A Spiraling Curriculum Project on Factoring for Grade 7-Algebra 2

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A Spiraling Curriculum Project on Factoring for Grades 7-Algebra 2
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A thesis submitted to the Department of Education and Human Development
of The College at Brockport, State University of New York,
in partial fulfillment of the requirements for the degree of Master of Science.
<December 2019>
Abstract

This curriculum project was designed in response to the shift to the New York State Next Generation Standards to help teachers implement the updated standards for factoring in grades 7-12. These standards depend on spiraling curricula, but this aspect is often lost in implementation. This curriculum project provides units on factoring spanning grades 7, 8 (Pre-Algebra), Algebra I and Algebra II. This project has an emphasis on Mathematics Practice Standards 7 and 8.
Table of Contents:

Chapter 1: Introduction ................................................................. 4
Chapter 2: Literature Review ......................................................... 5
  The History ........................................................................ 5
  Spiraling Curriculum ............................................................ 6
  Cognitive Load Theory .......................................................... 7
Chapter 3: Curriculum Project ...................................................... 8
  Introduction ....................................................................... 8
  Curriculum ....................................................................... 9
Chapter 4: Validity of Curriculum ................................................ 97
Chapter 5: Conclusion ................................................................. 100
References ........................................................................... 102
Appendix: Unit Plan Answer Keys .............................................. 103
**Introduction**

Factoring of polynomials is one of the highly emphasized topics in the Algebra 1 and Algebra 2 curriculum. Factoring is evident throughout solving quadratics, graphing polynomials, simplifying rational expressions, and within many more topics and is one of the topics students struggle with the most. By implementing a spiraling curriculum and placing an emphasis on student learning factoring may become a more enjoyable topic for students.

The Cognitive Load Theory (CLT) suggests that students are more likely to remember a concept if they are exposed to the concept multiple times. Spiraling curriculum relies heavily on this theory. Spiraling curriculum is a curriculum that constantly revisits topics throughout a period of time. Meaning, spiraling curriculum is one way to provide curriculum and implement the CLT. Learning, as defined by CLT, is a permanent change in long term memory (Van Merrienboer 2005). This would indicate what is learned can be used to learn more mathematics, and this is concept of a spiraling curriculum.

This curriculum project uses intentional spiraling in middle and high school math classes to explore factoring. The implementation of the Common Cores Standards has led to a spiraling effect within the curriculum. Meaning, factoring starts at a young grade level and spirals and becomes more challenging as a student continues through schooling. As a teacher of Math 7, Math 8, Algebra, Geometry, and Algebra 2, it is evident that factoring has a spiral effect throughout all of those subjects. With spiraling curriculum students are able to retain more information over a longer period of time. This is primarily because there are multiple exposures
(over multiple grade levels) and each exposure becomes increasingly difficult in the level of engagement required.

**Literature Review**

The year was 1959 when Jerome Bruner, who believed in the spiraling curriculum, changed the course of Mathematics education through his philosophies on education. “In 1959, Bruner brought together scholars from many academic disciplines—all content experts but no professional educators—at a 10-day conference to focus on redesigning curriculum and thereby redesigning the foundation of American schools” (Gibbs, 2014, p. 42). By having this conference, Bruner paved the way for the spiraling curriculum to begin to be implemented in schools.

“Topics are continually introduced to the curriculum but leave more slowly” (Polikoff, 2012, pg 230). Through such instructional practices there is horizontal and vertical alignment which allows for students to grow as learners, develop confidence with mathematics, and develop perseverance and problem-solving skills. “A properly aligned curriculum helps learners master these generic skills in the most balanced, efficient, and effective manner through the acquisition of mathematical knowledge and concepts (expressed in the form of learning targets)” (Leung et al, 2014, p. 116). Furthermore, the generic skills that help to form a student’s mathematical identify are incredibly important for the motivation of students within a mathematics course. Student self-perception, confidence, attitudes and beliefs, and anxiety are all linked to persistence and motivation to study mathematics (Benken et al., 2015, p. 15).

Spiraling curriculum is what most teachers use within their classrooms and is essentially second nature since the ideology of spiraling is embedded within the Common Core State Standards (CCSS). As of 2019 there were 41 states in the United States (US) that adopted the CCSS. “Content-standards documents in the United States tend to introduce topics at the same
rate as curriculum frameworks from other countries do, but the topics tend to linger in the U.S. curricula for many grade levels” (Polikoff, 2012, p. 231). With each year, unit, and course, students are tackling more challenging problems that revolve around the same content standards that they were previously exposed to. “It is not enough for educators to examine only one point in a students’ academic program; all aspects must be considered together, and alignment across a students’ entire mathematics education is essential (Benken et al., 2015, p. 15).” Spiraling the curriculum means to continually revisit the material. Whereas scaffolding the curriculum means giving students supports within an activity/lesson/worksheet that they will need to be successful. For instance, creating a more open-ended question versus giving some direction with a question. Within a spiraling curriculum, teachers are able to scaffold and scaffold material as necessary based on the needs of the students. Students are able to activate prior knowledge on a subject, which is an important piece in terms of the retention of the concept.

The ideology of a spiraling curriculum applies the concepts of CLT. “Cognitive load theory is premised on the well-established idea that our working memory has limited capacity to process novel information from the environment. It can, however, access extremely large amounts of previously processed and organized information from long-term memory (Baddley, 1992). Cognitive load theory essentially explores the instructional consequences of our limited working memory capacity (Sweller, van Merrienbier, & Paas, 1998)” (Russo, Hopkins, 2017, pg 21). CLT suggests that with repeated practice or repeated exposure to a certain concept, one will commit the concept to memory because it will turn into long term memory. However, this transition only occurs if there is repeated exposure to the content. This is also important because typically in a mathematics classroom, educators tend to move very quickly through the material in order to teach students all of the content prior to a state exam or regents exam. This, however,
presents an issue because “If this permanent change [through repeated exposure] in a student’s long term memory does not occur before they move onto a new unit or mathematical topic, the student will have to repeat the learning process all over again” (Elkins, 2015, p.3).

When examining the Common Core Standards, it is evident that factoring is a standard found within each core level. For instance, 7th grade, Algebra, and Algebra 2. Accessed from http://www.corestandards.org/Math/Practice/ below are the standards on factoring that are throughout the mathematics curriculum and are vertically aligned. At the sixth grade level, the standard is “CCSS.MATH.CONTENT.6.NS.B.4 Find the greatest common factor...” This standard indicates that students simply need to find the greatest common factor of a whole number. This standard is then extended at the seventh grade level to “CCSS.MATH.CONTENT.7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.” This standard allows students to take their knowledge of finding the greatest common factor of whole numbers and apply it to finding the greatest common factor of expressions. For example, if at the sixth grade level students are finding the factor of 36+8, at the seventh grade level students would be finding the factors of (36x+8). Thus, invoking their prior knowledge of the greatest common factor and using it to extend into more rigorous content with the same underlying concepts.” The standard is then taken to algebra with “CCSS.MATH.CONTENT.HSA.SSE.B.3.A Factor a quadratic expression to reveal the zeros of the function it defines.” and extended to geometry where students are expected to factor quadratics to find the measure of angles. At the Algebra 2 level students are expected to factor trinomials and polynomials through the methods of difference of two perfect squares, greatest common factor (gcf), sum and difference of perfect cubes, trinomial factoring, slide and divide,
and factor by grouping. All of their base knowledge with factoring comes from their seventh grade exposure to the concept.

Overall, there was a large change in the mathematics curriculum when spiraling curriculum was viewed as necessary. As evident by the CLT, spiraling curriculum is an important aspect in student retention of the material. By starting factoring in seventh grade, the foundation for factoring in higher level mathematics is set. By continuous re-visitation of factoring students are able to recall their prior information and then expand on it (whether with new types of factoring or with engaging with challenging problems). The Common Core State Standards have set the curriculum up so that there is a natural spiral within it. This should help to improve retention with the constant re-visitation of the standards throughout a student's learning experience.

**Curriculum**

**Introduction:**

All of the lessons within align with the CLT and demonstrate a spiraling curriculum through the grade levels. Each lesson beings with a warm up which is either a practice activity or a question to activate prior knowledge from either a previous grade level or a previous grade. By spiraling the different types of factoring throughout many grade levels, the curriculum is applying the CLT. This is because students are practicing the material not only over multiple days within a unit, but also over multiple years. In addition, this curriculum is scaffolded by starting at a less challenging question and working up to more difficult/ challenging questions.

These lessons have been used, and are designed for other teachers to use, in a Mathematics classroom. The focus of each one is to develop the skills of factoring. Starting at the 7th grade level, students develop the skills necessary to be able to fully factor expressions and
polynomials in the upper level mathematics courses. This knowledge of factoring will lead students into the applications of factoring. Such as, solving quadratic equations and creating polynomial equations based on the zeros of a function.

This curriculum was designed using with a mathematics department in an upstate, NY high school. This school uses eMathinstruction to guide instruction, and has been cited in specific lessons. This curriculum can be modified to meet the needs of each unique classroom by adjusting the warm up questions and homework questions to meet the needs of the students. Furthermore, teachers are able to further differentiate the material in order to increase the level of difficulty of the material. It is at an individual teacher’s discretion to do this because not all classrooms are the same. Each classroom is unique, thus, the adjustments and the connections made from these adjustments will also be unique.
Curriculum:
Keys to the following lesson plans are included in the appendix.

7th grade:

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Lesson Objective</th>
<th>Connection to Spiraling Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The Distributive Property</td>
<td>Students will be able to apply the distributive property in order to expand and simplify expressions.</td>
<td>This lesson is important to the spiraling curriculum because it is creating the foundation that students will use in order to check whether or not they factored an expression/equation correctly.</td>
</tr>
<tr>
<td>2</td>
<td>Factoring Expressions</td>
<td>Students will be able to determine the greatest common factor and use this in order to factor an expression. Students will identify the structure of an expression and rewrite it in an equivalent form.</td>
<td>Identifying the greatest common factor is an essential part of factoring in Algebra, and Algebra 2. This is the base that students will develop in order to start their foundational knowledge of factoring.</td>
</tr>
</tbody>
</table>

Common Core Standards:
CCSS.MATH.CONTENT.7.EE.A.1
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure
Lesson 1:

Warm Up:
Complete the following operations:
1.) 8(3)  
2.) 8(3x)  
3.) 3x(4)

Notes:

**Distributing Expressions**

The Distributive Property:

Ex: Expand 8(3x + 4)

Visualize it:

Ex: Expand 2(4x − 2)

Visualize It:
Ex: $8(9x - 2y)$  

Ex: $10(3x + 2)$

Ex: $\frac{1}{2}(4x + 8y)$  

Ex: $-5(3x - 4)$
Homework:

1.) $9(-4p - y)$

2.) $-4(6n - 7)$

3.) $5(n + 6)$

4.) $-5(10x + 1)$

5.) $8(7z - 6s + 6)$

6.) $-7(-q - 9 - 2p)$

7.) $3(-5 + 11p)$

8.) $-6(-4a + w)$
9.) \(9(-6d + 5 - 3y)\)  
10.) \(-3(6x + 2)\)

11.) Write an expression to represent the product of 3 and \(\frac{5}{4}n + 1.8\)
Connection to Spiraling Curriculum:

By having students work on the distributive property in 7th grade, one is teaching students a method that they will be able to use in order to check their factoring work. In addition, it is the start to teaching the “box method” which in turn will be beneficial in helping students to visualize what they are factoring. Furthermore, by starting to introduce the box method now, students will find it easier when they are asked to multiply polynomials at the pre-algebra level and up. The box method when distributing will also be beneficial when having students factor quadratic expressions when the leading coefficient is a number other than 1.
Lesson 2
Warm Up:

Simplify the following expressions:
1.) 8(7x+4)  
2.) -8(x-5)

Notes:

Factoring Expressions

Factoring:

Ex: Determine the greatest common factor between each of the given numbers:
a.) 36 and 60  
b.) 40 and 100

c.) 54 and 144  
d.) 75x and 90

In order to factor expressions, we must first find the ____________________________ of all terms in the expression.

