} \\
   \text{Round 2} & : \frac{2 - 160}{0 - 5} = \frac{-158}{-5} \\
   \text{Round 3} & : \frac{2 - 4816}{0 - 5} = \frac{-4814}{-5}
   \end{align*}
   \]

Homework: Think of another example of a real life situation that can be modeled by an exponential function.
Lesson 3: Exponential Growth and Decay

**Exercise #1:** The number of people who have heard a rumor often grows exponentially. Consider a rumor that starts with 3 people. Every hour, the number of people who know the rumor **doubles**.

(a) Fill in the table below for the total number of people, \(N\), who knew the rumor after it has spread a certain number of hours, \(h\).

<table>
<thead>
<tr>
<th>(h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N(h))</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
<td>96</td>
<td>192</td>
</tr>
</tbody>
</table>

(b) What operation is happening to the total number of people who knew the rumor as the hours go by? **Multiplying**

(c) Graph this situation. Label the \(y\)-intercept with its coordinates.

(d) Why are we only graphing in the first quadrant? **Because we only want positive values.**

(e) Write a formula giving the number of people who know the rumor, \(N(h)\), if you know the number of hours, \(h\), it has been spreading.

\[
N(h) = 3(2)^h
\]

(f) What significance do the two values used in (e) have in context of the problem? 3 is the initial amount of people who heard the rumor. 2 shows that the amount of people doubles.

(g) Use this to predict how many students will know the rumor after 40 hours.

\[
N(40) = 3(2)^{40}
\]

Is this reasonable?
Exercise #2: Jean (from Finland) is heading towards a windmill in a very peculiar way. He starts 160 feet from the windmill. On his first trip he walks half the distance to the windmill. On his next trip he walks half of what is left. On each consecutive trip he walks half of the distance he has left. We are going to model the distance, \( D \), that Jean has remaining to the windmill after \( n \)-trips.

(g) Fill in the table below for the amount of distance that Jean has left after \( n \)-trips.

<table>
<thead>
<tr>
<th>( n ) (trips)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D ) (ft)</td>
<td>160</td>
<td>80</td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>2.5</td>
<td>1.25</td>
</tr>
</tbody>
</table>

(h) What do you notice is happening to the distance?

It is being halved

Write this as multiplication instead:

\[ \times \frac{1}{2} \]

(i) Graph this situation. Label the y-intercept with its coordinates.

(j) Write a function giving the distance Jean has left, \( D(n) \), if you know the number of trips, \( n \), he has taken.

\[ D(n) = 160 \left( \frac{1}{2} \right)^n \]

(k) What significance do the two values you used in (d) have in context of the problem?

160 is the initial distance and \( \frac{1}{2} \) represents the distance being halved.

(l) When will Jean finally reach the windmill?

He will get very close but never reach.
Homework:

Match each situation below to its graph.
1. Mary had a bag of 100 apples. Each day, half are taken away. ________
2. Two mice decide they want to start a family. Each day, the family doubles. ________
3. Jenna is addicted to her phone. On Monday, she sent 10 snapchats. Every day, she sends triple the amount of snap chats than the day before. ________
4. At the start of hunting season, the population of deer in one area was 400. Each week, the population decreased by one-third. ________
5. A group of students decided to pay it forward and do good deeds for the community. The function \( D = 20(3)^n \) represents the number of student deeds, \( D \), for the number of days, \( n \). Explain what 20 and 3 represent in the context of this problem.

6. A piece of paper is 0.01 centimeters (cm) thick. When you fold it once, it becomes 0.02 centimeters thick. If you fold it again, it doubles again to 0.04 centimeters thick.

<table>
<thead>
<tr>
<th>( f )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(f) )</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
<td>0.32</td>
<td>0.64</td>
</tr>
</tbody>
</table>

(m) Graph this situation.

(n) State the coordinates of the y-intercept:

(o) Why are we only graphing in the first quadrant?

(p) What do you notice is happening to the thickness?

(q) Write a function giving the thickness of the paper, \( T(f) \), if you know the number of folds, \( f \).

\[ T(f) = 0.01 \times 2^f \]

(r) What significance do 0.01 and 2 have relating back to the original problem? 0.01 is the initial thickness, and it doubles with each fold.