Then, we _______________ each term in the expression by this number.

We write the __________________ on the outside of a set of parenthesis and put the quotient(s) inside the parenthesis.

We can always check if we factored correctly by using the
______________________________.
Ex: Factor $4x + 30$  

Check:

Visualize it:

Ex: Factor $15x + 40$  

Check:

Visualize It:

Ex: Factor $4x + 16$  

Check:

Ex: Factor $9x + 12$  

Check:
Ex: Factor $3x^2 + 6x + 12$  
Check:

Ex: Factor $5x^2 + 10x + 30$  
Check
Homework

Directions: Factor and check each of the following expressions.

1.) $6x + 12$                               2.) $-3x - 18$

3.) $10x + 15$                               4.) $-5x + 15$

5.) $12x - 24$                               6.) $-16x + 48$

7.) $2x + 8$                                 8.) $4x - 38$

5.) $2x^2 + 10x + 12$                       6.) $3x^2 + 6x - 12$

7.) $-5x^2 - 75x + 95$                      8.) $10x^2 + 15x + 20$
Connection to Spiraling Curriculum:

This lesson starts out by asking students to identify the greatest common factor between two numbers. Thus evoking their prior knowledge from the 6th grade curriculum, where students are taught how to find the greatest common factor between two numbers. By starting out the lesson this way, it is giving all students an entry point into the 7th grade material and implementing the Cognitive Load Theory by having students work through the continued practice. Furthermore, it is a stepping stone into finding the greatest common factor in an expression rather than between two numbers. In the visualize it portion of the notes, students are able to make a connection to the lesson from the previous day in which they were asked to simplify expressions using the distributive property. They are also able to make connections from the previous lesson by checking their work in order to make sure that they factored correctly.
### Pre-Algebra:

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Lesson Objective</th>
<th>Connection to Spiraling Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Factoring a GCF</td>
<td>Students will be able to identify the GCF of multiple terms. Students will be able to factor a polynomial with the use of a GCF</td>
<td>This lesson ties to prior knowledge from 7th grade. In the 7th grade lessons students were first introduced to the GCF of two different numbers. In addition, students were asked to identify the GCF of a binomial. This lesson ignites this prior knowledge by asking students to do the same thing, but with more challenging terms.</td>
</tr>
<tr>
<td>2</td>
<td>Difference of Two Perfect Squares</td>
<td>Students will be able to examine a binomial and determine if it can be restructured using the difference of perfect squares model. Students will be able to factor using difference of two perfect squares.</td>
<td>This lesson ties to prior knowledge from 8th grade because students will need to evaluate their perfect squares and identify if terms are perfect squares.</td>
</tr>
<tr>
<td>3</td>
<td>Difference of Two Perfect Squares Day 2</td>
<td>Students will be able to examine a binomial and determine if it can be restructured using the difference of perfect squares model. Students will be able to factor using difference of two perfect squares.</td>
<td>This lesson ties to prior knowledge from 8th grade because students will need to evaluate their perfect squares and identify if terms are perfect squares.</td>
</tr>
<tr>
<td>4</td>
<td>Trinomial Factoring</td>
<td>Students will be able to identify factors of a trinomial and use these factors to restructure the trinomial in a different form. Students will be able to check their factoring by multiplying the factors together.</td>
<td>This lessons ties to prior knowledge from elementary year by asking students to think about factors of the ending term in a trinomial and determining which factors work to add to the b value in the trinomial. Which ties to the students’ knowledge of multiplication.</td>
</tr>
<tr>
<td>5</td>
<td>Trinomial Factoring Day 2</td>
<td>Students will be able to identify factors of a trinomial and use these factors to restructure the trinomial in a different form. Students will be able to check their factoring by multiplying the factors together.</td>
<td>This lessons ties to prior knowledge by asking students to think about factors of the ending term in a trinomial and determining which factors work to add to the b value in the trinomial.</td>
</tr>
<tr>
<td>6</td>
<td>Putting it Together</td>
<td>Students will be able to distinguish between situations that require GCF, difference of two perfect squares, and trinomial factoring. Students will apply their knowledge by factoring various types of polynomials</td>
<td>The focus of this lesson is to take all of the knowledge gained in the five days prior to this lesson and put it together for students to distinguish and decide when to use specific types of factoring.</td>
</tr>
<tr>
<td>7</td>
<td>Factoring Trinomials when a&gt;1 Method 1</td>
<td>Students will be able to identify factors of trinomials with a leading coefficient other than 1. Students will rewrite a trinomial with a different structure utilizing the factors.</td>
<td>This concept is new to students but it is the basis for factoring by grouping.</td>
</tr>
<tr>
<td>Day 8</td>
<td>Factoring Trinomials when a &gt; 1 Method 2</td>
<td>Students will be able to identify factors of trinomials with a leading coefficient other than 1. Students will rewrite a trinomial with a different structure utilizing the factors.</td>
<td>This concept is new to students but uses the prior knowledge of factoring typical trinomials with a = 1.</td>
</tr>
<tr>
<td>Day 9</td>
<td>Factoring Completely Day 1</td>
<td>Students will factor polynomial expressions using one or many factoring methods.</td>
<td>This lesson takes all of the prior knowledge for this unit and combines them into questions where multiple forms of factoring may be required.</td>
</tr>
<tr>
<td>Day 10</td>
<td>Factoring Completely Day 2</td>
<td>Students will factor polynomial expressions using one or many factoring methods.</td>
<td>This lesson takes all of the prior knowledge for this unit and combines them into questions where multiple forms of factoring may be required.</td>
</tr>
<tr>
<td>Day 11</td>
<td>Factoring Completely Day 3</td>
<td>Students will factor polynomial expressions using one or many factoring methods.</td>
<td>This lesson takes all of the prior knowledge for this unit and combines them into questions where multiple forms of factoring may be required.</td>
</tr>
</tbody>
</table>

Common Core Standards:

- **CCSS.MATH.PRACTICE.MP7** Look for and make use of structure

- **CCSS.MATH.CONTENT.HS.A.SSE.A.2**
  Use the structure of an expression to identify ways to rewrite it.

- **CCSS.MATH.CONTENT.HS.A.SSE.B.3**
  Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

- **CCSS.MATH.CONTENT.HSF.IF.C.8**
  Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
Lesson 1: Factoring a GCF

**Warm Up:** Find the GCF for each of the following:

1.) 6 & 18  
2.) 9 & 15  
3.) 24 & 36  
4.) 25 & 10

*Divide each of the following. Remember when dividing, _______________ exponents.*

5.) \( \frac{8y^2}{4y} \)  
6.) \( \frac{25x^2y}{5x^2y} \)  
7.) \( \frac{24x^5}{36x^2} \)

*Find the GCF of each set of monomials.*

8.) 12x, 48y  
9.) 60r^2s^4, 30 r^3s

If the variables are different, the GCF is ______________________________.  
If the variables are the same the GCF is ______________________________.

*Factor the following polynomials using Greatest Common Factor.*

1.) _______________________________  
2.) _______________________________  
3.) _______________________________
10.) 14m + 35n

GCF: ____________

Factored Form: __________________________

11.) 4x^4 + 24x^3

GCF: ____________

Factored Form: __________________________

12.) 20x^3 + 35x^2 – 12x

GCF: ____________

Factored Form: __________________________

* You can check your answers by using the __________________________!!
**Connection to Spiraling Curriculum:**

This lesson demonstrates spiraling curriculum because students are asked to pull their prior knowledge from 7th grade in order to factor using the greatest common factor method.

Furthermore, the lesson is scaffolded because students are first asked to recall how to find the greatest common factor (GCF) and then this is taken further into dividing monomials, which is part of the factoring process. In addition, students are asked to practice the distributive property in order to check their responses.
Lesson 2:

Warm Up:

1.) Distribute
   \[3x(x^2 + 7x - 9)\]

2.) Factor
   \[4x^2 + 6x\]

Multiply:

1.) \((x + 3)(x - 3)\)

2.) \((b + 5)(b - 5)\)

3.) \((y + 8)(y - 8)\)

4.) \((a + 7)(a - 7)\)

What do you notice happens to the middle term in both examples?

Factoring the difference of second Perfect Squares goes in the opposite direction.

Perfect Square:

Name the first 20 perfect squares.

Does anything multiply itself to give us \(x^2\)?

Does anything multiply by itself to give us \(y^2\)?

Does anything multiple by itself to give us \(a^2\)?
In order to factor by difference of perfect squares, you must have:

1.) ______________________

2.) ______________________

3.) ______________________

Then, to factor you ______________________

Factor:

1.) \( x^2 - 25 \) \hspace{2cm} 2.) \( y^2 - 49 \)

3.) \( d^2 - 225 \) \hspace{2cm} 4.) \( x^2 - 169 \)
Factor by difference of perfect squares. Check by distributing

1.) $a^2 - 324$

2.) $x^2 - 64$

3.) $y^2 - 100$

4.) $b^2 - 196$
Lesson 3:
Warm Up:
Factor the following expressions:

1.) $4x^2 + 24x + 6$

2.) $x^2 - 64$

Difference of Perfect Squares Day 2

WE know that $x \cdot x = x^2$ and $y \cdot y = y^2$, but is there any way to multiply a variable by itself to give us $x^3$? What about $x^4$?

Variables are only perfect square if they have _______________ exponents.

List some examples here:

Can you factor $4x^2 - 81$?

Is there a subtraction sign? Is there an even number exponent on the variable? Are all the numbers perfect squares?

$4x^2 - 81 = (2x + 9)(2x - 9)$

What about $16a^2 - 49$?
Classwork, factor and check:

1.) \(9x^2 - 100\)  
2.) \(y^4 - 361\)  
3.) \(4x^6 - 36\)

4.) \(16x^2 - 64y^2\)  
5.) \(x^8 - 144\)  
6.) \(\frac{1}{4}x^2 - 289\)
Homework

Factor by difference of perfect squares. Check by distributing

1.) \(4y^6 - 9\)

2.) \(16x^2 - 25y^4\)

3.) \(49b^8 - 256a^2\)

4.) \(9x^2 - 1024y^2\)
Connection to spiraling curriculum for lesson 2 and 3:

This is one concept looked at over the course of two days. By splitting this concept, the curriculum is implementing the Cognitive Load Theory and considering that students learn material best when information is chunked. Students have to recall their prior knowledge of perfect squares in order to implement difference of perfect squares factor. In addition, the second day of the concept (lesson 3) looks at more challenging questions than the first day (lesson 2) does, which is a demonstration of scaffolding within the lesson.
Lesson 4:
Warm Up:
Factor the following expressions and check your answers:
1.) \(x^2 - 16\)  
2.) \(25z^2 - 64\)  
3.) \(3x^3 - 15x^2 - 36x\)

Factoring Trinomials Round 1
Factoring Trinomials is just like reversing the box or distributing methods we did with pairs of binomials.

Multiply each of the following using whatever method you choose.
1.) \((x + 2)(x + 7)\)  
2.) \((x - 2)(x - 7)\)  
3.) \((x + 2)(x - 7)\)  
4.) \((x - 2)(x + 7)\)

*What similarities do you notice between the answers to these examples?*

*What is the only difference?*

*Are there similarities between these examples?*
How to Factor Trinomials

Step 1: __________________________________________

Step 2: __________________________________________

If last sign is _____________ the signs are _______________________

Signs are the same as that of the _______________________________

If last sign is __________________, the signs are _________________

Step 3:

______________________________________________________________________________

Step 4: ________________________________________________

Factor and check each of the following trinomials

5.) \( x^2 + 5x + 6 \) 6.) \( x^2 - 6x + 8 \)

7.) \( x^2 + 2x - 3 \) 8.) \( x^2 - 3x - 28 \)
Homework

Factor each of the following trinomials. Don’t forget to check your answers!

1.) \(x^2 + 13x - 30\)

2.) \(x^2 - 4x - 12\)

3.) \(x^2 + 13x + 22\)

4.) \(x^2 - 15x + 44\)
Lesson 5:

Warm Up:
Factor the following and check:
1.) \( x^2 - 36 \)  
2.) \( 25x^2 - 121 \)

Factoring Trinomials Round 2
Factor the following trinomials. Check your answers using the distribution method.

1.) \( a^2 - 2a - 15 \)  
2.) \( x^2 + 10x + 25 \)

3.) \( y^2 - 10y + 24 \)  
4.) \( x^2 + 3x - 28 \)

5.) \( x^2 - 6x + 9 \)  
6.) \( x^2 + 5x - 24 \)
Connection to Spiraling Curriculum lessons 4 and 5:
By looking at the material for factoring a trinomial over the course of two days, students are receiving the information in a chunked form. Meaning, the CLT is represented within these two lessons. Furthermore, the spiraling aspect of the curriculum is represented because the students are looking at the material over various days. The lessons are also scaffolded which is represented by the various types of questioning used throughout. The second day of trinomial factoring (lesson 5) is meant for students to practice on their own as a way to review and reiterate the material.
Lesson 6:

When do I use GCF, DOS or Trinomial Factoring??

Directions:

1.) ________________________________

__________________________________

__________________________________

__________________________________

2.) ________________________________

3.) ________________________________

1.) 3x + 3

Type of Factoring: ____________________

Step 1: I found the ________________

Step 2: I made one ________________

Step 3: I then _________ each term

by ____________________

2.) 121 – 4x^2

Type of Factoring: ____________________

Step 1: I found the ________________

Step 2: Then I found what ________________

Step 3: Then I found what ________________

The signs are _____ and ________

3.) x^2 – 6x - 16

Type of Factoring: ____________________

Step 1: First I made ________________

Step 2: Then I determine the signs by

__________________________

Step 3: I chose the numbers by

__________________________
Factor a GCF

1.) \(3a + 3b\)  
2.) \(25c - 30d\)

3.) \(y^3 + 5y\)  
4.) \(3x^3 + 18x^2 + 9x\)

Factor using Difference of 2 Perfect Squares

5.) \(x^2 - 16\)  
6.) \(m^2 - n^2\)

7.) \(49x^2 - 121\)  
8.) \(x^4 - 0.09y^2\)
Factor each trinomial

9.) \( x^2 - 5x + 6 \)  
10.) \( x^2 + 8x + 16 \)

11.) \( x^2 - 19x - 42 \)  
12.) \( x^2 -3x - 18 \)
Homework

Factor each of the following trinomials using either GCF, DOS or trinomials.

1.) $12y - 6$  
2.) $64 - x^2$

3.) $6x^3 + 18x$  
4.) $x^2 + 10x + 24$

5.) $x^2 - 14x - 32$  
6.) $49x^2 - 121$
**Connection to Spiraling Curriculum:**

Lesson 6 is a review of all of the types of factoring methods that have been represented up until that point. As a result, all of the material has been spiraled into this lesson. By allowing students to practice what they have learned rather than moving on to the next topic, students are able to fully grasp the material and convert it to their long term memory due to the continued practice. Meaning, the CLT is further implemented here.
Lesson 7:
Warm Up:
Factor the following expressions:
1.) \( x^2 - 5x + 6 \)  
2.) \( x^2 - x - 12 \)

Factoring Trinomials with Coefficient other than 1 (Day 1)- Method 1

1.) \( 3x^2 - 10x - 8 \) (from the video)

Try some more…. 

2.) \( 2x^2 - 11x + 5 \) 
3.) \( 3x^2 - 5x + 2 \)

4.) \( 5n^2 + 2n - 3 \) 
5.) \( 2m^2 + m - 21 \)
Lesson 8:
Warm Up:
Factor the following expressions:

1.) \(x^2 - 25\)  
2.) \(3x^2 - 5x - 6\)

Factoring Trinomials with Coefficient other than 1 Method 2

1.) \(6x^2 - 2x - 4\)
2.) \(2x^2 + 5x + 2\)
3.) \(3x^2 + 10x - 8\)
4.) \(4x^2 + 15x + 4\)
5.) \(3x^2 - 14x + 8\)
Connection to Spiraling Curriculum lessons 7 and 8:

Lessons 7 and 8 look at two different ways to factor trinomials when the leading coefficient is a number other than one. By looking at two different methods, one is giving students choice in the method that they use. Each method has a spiraled piece focused in on it. The method implements in lesson 7, which is referred to in my classroom as the reverse box method, utilizes the ideology of the greatest common factor method. As a result, students are still practicing with the greatest common factor. The method implemented in lesson 8, often referred to as slide and divide, focuses on regular trinomial factoring, which results in continued practice of that concept. With the practice over the multiple days, the spiraling of the curriculum is in effect. Furthermore, by looking at the different methods over the course of two days, rather than one, the material is chunked, which is relating to the CLT.
Lesson 9

Factoring completely!

Complete factoring means that you must factor __________________________!

Step 1: __________________________________________

Step 2: __________________________________________

Step 3: __________________________________________

Step 4: __________________________________________

_____________________________________________________________________

Factor each of the following completely (MORE THAN ONCE!!)

1.) by² - 4b  
2.) 3x² - 6x - 24

3.) 4d² - 6d + 2  
4.) 18y² - 32

5.) 8x² - 98  
6.) x⁴ - 1
Homework

*Factor each polynomial completely.*

1.) $4x^2 - 4$
2.) $d^3 - 8d^2 + 16d$

3.) $16x^2 - 16x + 4$
4.) $ax^2 - ay^2$

5.) $x^4 - 16$
Lesson 10:

Factoring completely ~ Here we go….AGAIN!!

Complete factoring means that you must factor __________________________! 

Step 1: ____________________________________________________________

Step 2: ____________________________________________________________

Step 3: ____________________________________________________________

Factor each of the following completely (MORE THAN ONCE!!)

1.) $18m^2 - 8$ 
2.) $3x^2 + 6x + 3$

3.) $x^3 + 7x^2 + 10x$ 
4.) $4r^2 - 4r - 48$

5.) $y^4 - 81$ 
6.) $16x^2 - x^2y^4$
Homework

Factor completely.

1. $2a^2 - 2b^2$

2. $ax^2 - ay^2$

3. $3x^2 + 6x + 3$
Connection to Spiraling Curriculum lessons 9 and 10:
Lessons 9 and 10 look at factoring completely. By examining the concept over the course of two days, the students are practicing the material over multiple days. Thus allowing them the opportunity to convert their knowledge from short term memory to long term memory (The CLT) which is possible because the content is spiraled.
Lesson 11:

Factor the following:

1) $10x^2 + 15x$  

2) $24x^2y^3 - 30x^2y$

3) $x^2 - 36$  

4) $121 - y^2$

5) $49y^2 - 1$  

6) $64x^2 - 9y^2$

7) $x^2 - 12x + 35$  

8) $y^2 - 5y - 24$

9) $8x^2 - 2x - 1$  

10) $x^2 + 8x + 12$

11) $x^2 + 6x - 27$  

12) $9x^2 + 18x + 5$

Always look for a GCF first!!!! You might be able to factor again. Keep factoring until you can’t factor anymore!
13) $4x^2 + 32x + 28$  
14) $3x^2 - 27$

15) $5y^2 - 30y + 40$  
16) $x^4 - 16$

17) $4x^3 + 10x^2 + 6x$  
18) $6x^2 + 18x - 60$
Connection to Spiraling Curriculum:
This lesson is meant to be a review of all of the prior lessons. By having this review students are able to receive extended exposure to the content they have learned and the objectives that they have been expected to obtain. With this multiple exposure technique, students are able to convert their understandings from short term memory to long term memory, which is a direct application of the Cognitive Load Theory. In addition, with the multiple practice exposure, the spiraling curriculum ideology is directly implemented.
Algebra:

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Lesson Objective</th>
<th>Connection to Spiraling Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Factoring a GCF</td>
<td>Students will be able to identify the GCF of multiple terms. Students will be able to factor a polynomial with the use of a GCF</td>
<td>This lesson ties to prior knowledge from 7th grade. In the 7th grade lessons student were first introduced to the GCF of two different numbers. In addition, students were asked to identify the GCF of a binomial. This lesson ignites this prior knowledge by asking students to do the same thing, but with more challenging terms.</td>
</tr>
<tr>
<td>2</td>
<td>Difference of Two Perfect Squares</td>
<td>Students will be able to examine a binomial and determine if it can be restructured using the difference of perfect squares model. Students will be able to factor using difference of two perfect squares.</td>
<td>This lesson ties to prior knowledge from 8th grade because students will need to evaluate their perfect squares and identify if terms are perfect squares.</td>
</tr>
<tr>
<td>3</td>
<td>Trinomial Factoring</td>
<td>Students will be able to identify factors of a trinomial and use these factors to restructure the trinomial in a different form. Students will be able to check their factoring by multiplying the factors together.</td>
<td>This lesson ties to prior knowledge from pre-algebra by asking students to think about factors of the ending term in a trinomial and determining which factors work to add to the b value in the trinomial. This also related to their elementary school years by having knowledge of their multiplication facts.</td>
</tr>
<tr>
<td>4</td>
<td>Factoring Trinomials when a&gt;1 “Slide and Divide”</td>
<td>Students will be able to identify factors of trinomials with a leading coefficient other than 1. Students will rewrite a trinomial with a different structure utilizing the factors.</td>
<td>This concept is new to students but uses the prior knowledge of factoring typical trinomials with a=1.</td>
</tr>
<tr>
<td>5</td>
<td>Factoring Completely Day 1</td>
<td>Students will factor polynomial expressions using one or many factoring methods.</td>
<td>This lesson takes all of the prior knowledge for this unit and combines them into questions where multiple forms of factoring may be required.</td>
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</tbody>
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Common Core Standards:

**CCSS.MATH.PRACTICE.MP7 Look for and make use of structure**

**CCSS.MATH.CONTENT.HSA.SSE.A.2**
Use the structure of an expression to identify ways to rewrite it.

**CCSS.MATH.CONTENT.HSA.SSE.B.3**
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

**CCSS.MATH.CONTENT.HSF.IF.C.8**
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
Lesson 1:

Finding a Greatest Common Factor (GCF)

When two numbers are multiplied, the result is called their.

The numbers that are multiplied are called _______________________.

Example: List all the factors of 80

GCF Factoring

What does GCF stand for?

Example: Identify the GCF of each pair of numbers below.

a) 12 and 18  
   b) 20 and 60  
   c) 36 and 60

Example: Identify the GCF of each pair of monomials below.

a) $x^5$ and $x^3$  
   b) $5a^3$ and $15a$  
   c) $8xy$ and $6xz$

Practice: Identify the GCF of each pair of monomials below.

a) $10x^2$ and $15xy^2$  
   b) $25xyz$ and $36xyz$  
   c) $21ab$ and $14a^2b$
To factor a polynomial using a GCF:

1. ________________________________________________
2. ________________________________________________
3. ____________________________________________________________________

Ex. 1. Factor $6x^2 + 8$ Check

Ex. 2. Factor $5x^2 + 25x$ Check

Ex. 3. Factor $ax - 5ab$ Check

Ex. 4. Factor $9x^3 + 36x^5$ Check

Ex. 5. Factor $6c^3d - 12c^2d^2 + 3cd$ Check
Practice: Factor the expression.