(s) Using your function, determine how thick the paper would be after 12 folds. Use proper units. Verify on the table on your calculator.

\[ T(12) = 0.01 \times 2^{12} = 40.96 \text{ cm} \]
Lesson 4: Exponential Models and Percents

<table>
<thead>
<tr>
<th>Exercise #1: Fill in the table for each of the functions below:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function</strong></td>
</tr>
<tr>
<td>$y = 2^x$</td>
</tr>
<tr>
<td>$y = \left(\frac{1}{3}\right)^x$</td>
</tr>
<tr>
<td>$y = 4 \left(\frac{3}{4}\right)^x$</td>
</tr>
<tr>
<td>$y = 3 \left(\frac{3}{2}\right)^x$</td>
</tr>
<tr>
<td>$y = 4.6 \times (1.5)^x$</td>
</tr>
<tr>
<td>$y = 50 \times (0.75)^x$</td>
</tr>
</tbody>
</table>

Exercise #2: You are going shopping and you find new pair of shoes that you have to buy. What percent are you actually paying for the shoes if these things happen to the price?

- Decrease by 50%
- Increase by 50%
- Decrease by 20%
- Decrease by 25%
- Decrease by 10%
- Percent growth of 5%
**Exercise #3:** Examine an example of an exponential growth and then an exponential decay function involving percents.

### Growth:

$$f(t) = C \cdot (1 + r)^t$$

You found a beanie baby from your childhood from the year 2000. The beanie baby was bought for $10. The value of the beanie baby since then increased 1% per year.

Write an equation that can be used to model the value of the beanie baby, $b(t)$, over $t$ years.

$$b(t) = 10 \cdot (1.01)^t$$

According to your equation, how much is your beanie baby worth now in 2019?

$$b(19) = 10 \cdot (1.01)^{19} = 120810 \ldots$$

$1208$

### Decay:

$$f(t) = C \cdot (1 - r)^t$$

You purchase a brand new Jeep for $50,000. It is known that Jeeps depreciate at about 10% per year.

Write an equation to model the price of the Jeep, $P(t)$, over $t$ years.

$$P(t) = 50,000 \cdot (0.90)^t$$

If you decide to sell your Jeep in 7 years, how much will it be worth?

$$P(7) = 50,000 \cdot (0.90)^7$$

$P(7) = 23914.85$

$\$23,914.85$
Homework:
The current population of a town is 10,000. If the population, \( P \), increases by 20% each year, which equation could be used to find the population after \( t \) years?

1) \( P = 10,000(0.2)^t \)
2) \( P = 10,000(0.8)^t \)
3) \( P = 10,000(1.2)^t \)
4) \( P = 10,000(1.8)^t \)

2. The country of Benin in West Africa has a population of 9.05 million people. The population is growing at a rate of 3.1% each year. Which function can be used to find the population 7 years from now?

1) \( f(t) = (9.05 \times 10^6)(1 - 0.31)^t \)
2) \( f(t) = (9.05 \times 10^6)(1 + 0.31)^t \)
3) \( f(t) = (9.05 \times 10^6)(1 + 0.031)^t \)
4) \( f(t) = (9.05 \times 10^6)(1 - 0.031)^t \)

3. Anne invested $1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) \( A = 1000(1 - 0.013)^2 \)
2) \( A = 1000(1 + 0.013)^2 \)
3) \( A = 1000(1 - 1.3)^2 \)
4) \( A = 1000(1 + 1.3)^2 \)

4. A student invests $500 for 3 years in a savings account that earns 4% interest per year. No further deposits or withdrawals are made during this time. Which statement does not yield the correct balance in the account at the end of 3 years?

1) \( 500(1.04)^3 \)
2) \( 500(1 - 0.04)^3 \)
3) \( 500(1 + .04)(1 + .04)(1 + .04) \)

4) \( 500 + 500(.04) + 520(.04) + 540.8(.04) \)

5. Kathy plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. Which equation represents the value, \( v \), of the car after 3 years?
   1) \( v = 21,000(0.14)^3 \)
   2) \( v = 21,000(0.86)^3 \)
   3) \( v = 21,000(1.14)^3 \)
   4) \( v = 21,000(0.86)(3) \)

6. The New York Volleyball Association invited 64 teams to compete in a tournament. After each round, half of the teams were eliminated. Which equation represents the number of teams, \( t \), that remained in the tournament after \( r \) rounds?
   1) \( t = 64(r)^{0.5} \)
   2) \( t = 64(-0.5)^r \)
   3) \( t = 64(1.5)^r \)
   4) \( t = 64(0.5)^r \)

7. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation \( y = 5000(0.98)^x \) represents the value, \( y \), of one account that was left inactive for a period of \( x \) years. What is the \( y \) intercept of this equation and what does it represent?
   1) 0.98, the percent of money in the account initially
   2) 0.98, the percent of money in the account after \( x \) years
   3) 5000, the amount of money in the account initially
   4) 5000, the amount of money in the account after \( x \) years

8. Rhonda deposited $3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find \( B \), her account balance after \( t \) years.

\[
B = 3000(1.042)^t
\]
10. The number of carbon atoms in a fossil is given by the function \( y = 5100(0.95)^x \), where \( x \) represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

5% decrease, the decay rate is 0.95 which represents the carbon atoms losing 5% each year.

11. The value, \( v(t) \) of a car depreciates according to the function \( v(t) = P(0.85)^t \), where \( P \) is the purchase price of the car and \( t \) is the time, in years, since the car was purchased. State the percent that the value of the car decreases by each year. Justify your answer.

15% decrease

\[
0.85 \rightarrow 85\% \\
\frac{100 - 85}{15} = 150% 
\]
Summative Assessment

Name:

Multiple Choice #1-7

1. Which function is shown in the graph to the right?
   - (1) $f(x) = 2(3)^x$
   - (2) $f(x) = 3(2)^x$
   - (3) $f(x) = 2^x$
   - (4) $f(x) = 3^x$

2. Milton had his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where $x$ is the number of years since he made his investment. Which statement best describes the rate of change of the value of his portfolio?
   - (1) It decreases 78% per year.
   - (2) It decreases 22% per year.
   - (3) It increases 78% per year.
   - (4) It increases 22% per year.

3. Which function below has a y-intercept at (0, 8)?
   - (1) $y = 8(1.5)^x$
   - (2) $y = 5^x + 8$
   - (3) $y = 5(8)^x$
   - (4) $y = 8^x - 1$

4. Which table of values represents an exponential relationship?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-15</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-9</td>
<td>1</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
<td>2</td>
<td>30</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>3</td>
<td>40</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

- (1) (2) (3) (4)
5. A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

![Graphs of population growth](image)

6. The table below represents the function $F$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(x)$</td>
<td>9</td>
<td>17</td>
<td>65</td>
<td>129</td>
<td>257</td>
</tr>
</tbody>
</table>

The equation that represents this function is:

- (1) $F(x) = 3^x$
- (2) $F(x) = 3x$
- (3) $F(x) = 2^x + 1$
- (4) $F(x) = 3^x - 9$

7. Krystal was given $3000 when she turned 2 years old. Her parents invested it at a 2% interest rate compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Krystal had in the account when she turned 18?

- (1) $3000(1 + 0.02)^{16}$
- (2) $3000(1 - 0.02)^{16}$
- (3) $3000(1 + 0.02)^{18}$
- (4) $3000(1 - 0.02)^{18}$
Short Response #8-11: Be sure to show all work.

8. Determine which exponential function has a greater average rate of change over the interval \(-1 \leq x \leq 2\).

\[
\frac{4 - 32}{-1 - 2} = \frac{-28}{-3} = \frac{28}{3}
\]

9. The breakdown of a sample of a chemical compound is represented by the function \(p(t) = 300(0.5)^t\), where \(p(t)\) represents the number of milligrams of the substance and \(t\) represents the time, in years. In the function \(p(t)\), explain what 0.5 and 300 represent.

300 represents the initial amount of the chemical compound.
0.5 represents that the chemical compound amount is halved each year.

10. Linda deposited $3000 in an account in the Merrick National Bank, earning 3.5% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find \(B\), her account balance after \(t\) years.

\[
B = 3,000(1.035)^t
\]
11. An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Downloads</td>
<td>120</td>
<td>180</td>
<td>270</td>
<td>405</td>
</tr>
</tbody>
</table>

Write an exponential equation that models this data.

\[ y = 80(1.5)^x \]

Use the model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download.

\[ y = 80(1.5)^{26} \]

\[ y = 3030140.19529 \ldots \]

3,030,140 downloads

Would it be reasonable to use this model to predict the number of downloads past one year? Explain your reasoning.

This model would not be good at predicting downloads for one year because the number of downloads would be far too high.