1. $27x^6 + 9x$
2. $15x^4 - 10x^2$
3. $3a^5b^3 - 12a^2b^4$

4. $p + prt$
5. $8z + 8$
6. $c^3 - c^2 + 2c$

7. The perimeter of a rectangle is represented by $2l + 2w$. Express the perimeter as the product of two factors.
Homework

1. Identify the greatest common factor for each of the following sets of monomials.
   (a) $2x^3, 6x^2, \text{ and } 12x$  
   (b) $16t^2, 48t, \text{ and } 80$  
   (c) $8t^5, 12t^3, \text{ and } 16t$

2. Which of the following is the greatest common factor of the terms $36x^2y^4$ and $24xy^7$?
   (1) $12xy^4$  
   (2) $24x^2y^7$  
   (3) $6x^2y^3$  
   (4) $3xy$

3. Write each of the following as equivalent products of the polynomial’s greatest common factor with another polynomial (of the same number of terms). The first is done as an example.
   (a) $18 - 12x$  
   (b) $6x^3 + 12x^2 - 3x$  
   (c) $x^2 - x$
   (d) $10x^2 + 35x - 20$  
   (e) $21x^3 - 14x$  
   (f) $36x - 8x^2$

4. Which of the following is not a correct factorization of the binomial $10x^2 + 40x$?
   (1) $10x(x + 4)$  
   (2) $10(x^2 + 4x)$  
   (3) $5x(2x + 4)$  
   (4) $5x(2x + 8)$

5. The area of a rectangle is represented by the polynomial $16x^2 + 56x$. The width of the rectangle is given by the binomial $2x + 7$.
   a.) What is the length of the rectangle?
   b.) If the length is 80, what is the perimeter of the rectangle?
7.) Which of the following is not a factor of \(4x^2 + 12x\)?

(1) \(x + 3\) \hspace{2cm} (3) \(3x\)

(2) \(x\) \hspace{2cm} (4) \(4\)
**Connection to Spiraling Curriculum:**

This lesson demonstrates spiraling curriculum because students are asked to pull their prior knowledge from 7th grade and exposure in Pre-Algebra in order to factor using the greatest common factor method. Students are asked to practice the distributive property in order to check their responses which is also evoking prior knowledge. With this continued practice over multiple grade levels, the spiraling curriculum ideology is directly implemented.

Note: This lesson was adapted from Kirk Weiler’s eMathinstruction.
Lesson 2:

Factoring a Difference of Perfect Squares

Review: Squaring the following monomials.

1. \((3x)^2\)  2. \((7xy^2)^2\)  3. \(\left(\frac{2}{5}ab\right)^2\)  4. \((9x^3)^2\)

What are “perfect square numbers” and why are they called that?

<table>
<thead>
<tr>
<th>Number</th>
<th>Perfect Square</th>
<th>Number</th>
<th>Perfect Square</th>
<th>Number</th>
<th>Perfect Square</th>
<th>Term</th>
<th>Perfect Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>10</td>
<td>3b</td>
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<tr>
<td>3</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>11</td>
<td>11</td>
<td>c^2</td>
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</tr>
<tr>
<td>4</td>
<td>16</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>12</td>
<td>x^3</td>
<td>x^3</td>
</tr>
</tbody>
</table>

What do you notice about the exponents when monomials are squared?

Review: Multiply the following binomials.

1. \((x - 3)(x + 3)\)  2. \((2x - 5)(2x + 5)\)  3. \((a^2 - 6b)(a^2 + 6b)\)
In order to use the Difference of Perfect Squares factoring method we must have:

1. ________________________________________________
2. ________________________________________________
3. ________________________________________________

Ex. 1. \( m^2 - 4 \) Check

Ex. 2. \( a^6 - 81 \) Check

Ex. 3. \( 9x^2 - 25z^4 \) Check

Practice: Factor.

1. \( x^2 - 100 \) 2. \( a^2 - b^2 \)
3. \( 49 - y^4 \) 4. \( 25r^2 - 64 \)
5. $4x^2 - 81y^2$

6. $4x^4 - y^6$

7. $x^2 - 16$

8. $x^2 - 100$

9. $x^2 - 1$

10. $x^2 - 25$

11. $4 - x^2$

12. $9 - x^2$

13. $4x^2 - 1$

14. $16x^2 - 49$

15. $1 - 25x^2$

16. $x^2 - 9y^2$

17. $81 - 4t^2$

18. $x^4 - 36$
Connection to Spiraling Curriculum:
Within this lesson students are asked to recall information from 8th grade, focusing on their knowledge of perfect squares. In addition, it is connected to the pre-algebra curriculum because students had that prior practice of difference of two perfect squares factoring. Furthermore, by having students practice multiplying two binomials (that result in a difference of perfect squares) they are able to visually see the connection prior to the actual factoring portion of the notes. With this continued practice of the skill over the course of multiple years, there is a direct connection to the Spiraling Curriculum and the CLT. This is due to the fact that the students are receiving the multiple practice exposure and the material is getting more challenging as time goes on.
Lesson 3:  

Factoring a Trinomial

To factor a trinomial of the form $x^2 + bx + c$ using the “unbox” method:

1. ________________________________
2. ________________________________
3. ________________________________

Ex. 1. Factor $x^2 + 6x + 5$

Ex. 2. Factor $x^2 - 3x - 10$

Ex. 3. Factor $x^2 - 10x + 24$

Practice: Factor each trinomial.

1. $x^2 + 6x - 27$  
2. $x^2 - 6x + 9$
3. $x^2 - 18x + 72$

4. $x^2 - 13x - 48$

5. $x^2 + 20x + 100$

6. $x^2 - 7x + 6$
Lesson 3 Homework

Factor:

1. $a^2 + 3a + 2$

2. $x^2 + 8x + 7$

3. $y^2 - 6y + 8$

4. $y^2 - 2y - 8$
5. \( x^2 - 9x + 8 \)

6. \( x^2 + 11x + 24 \)

Factor using the GCF method.

7) \( 10x - 15x^3 \)

8) \( 12y^2 - 4y \)

9) \( 9b^3 - 6b^2 - 3b \)
Connection to Spiraling Curriculum:
This lesson is taught to students with an emphasis on the undoing the box method. This helps students make the connection with the box method and factoring. By doing this, students are getting multiple exposures and are able to expand their learning, which is a direct connection to the CLT. Furthermore, part of this lesson focuses on reviewing the two prior types of factoring that were taught (GCF and Difference of Two Perfect Squares). By doing this repeated practice, the spiraling curriculum is evident. If students had previously taken the pre-algebra course then this would be their second exposure to trinomial factoring. Again, with the multiple exposures over multiple years, the spiraling curriculum aspect is evident. As time goes on, students are faced with more challenging questions, which is part of the cognitive load theory.
Lesson 4:

Factoring a Trinomial with Leading Coefficient > 1
(Slide and Divide and Grouping)

Factor:

1. \(4x^2 - 19x - 5\)  
2. \(9x^2 - 2x - 7\)

3. \(12x^2 - 7x - 10\)  
4. \(7x^2 + 53x + 28\)

5. \(4x^2 - 17x + 4\)  
6. \(5x^2 - 18x + 9\)

7. \(4x^2 - 35x + 49\)  
8. \(6x^2 - 11x - 35\)
Homework

Factor:

1. $5x^2 + 34x + 24$

2. $4x^2 + 38x + 70$

3. $2x^2 - 3x - 9$

4. $5x^2 + 23x + 24$

5. $4x^2 + 22x + 10$

6. $4x^2 - 8x - 45$
**Connection to Spiraling Curriculum:**
This lesson goes over two different ways to factor a trinomial with a leading coefficient that is not equal to one. In the first method, referred to as slide and divide, students are asked to manipulate the equation so that they can factor it as a normal trinomial. By doing this, students are getting multiple exposures to the trinomial factoring methods. The second method, is a method in which they manipulate the equation to factor by grouping, which requires knowledge of GCF factoring. In both cases, students are reviewing a type of factoring that they have already gone over. Thus, with the exposure over multiple days, the CLT is directly implemented through the use of the spiraling curriculum and students have the opportunity to convert their knowledge of factoring from short term to long term memory.
Lesson 5: Factoring Completely

THE GCF METHOD
REMEMBER:
1.) Find GCF
2.) Divide the GCF from each term

1) $10x^2 - 5x$  
2) $2x^4 - 6x^3 + 8x^2$

FACTORIZING THE DIFFERENCE OF 2 PERFECT SQUARES
Remember to find the square root of each term and place in parentheses. Make sure the sign in between is a ______________   __________.
REMEMBER:
1.) Take the square root of each term
2.) Set up (+)(-)
3.) Put square root of 1st and 2nd term in ( ).

3) $x^2 - 121$  
4) $16x^4 - 9y^2$

FACTORIZING TRINOMIALS and SLIP & DIVIDE

5) $x^2 + 7x - 18$  
6) $5x^2 - 14x - 3$

Putting it all together – factoring completely

When factoring, we should always look for ___________________________ first.

Sometimes, we can factor more than once. Continuing to factor until we can’t factor anymore is called _________________________________.

73
Practice: Factor Completely – LOOK FOR A ___________ FIRST!!!

7) $5x^2 - 45$
8) $3x^2 - 6x - 24$
9) $ax^2 + 3ax + 2a$

10) $st^2 - s$
11) $x^3 - 8x^2 + 12x$
12) $x^4 - 16$

Try these on your own!! 😊

13) $2x^2 - 32$
14) $16x^2 - 16x + 4$
15) $ax^2 + 3ax$
**FACTORING MAZE**

You can move between squares (diagonally too) if they share a common factor

<table>
<thead>
<tr>
<th><em><strong>START</strong></em></th>
<th>$3x^2 + 6x$</th>
<th>$x^2 - 1$</th>
<th>$2x^2 + 8x$</th>
<th>$3x^2 - 3x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 - x - 6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 - 2x$</td>
<td>$3x^2 - 12x$</td>
<td>$x^2 + x - 6$</td>
<td>$x^2 + 3x + 2$</td>
<td>$x^2 - x - 12$</td>
</tr>
<tr>
<td>$x^2 + 2x - 3$</td>
<td>$x^2 + 3x + 2$</td>
<td>$2x^2 - 32$</td>
<td>$x^3 + 3x^2 - 4x$</td>
<td>$x^2 - 3x + 4$</td>
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<tr>
<td>$x^3 - 16x$</td>
<td>$x^2 - 5x + 6$</td>
<td>$x^2 - 25$</td>
<td>$x^2 - 3x + 2$</td>
<td>$x^2 - 2x - 15$</td>
</tr>
<tr>
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<td>$2x + 6$</td>
<td>$x^2 + 2x$</td>
<td>$3x^2 - 12$</td>
<td>$x^3 + x^2 - 20x$</td>
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<tr>
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<td>$x^2 - 3x - 4$</td>
<td>$4x^2 - 3x$</td>
<td>$x^2 + 7x + 6$</td>
<td>$x^2 - 9$</td>
</tr>
</tbody>
</table>
Lesson 5 Homework:

Factor completely.

1. $2a^2 - 2b^2$
2. $ax^2 - ay^2$

3. $3x^2 + 6x + 3$
4. $x^3 + 7x^2 + 10x$

5. $6y^2 - 14y + 4$
6. $18a^2 - 3a - 6$
**Connection to Spiraling Curriculum:**

This lesson is a review of all of the types of factoring that were covering during this unit. This is a direct connection to the CLT because students are practicing the material over multiple days. Meaning, they have the opportunity to convert their learning from short term memory to long term memory. In addition, this lesson is of spiraled curriculum because it is a continuation of practice of a lot of the material.
Algebra 2:

<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Lesson Objective</th>
<th>Connection to Spiraling Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Factoring Review- GCF, Difference of Perfect Squares, Trinomial</td>
<td>Students will be able to identify the GCF of multiple terms. Students will be able to factor a polynomial with the use of a GCF. Students will be able to examine a binomial and determine if it can be restructured using the difference of perfect squares model. Students will be able to factor using difference of two perfect squares. Students will be able to identify factors of a trinomial and use these factors to restructure the trinomial in a different form. Students will be able to check their factoring by multiplying the factors together.</td>
<td>This lesson ties to prior knowledge from 7th grade. In the 7th grade lessons students were first introduced to the GCF of two different numbers. In addition, students were asked to identify the GCF of a binomial. This lesson ignites this prior knowledge by asking students to do the same thing, but with more challenging terms. This lesson ties to prior knowledge from 8th grade because students will need to evaluate their perfect squares and identify if terms are perfect squares. This lesson ties to prior knowledge by asking students to think about factors of the ending term in a trinomial and determining which factors work to add to the b value in the trinomial.</td>
</tr>
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<td>2</td>
<td>Sum and Difference of Perfect Cubes</td>
<td>Students will be able to factor the sum and difference of perfect cubes. Students will be able to identify when to use the sum or difference of perfect cubes.</td>
<td>This is a new concept for students but utilizes knowledge from 8th grade of identifying perfect cubes.</td>
</tr>
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<td>3</td>
<td>Factoring Trinomials when (a\neq1) “Slide and Divide”</td>
<td>Students will be able to identify factors of trinomials with a leading coefficient other than 1. Students will rewrite a trinomial with a different structure utilizing the factors.</td>
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<td>Factoring Completely</td>
<td>Students will factor polynomial expressions using one or many factoring methods.</td>
<td>This lesson takes all of the prior knowledge for this unit and combines them into questions where multiple forms of factoring may be required.</td>
</tr>
<tr>
<td>5</td>
<td>Factoring by Grouping</td>
<td>Students will be able to identify polynomials that can be factored with the grouping method.</td>
<td>This lesson will utilize and expand on students’ prior knowledge of GCF factoring.</td>
</tr>
<tr>
<td>6</td>
<td>Review Chart</td>
<td>Students will identify when to use which type of factoring method. Students will be able to explain how to use each type of factoring method.</td>
<td>Students will use all of their prior knowledge of factoring in order to create a summary chart. Within this chart students will come up with examples and demonstrate the factoring methods and when to use which method.</td>
</tr>
</tbody>
</table>

Common Core Standards:  
CCSS.MATH.PRACTICE.MP7 Look for and make use of structure  
CCSS.MATH.CONTENT.HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.
CCSS.MATH.CONTENT.HSA.SSE.B.3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

CCSS.MATH.CONTENT.HSF.IF.C.8
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
Lesson 1:

New Method! Sum and Difference of Perfect Cubes

First, what are our perfect cubes?

Steps for Sum of Different Cubes:

1.) To get the binomial, take the ________________ or the original two terms

2.) __________ the first term, __________ the two terms, __________ the last term.

3.) The signs will always be

In general:

Ex: $x^3 + 27$

Ex: $125x^3 + y^6$
**Difference of Perfect Cubes:**

Note, the only difference from the Sum of Perfect Cubes is the signs.

In general:

Ex: $x^3 - 64$

Ex: $8r^3 - 343s^2$

So, let's practice. Factor completely $y^6 + 3y^3 - 40$
Connection to Spiraling Curriculum:
This lesson introduces a brand new type of factoring, but has students recall knowledge from 8th grade by listing their perfect cubes and finding the cube root of values in order to factor with sum and difference of cubes. Thus, the curriculum from 8th grade has spiraled into the curriculum for Algebra 2.
Lesson 2:
Name: __________________________________________ Date: ___

FACTORED WITH A > 1

SLIDE AND DIVIDE!!!
Steps: 1.) Multiply ___ and _____ to create a new trinomial
2.) Factor as normal
3.) Divide each factor by c.

Ex: \(5x^2 - 14x + 8\)

Ex: \(3x^2 + 19x - 40\)

Ex: \(2x^2 - 15x + 18\)

Ex: \(15x^2 + 13x + 2\)
Ex: $12x^2 + 8x - 15$

Ex: $10x^2 + 13x - 30$
Lesson 2 Homework

Name: ____________________________________________ Date:____

For questions 1-8, write each of the following trinomials in factored form.

1) \(x^2 - 7x - 18\) 

2) \(18x^2 - 25x + 8\)

3) \(5x^2 - 41x + 8\) 

4) \(3x^2 + 4x - 20\)

5) \(2x^2 - 29x - 15\) 

6) \(x^2 - 17x + 30\)

7) \(7x^2 + 39x + 20\) 

8) \(20x^2 - 11x - 42\)
9) Consider the trinomial $12x^2 + 7x - 10$.

(a) Does this trinomial have a greatest common factor that could be “factored out?”

1) $(x - 9)(x + 2)$  2) $(9x - 8)(2x - 1)$  3) $(5x - 1)(x - 8)$  4) $(x - 2)(3x + 10)$
5) $(x - 15)(2x + 1)$  6) $(x - 2)(x - 15)$  7) $(7x + 4)(x + 5)$  8) $(5x + 6)(4x - 7)$
9) (a) No
**Connection to Spiraling Curriculum:**

This lesson goes over two different ways to factor a trinomial with a leading coefficient that is not equal to one. In the first method, referred to as slide and divide, students are asked to manipulate the equation so that they can factor it as a normal trinomial. This is a review of both of the methods from their Algebra 1 exposure. In both cases, students are reviewing a type of factoring that they have already gone over. Thus, with the exposure over multiple years, the CLT is directly implemented through the use of the spiraling curriculum and students have the opportunity to convert their knowledge of factoring from short term to long term memory.
When an expression is **completely factored**, it is written as a product of polynomials, none of which can be factored further.

Ex: Consider the trinomial $2x^2 - 4x - 6$.

(a) Consider the following products. Verify through multiplication that both are representation of factors of the given trinomial.

\[
\begin{align*}
(2x - 6)(x + 1) & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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(c) \(10x^2 + 55x - 105\)  
(d) \(12x^2 + 57x - 15\)

Factor each of the following expressions:

(a) \(6x^2 - 13x + 6\)  
(b) \(12x^2 + 29x - 8\)
Lesson 3 Homework:
Name:_____________________________________________ Date:____

Completely factor each of the following expressions:

1. $2x^2 - 14x - 36$
2. $3x^2 - 192$

3. $8x^2 + 12x - 8$
4. $5x^2 + 70x + 245$

5. $10x^3 - 26x^2 - 12x$
6. $28x - 7x^3$

7. $15x^2 - 110x + 120$
8. $12x^2 - 20x + 3$

9. $45x - 20x^3$
10. $8x^2 + 67x + 24$
11. \(20x^2 + 112x - 48\)  
12. \(90x^3 - 90x^2 + 20x\)

13. \(8x^2 + 30x + 28\)  
14. \(27x^2 - 3\)

15. \(18x^2 - 39x - 15\)
**Connection to Spiraling Curriculum:**
This lesson is a review of all of the types of factoring that were covering during this unit. This is a direct connection to the CLT because students are practicing the material over multiple days. Meaning, they have the opportunity to convert their learning from short term memory to long term memory. In addition, this lesson is of spiraled curriculum because it is a continuation of practice of a lot of the material.

**This lesson was adapted from Kirk Weiler’s eMathinstruction.**
Lesson 4:

Name:________________________________________________ Date:___

**FACTORING BY GROUPING**

When we are given a polynomial to factor, and we have more than 3 terms with no GCF, we try factoring by grouping.

Steps:

1.) Group
2.) Take _____ of both binomials
3.) Factor the common result out of the entire expression
4.) Factor completely

Ex: $5a^3 - a^2 + 5a - 1$

Ex: $2x^3 + 10x^2 + 7x + 21$

Ex: $x^3 + x^2 - 4x - 4$

Ex: $8x^4 + 24x^3 + 125x + 375$
Ex: $x^2 - ax - bx + ab$

Ex: $4x^3 - 8x^2 - x + 2$

Ex: $x^4 + 5x^3 + 8x + 40$
Connection to Spiraling Curriculum:

This lesson introduces a new type of factoring to students. However, without knowing it, they have been practicing this method when they use one of the methods to factor a trinomial with a leading coefficient greater than one. Thus, students have been exposed to this method in Pre-Algebra, Algebra, and now again in Algebra 2. Since it is viewed in all three courses, this is a direct view of the spiraling curriculum. Due to the multiple exposure of this method that students are receiving, students will be able to convert their knowledge of this concept and procedure from short term to long term memory. Which, is a direct application of the CLT.
Lesson 5:

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</table>
Connection to Spiraling Curriculum:
This lesson is a review of all of the types of factoring that were covering during this unit. This is a direct connection to the CLT because students are practicing the material over multiple days. Meaning, they have the opportunity to convert their learning from short term memory to long term memory. In addition, this lesson is of spiraled curriculum because it is a continuation of practice of a lot of the material.

Connection to the Classroom

This curriculum project was reviewed by a 15 year Algebra and 8th grade teacher at a small rural school district in The Finger Lakes Region of New York and a 22 year 7th grade and Pre-Algebra teacher at the same small rural school district in The Finger Lakes Region of New York. The feedback was given based on the grade level being examined.

For the 7th grade curriculum the feedback was as follows:

- I really like how the curriculum ties in the distributive property and uses it as a checking method when factoring. *
- I like that there is a visualize it section for students that are more visual learners. I think this will be beneficial to students that need to see the material in a different way.
- The material starts out less challenging and gets more challenging as time goes on. *
- Maybe incorporate some questions where the variables are something other than an x.
- For the students that need to be challenged you could incorporate a question where they need to divide out a GCF that has a variable. This would be a pre-lesson to future work with GCF.
For the Pre-Algebra/8th grade curriculum feedback was as follows:

- I like that the concepts are all looked at over multiple days. This is a great way to get students practicing the material.

- I like that there are two different ways to factor with a trinomial. Especially since the two ways both pull from concepts that the students had already learned. And also lead into future lessons on factoring by grouping.

- I like that the warm up questions look at lessons not from the previous day but from many days prior.

- In the GCF lesson has a great flow to it.

- I love that there is a lesson that has students decide which method of factoring to use and they have to explain why. This is a great way to make sure that they understand the concepts.

For the Algebra curriculum feedback was as follows:

- I love that each lesson connects to prior lessons either in the same unit or from a previous grade level.

- I love the maze that reviews all of the different methods.

- On the slide and divide method you should also add in a section that review the other form of factoring a trinomial with a gcf greater than one. That would allow for further spiraling.

- I like that there is the visual of the factor charts. I think that this helps to break down the concepts.
- I really like that there is the practicing of multiplying polynomials with the factors of perfect squares. I think that the curriculum was a great way to conceptualize what is occurring and showing the connection that multiplying the polynomials has with factoring them.

For the Algebra II curriculum the feedback was as follows:

- I like the review lesson for students to look back to on all of their year of factoring. *

- For the most part I like the flow of the curriculum and how the sum and difference of cubes come into the curriculum after reviewing the difference of perfect squares. My suggestion to better the flow would be to put the trinomials with a>1 closer to the front of the unit because it is also a review lesson.

- On the slide and divide section create a portion of the notes to review the other method for factoring trinomials. This would lead nicely into factoring by grouping and help to build that conceptual understanding.

- I absolutely love the review chart that summarizes all of the methods of factoring, when to use it, and some examples. That is a great reference for students. *

Overall feedback:

-This is a curriculum that considers students’ level of engagement, and allows them to practice concepts over multiple periods of time. I will definitely be implementing this curriculum.

-I love the vertical alignment of this curriculum. Great job!
Any comments with an asterisk are comments that align to the ideology of the Cognitive Load Theory and the objective of the Spiraling Curriculum. Primary components of the Spiraling Curriculum are that the level of difficulty increasing as time goes on, and students have continued practice with the material. The comments and feedback that were made apply directly to this ideology. For instance, the comment of “I love that each lesson connects to prior lessons either in the same unit or from a previous grade level.” In regards to the Algebra curriculum, makes that direct connection to the spiraling curriculum throughout the grade levels because spiraling curriculum does look at the material with revisiting concepts. Furthermore, the comment of “This is a curriculum that considers students’ level of engagement, and allows them to practice concepts over multiple periods of time. I will definitely be implementing this curriculum” is also a direct connection to the spiraling curriculum because the main goal is to have multiple exposures to the concepts over time. By implementing the spiraling curriculum students are able to make connections between grade levels, and pull form their prior knowledge. By doing this, students are able to make deeper connections with the material and expand their knowledge while deepening their understanding. Thus, with the multiple exposures, students are able to convert their learning from the short term memory to their long term memory. Many comments were made about how the material was broken up over multiple days. The reviewers of the material here are making connections to the Cognitive Load Theory since it requires students to look at small portions of material over a longer period of time.

**Conclusion**

This curriculum is presented to be a resource for teachers within their classrooms and for school districts to use a vertically aligned curriculum. While this curriculum is presented as a connection to the Common Core State Standards, there is also a direct connection to the Next
Generation Standards. Thus, this curriculum may be utilized throughout many years. The primary goal of this curriculum was to help students develop a deeper understanding of concepts by making the conversion from short term to long term memory. When students go from grade level to grade level with this long term memory, it allows students to make the connection of the content between grade levels and work deeper with it at the next grade level, while also creating a learning environment appropriate for the age level of the students.

After consideration of the feedback from the two veteran teachers, some changes will be made to the curriculum as a way to assist students further in developing the deeper connections. The curriculum can certainly be modified to meet the needs of students in individual districts or classrooms. This can be done through the use of the warm up problems and adjusting homework questions as needed. In order to determine the effectiveness of the curriculum, one would want to complete multiple cohort studies and determine if the level of proficiency scored on the state and regents exams on the standards dealing with factoring were truly improved.

In conclusion, the goal of this curriculum project was to provide a resource of a vertically aligned, spiraled curriculum that has an emphasis on the Cognitive Load Theory with a connection to the New York State mathematics standards. Since the feedback that was given by veteran teachers tied directly to the ideology of this curriculum project, it is evident that the goal was met.
References:


Appendix: Answer Keys

7th Grade:
Lesson 1:

Warm Up:
Complete the following operations:

1.) $8(3) = 24$
2.) $8(3x) = 24x$
3.) $3x(4) = 12x$

Notes:

Distributing Expressions

The Distributive Property: multiply a sum by multiplying each addedent.

Ex: $a(x+y)=ax+ay$

You multiply everything inside the parenthesis by what is on the outside of the parenthesis.

Ex: Expand $8(3x + 4) = 8(3x) + 8(4) = 24x + 32$

Visualize it:

\[
\begin{array}{c|c|c|}
8 & 3x & +4 \\
\hline
& 24x & 32 \\
\end{array}
\]

= $24x + 32$

Ex: Expand $2(4x - 2) = 2(4x) - 2(2) = 8x - 4$

Visualize It: Visualize it:

\[
\begin{array}{c|c|c|}
2 & 4x & -2 \\
\hline
& 8x & -4 \\
\end{array}
\]

= $8x - 4$

Ex: $8(9x - 2y)$

= $8(9x) - 8(2y)$

Ex: $10(3x + 2)$

= $10(3x) + 10(2)$
=72x – 16y  
Ex: \( \frac{1}{2} (4x + 8y) \)

= 1/2 (4x) + ½ (8y)  
= 2x + 4y

Ex: \(-5(3x – 4)\)

= -5(3x) -5(-4)  
= -15x +20

Homework:

1.) \(9(-4p – y)\)

\(=9(-4p) – 9(y)\)
\(= -36p – 9y\)

2.) \(-4(6n – 7)\)

\(=-4(6n) – 4(-7)\)
\(= -24n + 28\)

3.) \(5(n + 6)\)

\(=5(n) + 5(6)\)
\(=5n + 30\)

4.) \(-5(10x + 1)\)

\(=-5(10x) -5(1)\)
\(= -50x -5\)

5.) \(8(7z – 6s + 6)\)

\(=8(7z) – 8(6s) + 8(6)\)
\(=56z – 48s + 48\)

6.) \(-7(-q – 9 – 2p)\)

\(=-7(-q) – 7(-9) -7(-2p)\)
\(= 7q + 63 + 14p\)

7.) \(3(-5 + 11p)\)

\(=3(-5) +3(11p)\)
\(= -15 + 33p\)

8.) \(-6(-4a + w)\)

\(=-6(-4a) - 6(w)\)
\(= -24a – 6w\)

9.) \(9(-6d + 5 – 3y)\)

\(= 9(-6d) + 9(5) – 9(3y)\)
\(= - 54d + 45 – 27y\)

10.) \(-3(6x + 2)\)

\(=-3(6x) -3(2)\)
\(= -18x -6\)

11.) Write an expression to represent the product of 3 and \(\frac{5}{4}n + 1.8\)

\(=3((5/4)n +1.8)\)
\(=(15/4)n +5.4\)
Lesson 2
Warm Up:

Simplify the following expressions:

1.) \(8(7x+4)\)
   \[= 8(7x) + 8(4)\]
   \[= 56x + 32\]

2.) \(-8(x-5)\)
   \[= -8(x) -8(-5)\]
   \[= -8x + 40\]

Notes:

Factoring Expressions

Factoring:
To break an expression into parts that when multiplied together produce the original expression. This “undoes” the distributive property.

Ex: Determine the greatest common factor (GCF) between each of the given numbers:

b.) 40 and 100
   \(40: 1, 2, 4, 5, 8, 10, 20, 40\)
   \(100: 1, 2, 4, 5, 10, 20, 25, 50, 100\)
   GCF = 20

c.) 54 and 144
   \(54: 1, 2, 3, 6, 9, 18, 27, 54\)
   \(144: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144\)
   GCF = 18

d.) 75x and 90
   \(75: 1, 3, 5, 15, 25, 75\)
   \(90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90\)
   GCF: 15

In order to factor expressions, we must first find the greatest common factor (GCF) of all terms in the expression.

Then, we divide each term in the expression by this number.

We write the GCF on the outside of a set of parenthesis and put the quotient(s) inside the parenthesis.
We can always check if we factored correctly by using the **distributive property**.

**Ex: Factor** $4x + 30$

Check:

\[
2(2x+15) \\
\text{Check:} \\
2x + 15 \\
2 \times 4x + 30
\]

**Ex: Factor** $15x + 40$

Check:

\[
5( 3x + 10) \\
\text{Check:} \\
3x + 10 \\
5 \times 15x + 50
\]

**Ex: Factor** $4x + 16$

Check:

\[
4 ( x + 4) \\
\text{Check:} \\
x + 4 \\
4 \times 4x + 16
\]

**Ex: Factor** $9x + 12$

Check:

\[
3 (3x + 4) \\
\text{Check:} \\
3x + 4 \\
3 \times 9x + 12
\]

**Ex: Factor** $3x^2 + 6x + 12$

Check:

\[
3( x^2+2x+4) \\
\text{Check:} \\
X^2 + 3x + 4 \\
3 \times 3x^2 + 6x + 12
\]

**Ex: Factor** $5x^2 + 10x + 30$

Check:

\[
5(x^2+2x+3) \\
\text{Check} \\
X^2 + 2x + 3 \\
5 \times 5x^2 + 10x + 15
\]

**Homework**

Directions: Factor and check each of the following expressions.

1.) $6x + 12$

$6(x+2)$

2.) $-3x - 18$

$-3(x+6)$
3.) $10x + 15$

4.) $-5x + 15$

$5(2x+3)$

$-5(x-3)$

5.) $12x - 24$

6.) $-16x + 48$

$12(x-2)$

$-16(x-3)$

7.) $2x + 8$

8.) $4x − 38$

$2(x+4)$

$2(2x-19)$

9.) $2x^2 + 10x + 12$

10.) $3x^2 + 6x − 12$

$2(x^2+5x+6)$

$3(x^2+2x-6)$

11.) $-5x^2 − 75x + 95$

12.) $10x^2 + 15x + 20$

$-5(x^2+15x+19)$

$5(x^2+3x+4)$

Pre-Algebra
Lesson 1:
Factoring a GCF

Warm Up: Find the GCF for each of the following:

1.) 6 & 18

6: 1, 2, 3, 6
18: 1, 2, 3, 6, 9, 18
GCF: 3

2.) 9 & 15

9: 1, 3, 9
15: 1, 3, 5, 15
GCF: 3

3.) 24 & 36

24: 1, 2, 3, 4, 6, 8, 12, 24
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
GCF: 12

4.) 25 & 10

25: 1, 5, 25
10: 1, 2, 5, 10
GCF: 5
Divide each of the following. Remember when dividing, subtract exponents.

5.) \( \frac{8y^3}{4y} \)  
\[ 2y^2 \]

6.) \( \frac{25x^6y}{5x^2y} \)  
\[ 5x^3 \]

7.) \( \frac{24x^5}{36x^2} \)  
\[ \frac{2x^3}{3} \]

Find the GCF of each set of monomials.

8.) 12x, 48y  
12: 1, 2, 3, 4, 6, 12  
48: 1, 2, 3, 4, 12, 16, 24, 48  
GCF: 12

9.) 60r^2s^4, 30 r^3s  
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60  
30: 1, 2, 3, 5, 6, 10, 25, 30  
GCF: 30r^2s

If the variables are different, the GCF is a number.  
If the variables are the same the GCF is a number and a variable

Factor the following polynomials using Greatest Common Factor.

1.) Find the GCF of the coefficients

2.) Find the GCF of the variables

3.) Write the GCF outside the parenthesis, divide each term by the GCF and write that inside parenthesis.

10.) 14m + 35n  
GCF: 7  
Factored Form: 7(2m+5n)

11.) 4x^4 + 24x^3  
GCF: 4x^3  
Factored Form: 4x^3(4x+6)
12.) $20x^3 + 35x^2 - 12x$

GCF: $x$

Factored Form: $x(20x^2 + 35x - 12)$

* You can check your answers by using the distributive property!!

Lesson 2:

Warm Up:

2.) Distribute $3x(x^2 + 7x - 9)$

$3x^3 + 21x^2 - 27x$

2.) Factor: $4x^2 + 6x$

$2x(2x + 3)$

Factoring the difference of 2 Perfect Squares (Day 1)

Multiply:

1.) $(x + 3)(x - 3)$

2.) $(b + 5)(b - 5)$

$x^2 - 9$

3.) $(y + 8)(y - 8)$

4.) $(a + 7)(a - 7)$

$y^2 - 64$

$a^2 - 49$

What do you notice happens to the middle term in both examples?
It cancels out

Factoring the difference of PERFECT SQUARES goes in the opposite direction.

Perfect Square:
A number multiplied by itself.

Name the first 20 perfect squares.

1.) 1 11.) 121
2.) 4 12.) 144
3.) 9 13.) 169

Does anything multiply itself to give us $x^2$?

$x$

Does anything multiply by itself to give us $y^2$?

$y$

Does anything multiple by itself to give us $a^2$?
In order to factor by difference of perfect squares, you must have:

1.) A subtraction sign
2.) Coefficients that are perfect squares
3.) Even exponents

Then, to factor you write two sets of parenthesis, with one positive sign and one negative sign. Then, take the square root of the first and second terms given.

Factor:

1.) \(x^2 - 25\)  
\((x-5)(x+5)\)

2.) \(y^2 - 49\)  
\((y-7)(y+7)\)

3.) \(d^2 - 225\)  
\((d-15)(d+15)\)

4.) \(x^2 - 169\)  
\((x-13)(x+13)\)

Homework

Factor by difference of perfect squares. Check by distributing.

1.) \(a^2 - 324\)  
\((a-18)(a+18)\)

2.) \(x^2 - 64\)  
\((x-8)(x+8)\)
3.) \[ y^2 - 100 \]
\[(y-10)(y+10)\]

4.) \[ b^2 - 196 \]
\[(b-14)(b+14)\]

Lesson 3:
Warm Up:
Factor the following expressions:

1.) \[ 4x^2 + 24x + 6 \]
\[2(2x^2+12x+3)\]

2.) \[ x^2 - 64 \]
\[(x-8)(x+8)\]

Difference of Perfect Squares Day 2

We know that \( x \cdot x = x^2 \) and \( y \cdot y = y^2 \), but is there any way to multiply a variable by itself to give us \( x^3 \)? What about \( x^4 \)?

Variables are only perfect square if they have even exponents.

List some examples here:

Answers may vary.
\[ x^{10} \quad x^4 \quad x^6 \quad x^{12} \]

Can you factor \( 4x^2 - 81 \)?

Is there a subtraction sign? Is there an even number exponent on the variable? Are all the numbers perfect squares? Yes, yes, and yes!

\[ 4x^2 - 81 = (2x + 9)(2x - 9) \]

What about \( 16a^2 - 49 \)?
\[(4a-7)(4a+7)\]

Classwork, factor and check:

1.) \[ 9x^2 - 100 \]

2.) \[ y^4 - 361 \]

3.) \[ 4x^6 - 36 \]
(3x-10)(3x+10) \quad (y^2-19)(y^2+19) \quad (2x^3-6)(2x^3+6)

4.) \quad 16x^2 – 64y^2
\quad (4x-8y)(4x+8y)

5.) \quad x^8 – 144
\quad (x^4-12)(x^4+12)

6.) \quad \frac{1}{4}x^2 – 289
\quad \left(\frac{x}{2}-17\right)\left(\frac{x}{2}+17\right)

**Homework**

*Factor by difference of perfect squares. Check by distributing.*

1.) \quad 4y^{6} – 9
\quad (2y^3-3)(2y^3+3)

2.) \quad 16x^2 – 25y^4
\quad (4x-5y^2)(4x+5y^2)

3.) \quad 49b^8 – 256a^2
\quad (7b^4-16a)(7b^4+16a)

4.) \quad 9x^2 – 1024y^2
\quad (3x-32y)(3x+32y)

**Lesson 4:**
**Warm Up:**
Factor the following expressions and check your answers:

1.) \quad x^2 – 16
\quad (x-4)(x+4)

2.) \quad 25z^2 – 64
\quad (5z-8)(5z+8)
3.) $3x^3 - 15x^2 - 36x$

$3x(x^2-5x-12)$

**Factoring Trinomials Round 1**

Factoring Trinomials is just like reversing the box or distributing methods we did with pairs of binomials.

Multiply each of the following using whatever method you choose.

1.) $(x + 2)(x + 7)$

$x^2+9x+14$

2.) $(x - 2) (x - 7)$

$x^2-9x+14$

*What similarities do you notice between the answers to these examples?*

They have the same constant, and when multiplying it was the same numbers

*What is the only difference?*

The first is a positive $9x$ and the second is a negative $9x$

3.) $(x + 2)(x - 7)$

$x^2-5x-14$

4.) $(x - 2)(x + 7)$

$x^2+5x-14$

*Are there similarities between these examples?*

Both are multiplying to $-14$. The middle term has a different sign.

**How to Factor Trinomials**

Step 1: Make sure the expression/equation is in standard form

Step 2: Determine factors of the ending term that add to the middle term

If last sign is **positive**, the signs are the same (both positive or both negative)

Signs are the same as that of the **middle term**

If last sign is **negative**, the signs are different

Step 3: Write in two sets of parentheses. The first term is the square root of the leading term.
Step 4: check using the distributive property

Factor and check each of the following trinomials

5.) \( x^2 + 5x + 6 \)

<table>
<thead>
<tr>
<th>Factors of 6</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 6</td>
<td>7</td>
</tr>
<tr>
<td>2, 3</td>
<td>5</td>
</tr>
</tbody>
</table>

\((x+2)(x+3)\) \hspace{1cm} \((x-4) (x-2)\)

6.) \( x^2 - 6x + 8 \)

7.) \( x^2 + 2x - 3 \)

\((x+3)(x-1)\)

8.) \( x^2 - 3x - 28 \)

\((x-7)(x+4)\)

Homework

Factor each of the following trinomials. Don’t forget to check your answers!

1.) \( x^2 + 13x - 30 \)

\((x+10) (x+3)\)

2.) \( x^2 - 4x - 12 \)

\((x+2)(x-6)\)

3.) \( x^2 + 13x + 22 \)

\((x+11)(x+2)\)

4.) \( x^2 - 15x + 44 \)

\((x-11)(x-4)\)

Lesson 5:

Warm Up:
Factor the following and check:

1.) \( x^2 - 36 \)

2.) \( 25x^2 - 121 \)
(x-6)(x+6)  (5x-11)(5x+11)

**Factoring Trinomials Round 2**

Factor the following trinomials. Check your answers using the distribution method.

2.) \(a^2 - 2a - 15\)
\[(a-5)(a+3)\]

2.) \(x^2 + 10x + 25\)
\[(x+5)(x+5)\]

3.) \(y^2 - 10y + 24\)
\[(y-6)(y-4)\]

4.) \(x^2 + 3x - 28\)
\[(x+7)(x-4)\]

5.) \(x^2 - 6x + 9\)
\[(x-3)(x-3)\]

6.) \(x^2 + 5x - 24\)
\[(x+8)(x-3)\]

**Lesson 6:**

When do I use GCF, DOS or Trinomial Factoring??

**Directions:**

1.) Determine the type of factoring

GCF- do the terms have anything in common?

DOS- Is there a subtraction sign? Are there even exponents? Are the coefficients perfect squares?

Trinomial- must have three terms.

2.) Factor

3.) Check

1.) \(3x + 3\)
\[3(x+1)\]

**Type of Factoring:** GCF

Step 1: I found the GCF
Step 2: I made one parenthesis
Step 3: I then divided each term by the GCF
2.) 121 – 4x^2
   \( (11 - 2x)(11 + 2x) \)

   **Type of Factoring:** DOS

   Step 1: I found the square root of 121
   Step 2: Then I found the square root of 4x^2
   Step 3: Then I wrote the parenthesis
     The signs are + and -

3.) x^2 – 6x - 16
   \( (x - 8)(x + 2) \)

   **Type of Factoring:** Trinomial

   Step 1: First I made two parenthesis
   Step 2: Then I determine the signs by looking at what the ending term was
   Step 3: I chose the numbers by seeing what added to -6 and multiplied to -16

**Factor a GCF**

1.) 3a + 3b
   \( 3(a+b) \)

2.) 25c – 30d
   \( 5(5c-6d) \)

3.) y^3 + 5y
   \( y(y^2 + 5) \)

4.) 3x^3 + 18x^2 + 9x
   \( 3x(x^2 + 6x + 3) \)

**Factor using Difference of 2 Perfect Squares**

5.) x^2 – 16
   \( (x-4)(x+4) \)

6.) m^2 – n^2
   \( (m-n)(m+n) \)

7.) 49x^2 – 121
   \( (7x-11)(7x+11) \)

8.) x^4 – 0.09y^2
   \( (x^2-0.03y)(x^2+0.03y) \)

**Factor each trinomial**

9.) x^2 – 5x + 6
   \( (x-2)(x-3) \)

10.) x^2 + 8x + 16
    \( (x+4)(x+4) \)

11.) x^2 – 19x – 42
    \( (x-21)(x+2) \)

12.) x^2 -3x - 18
Homework

Factor each of the following trinomials using either GCF, DOS or trinomials.

1.) 12y - 6
2.) 64 - x^2
3.) 6x^3 + 18x
4.) x^2 + 10x + 24
5.) x^2 - 14x - 32
6.) 49x^2 - 121

Lesson 7:

Warm Up:
Factor the following expressions:

1.) x^2 - 5x + 6
2.) x^2 + x - 12

Factoring Trinomials with Coefficient other than 1 (Day 1) - Method 1

1.) 3x^2 - 10x - 8 (from the video)

X^2-10x-24
(x-4/3)(x-6/3)
(3x-4)(x-2)

Try some more….

2.) 2x^2 - 11x + 5

X^2-11x+10
(x-10/2)(x-1/2)
(x-5)(2x-1)

3.) 3x^2 - 5x + 2

x^2-5x+6
(x-3/3)(x-2/3)
(x-1)(3x-2)

4.) 5n^2 + 2n - 3

n^2+2n-15
(n+5/5)(n-3/5)
(n+1)(5n-3)

5.) 2m^2 + m - 21

m^2+m-42
(m+7/2)(m-6/2)
(2m+7)(m-3)

Lesson 8:

Warm Up:
Factor the following expressions:
1.) \( x^2 - 25 \)  
\( (x-5)(x+5) \)

2.) \( 3x^2 - 7x - 6 \)  
\( x^2-7x-18 \)  
\( (x-9/3)(x+2/3) \)  
\( (x-3)(3x+2) \)

Factoring Trinomials with Coefficient other than 1 Method 2

1.) \( 6x^2 - 2x - 4 \)  
\( (6x^2-6x)+(4x-4) \)  
\( 6x(x-1)+4(x-1) \)  
\( (6x-4)(x-1) \)

2.) \( 2x^2 + 5x + 2 \)  
\( (2x^2+4x)+(1x+2) \)  
\( 2x(x+1)+1(x+1) \)  
\( (2x+1)(x+1) \)

3.) \( 3x^2 + 10x - 8 \)  
\( (3x^2+12)+(-2x-8) \)  
\( 3x(x+4)-2(x+4) \)  
\( (3x-2)(x+4) \)

4.) \( 4x^2 + 15x - 4 \)  
\( (4x^2+16x)+(-x-4) \)  
\( 4x(x+4)-1(x-4) \)  
\( (4x-1)(x-4) \)

b) \( 3x^2 - 14x + 8 \)  
\( (3x^2-12x)+(-2x+8) \)  
\( 3x(x-4)-2(x-4) \)  
\( (3x-2)(x-4) \)

Lesson 9

**Factoring completely ~ Here we go!**

Complete factoring means that you must factor more than once!

Step 1: Look for a GCF

Step 2: If there is a GCF, factor it. If there not a GCF, determine the type of factoring.

Step 3: Factor what is left after you look for a GCF (or what you get when you factor out the GCF)
Step 4: Check using the distributive property or box method

Factor each of the following completely (MORE THAN ONCE!!)

1.) $by^2 - 4b$
   $b(y-2)(y+2)$

2.) $3x^2 - 6x - 24$
   $3(x-6)(x+4)$

3.) $4d^2 - 6d + 2$
   $2(2d^2-3d+1)$
   $2(d^2-3d+2)$
   $2(d-2/2)(d-1/2)$
   $2(d-1)(2d-1)$

4.) $18y^2 - 32$
   $2(3y-4)(3y+4)$

5.) $8x^2 - 98$
   $2(2x-7)(2x+7)$

6.) $x^4 - 1$
   $(x^2-1)(x^2+1)$
   $(x-1)(x+1)(x^2+1)$

Homework

Factor each polynomial completely. Circle your final answer.

1.) $4x^2 - 4$
   $4(x-1)(x+1)$

2.) $d^3 - 8d^2 + 16d$
   $d(d-4)(d-4)$

3.) $16x^2 - 16x + 4$
   $4(4x^2-4x+1)$
   $2(x^2-4x+4)$
   $2(x-2/4)(x-2/4)$
   $2(2x-1)(2x-1)$

4.) $ax^2 - ay^2$
   $a(x^2-y^2)$
   $a(x-y)(x+y)$

5.) $x^4 - 16$
   $(x^2-4)(x^2+4)$
   $(x-2)(x+2)(x^2+4)$

Lesson 10:

Factoring completely ~ Here we go….AGAIN!!

Complete factoring means that you must factor more than once!

Step 1: Look for GCF and factor it out
Step 2: Factor what is left after a GCF is removed or begin a different method
Step 3: Check using the distributive property and make sure that nothing more can be factored
Factor each of the following completely (MORE THAN ONCE!!)

1.) \(18m^2 - 8\)
\[2(2m-4)(2m+4)\]

2.) \(3x^2 + 6x + 3\)
\[3(x^2+2x+1)\]
\[3(x+1)(x+1)\]

3.) \(x^3 + 7x^2 + 10x\)
\[x(x^2+7x+10)\]
\[x(x+5)(x+2)\]

4.) \(4r^2 - 4r - 48\)
\[4(r^2-r-12)\]
\[4(r-4)(r+3)\]

5.) \(y^4 - 81\)
\[(y^2-9)(y^2+9)\]
\[(y-3)(y+3)(y^2+9)\]

6.) \(16x^2 - x^2y^4\)
\[x^2(4-y^4)\]
\[x^2(2-y^2)(2+y^2)\]

**Homework**

Factor completely.

1. \(2a^2 - 2b^2\)
\[2(a^2-b^2)\]
\[2(a-b)(a+b)\]

2. \(6y^2 - 14y + 4\)
\[2(3y^2-7y+2)\]
\[2(y^2-7y+6)\]
\[2(y-6/3)(y-1/3)\]
\[2(y-3)(3x-1)\]

3. \(18a^2 - 3a - 6\)
\[3(6a^2-a-2)\]
\[3(a^2-a-12)\]
\[3(a-4/6)(a+3/6)\]
\[3(3a-2)(2a+3)\]

**Lesson 11:**

1) \(10x^2 + 15x\)
\[5x(2x+3)\]

2) \(24x^2y^3 - 30x^2y\)
\[6x^2y(4y^2-5)\]

3) \(x^2 - 36\)
\[(x+6)(x-6)\]

4) \(121 - y^2\)
\[(11-y)(11+y)\]

5) \(49y^2 - 1\)
\[6) 64x^2 - 9y^2\]
(7y-1)(7y+1)  

7) $x^2 - 12x + 35$  
8) $y^2 - 5y - 24$  

(x-7)(x-5)  

(y-8)(y+3)  

9) $8x^2 - 2x - 1$  

$X^2-2x-8$  

$X^2-2x-8$  

$X^2-2x-8$  

$X^2-2x-8$  

$X^2-2x-8$  

$(x-4/8)(x+2/8)$  

$(2x-1)(4x+1)$  

$y^2 - 5y - 24$  

$(y-8)(y+3)$  

Always look for a GCF first!!!! You might be able to factor again. Keep factoring until you can’t factor anymore!

11) $x^2 + 6x - 27$  

$(x+9)(x-3)$  

12) $9x^2 + 18x + 5$  

$x^2+18x+45$  

$(x+15/9)(x+3/9)$  

$(3x+5)(3x+1)$  

4(x+7)(x+1)  

5(y^2-6y+8)  

5(y-4)(y-2)  

13) $4x^2 + 32x + 28$  

$4(x^2+8x+7)$  

$4(x+7)(x+1)$  

14) $3x^2 - 27$  

$3(x+3)(x-3)$  

15) $5y^2 - 30y + 40$  

$5(y^2-6y+8)$  

$5(y-4)(y-2)$  

16) $x^4 - 16$  

$(x^2-4)(x^2+4)$  

$(x-2)(x+2)(x^2+4)$  

$4x^3 + 10x^2 + 6x$  

$2x(2x^2+5x+3)$  

$2x(x^2+5x+6)$  

$2x(x+2/2)(x+3/2)$  

$2x(x+1)(3x+2)$  

17) $6x^2 + 18x - 60$  

$6(x^2+3x-10)$  

$6(x+5)(x-2)$  

Algebra:

Lesson 1:

Finding a Greatest Common Factor (GCF)

When two numbers are multiplied, the result is called their product
The numbers that are multiplied are called **factors**

**Example:** List all the factors of 80
1, 2, 4, 5, 8, 10, 16, 20, 40, 80

**GCF Factoring**

What does GCF stand for?
Greatest Common Factor

**Example:** Identify the GCF of each pair of numbers below.

a) 12 and 18  
GCF: 6  

b) 20 and 60  
GCF: 20  

c) 36 and 60  
GCF: 12

**Example:** Identify the GCF of each pair of monomials below.

a) $x^5$ and $x^3$  
$x^3$  

b) $5a^3$ and $15a$  
$5a$  

c) $8xy$ and $6xz$  
$2x$

**Practice:** Identify the GCF of each pair of monomials below.

a) $10x^2$ and $15xy^2$  
$5x$  

b) $25xyz$ and $36xyz$  
$xyz$  

c) $21ab$ and $14a^2b$  
$7ab$

**To factor a polynomial using a GCF:**

1. Find the GCF between the terms

2. Write the GCF outside parenthesis and divide each term by the GCF to fill in the inside of the parenthesis.

3. Check using the box method or distributing

**Ex. 1.** Factor $6x^2 + 8$  
Check

$$2(x^2 + 4)$$

$$6x^2 + 8$$

**Ex. 2.** Factor $5x^2 + 25x$  
Check

$$5x(x + 5)$$

$$5x^2 + 25x$$

**Ex. 3.** Factor $ax - 5ab$  
Check
\[(a(x - 5b) - ax - 5ab)\]

Ex. 4. Factor \(9x^3 + 36x^5\)  
\(9x^3(1+4x^2)\)  
Check \(9x^3 + 36x^5\)

Ex. 5. Factor \(6c^3d - 12c^2d^2 + 3cd\)  
\(3cd(2c^2 - 4cd + 1)\)  
Check \(6c^3d - 12c^2d^2 + 3cd\)

**Practice:** Factor the expression.

1. \(27x^6 + 9x\)  
2. \(15x^4 - 10x^2\)  
3. \(3a^5b^3 - 12a^7b^4\)  
\(9x(x^5 + 1)\)  
\(5x^2(3x^2 - 2)\)  
\(3a^2b^1(a^3 - 4b)\)

4. \(p + prt\)  
5. \(8z + 8\)  
6. \(c^3 - c^2 + 2c\)  
\(p(1 + rt)\)  
\(8(z + 1)\)  
\(c(c^2 - c + 2)\)

7. The perimeter of a rectangle is represented by \(2l + 2w\). Express the perimeter as the product of two factors.  
\(2(l + w)\)

8. Can the binomial \(3a + 5b\) be factored? Does that mean that for all values of \(a\) and \(b\), \(3a + 5b\) is a prime number? Justify your answer.  
No, there is nothing in common between 3 and 5. When \(a\) and \(b\) are multiples of each other, then it would be factorable.

**Homework**

1. Identify the greatest common factor for each of the following sets of monomials.  
   (a) \(2x^3, 6x^2,\) and \(12x\)  
   (b) \(16t^2, 48t,\) and \(80\)  
   (c) \(8t^5, 12t^3,\) and \(16t\)  
   \(2x\)  
   \(16\)  
   \(4t\)

2. Which of the following is the greatest common factor of the terms \(36x^2y^4\) and \(24xy^7\)?
3. Write each of the following as equivalent products of the polynomial’s greatest common factor with another polynomial (of the same number of terms). The first is done as an example.

(a) $18 - 12x$
   \[6(3-2x)\]

(b) $6x^3 + 12x^2 - 3x$
   \[3x(2x^2+4x-1)\]

(c) $x^2 - x$
   \[x(x-1)\]

(d) $10x^2 + 35x - 20$
   \[5(2x^2+7x-10)\]

(e) $21x^2 - 14x$
   \[7x(3x^2-2)\]

(f) $36x - 8x^2$
   \[4x(9-2x)\]

4. Which of the following is not a correct factorization of the binomial $10x^2 + 40x$?

(1) $10x(x+4)$
(2) $10(x^2 + 4x)$

(3) $5x(2x+4)$
(4) $5x(2x+8)$

5. Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor. Watch out for the subtraction problems (b) and (d).

(a) \[(x+5)(x+1)+(x+5)(x+8)\]
   \[(x+5)(x+1+x+8)\]

(b) \[(2x-1)(3x+5)-(2x-1)(x+4)\]
   \[(2x-1)(3x+5-x-4)\]

6. The area of a rectangle is represented by the polynomial $16x^2 + 56x$. The width of the rectangle is given by the binomial $2x + 7$.
   How would you represent the length of the rectangle? If the length is 80, what is the perimeter?

\[
\begin{align*}
8x &= 80 \
\text{so } x &= 10 \text{ then} \\
W &= 2(10)+7 = 27 \\
P &= 80+80+27+27
\end{align*}
\]
7. Which of the following is *not* a factor of $4x^2 + 12x$?

   (1) $x + 3$   (3) $3x$

   (2) $x$   (4) $4$

Lesson 2:

**Factoring a Difference of Perfect Squares**

Review: Squaring the following monomials.

1. $(3x)^2$  
   $9x^2$

2. $(7xy^2)^2$  
   $49x^2y^4$

3. $(\frac{2}{5}ab)^2$  
   $(4/25)a^2b^2$

4. $(9x^3)^2$  
   $81x^6$

What are “perfect square numbers” and why are they called that?  
*A number that is obtained by multiplying a number by itself.*

<table>
<thead>
<tr>
<th>Number</th>
<th>Perfect Square</th>
<th>Number</th>
<th>Perfect Square</th>
<th>Number</th>
<th>Perfect Square</th>
<th>Term</th>
<th>Perfect Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>25</td>
<td>9</td>
<td>81</td>
<td>a</td>
<td>a^2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>36</td>
<td>10</td>
<td>100</td>
<td>3b</td>
<td>9b^2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>7</td>
<td>49</td>
<td>11</td>
<td>121</td>
<td>c^2</td>
<td>c^4</td>
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<tr>
<td>4</td>
<td>16</td>
<td>8</td>
<td>64</td>
<td>12</td>
<td>144</td>
<td>x^3</td>
<td>x^6</td>
</tr>
</tbody>
</table>

Why is the exponent of each variable in the square of a monomial always an even number?  
*Because it is being multiplied by two when it is squared*  

Review: Multiply the following binomials.
1. \((x - 3)(x + 3)\)  \quad 2. \((2x - 5)(2x + 5)\)  \quad 3. \((a^2 - 6b)(a^2 + 6b)\)  
\(x^2-9\) \quad 4x^2-25 \quad a^4-36b^2

In order to use the **Difference of Perfect Squares** factoring method we must have:

1. A subtraction sign  
2. Even exponents  
3. Coefficients that are perfect squares

**Ex. 1.** \(m^2 - 4\)  
\((m-2)(m+2)\) \quad \text{Check}

**Ex. 2.** \(a^6 - 81\)  
\((a^3-9)(a^3+9)\) \quad \text{Check}

**Ex. 3.** \(9x^2 - 25z^4\)  
\((3x-5z^2)(3x+5z^2)\) \quad \text{Check}

**Practice: Factor.**

1. \(x^2 - 100\)  
\((x-10)(x+10)\)

2. \(a^2 - b^2\)  
\((a-b)(a+b)\)

3. \(49 - y^4\)  
\((7-y^2)(7+y^2)\)

4. \(25r^2 - 64\)  
\((5r-8)(5r+8)\)

5. \(4x^2 - 81y^2\)  
\((2x-9y)(2x+9y)\)

6. \(4x^4 - y^6\)  
\((2x^2-y^3)(2x^2+y^3)\)

7. \(x^2 - 16\)  
\((x-4)(x+4)\)

8. \(x^2 - 100\)  
\((x-10)(x+10)\)

9. \(x^2 - 1\)  
\((x-1)(x+1)\)

10. \(x^2 - 25\)  
\((x-5)(x+5)\)
Lesson 3:

Factoring a Trinomial

To factor a trinomial of the form $x^2 + bx + c$ using the “unbox” method:

1. Determine the factors of $c$
2. See what factors add to get the value of $b$
3. Write in parenthesis with the correct signs

Ex. 1. Factor $x^2 + 6x + 5$

$(x+5)(x+1)$

Ex. 2. Factor $x^2 - 3x - 10$

$(x-5)(x+2)$

Ex. 3. Factor $x^2 - 10x + 24$

$(x-6)(x-4)$

Practice: Factor each trinomial. ***MAKE FACTOR CHARTS!!!***

1. $x^2 + 6x - 27$
   $(x+9)(x-3)$

2. $x^2 - 6x + 9$
   $(x-3)(x+3)$
Lesson 3 Homework
Factor:

1. \( a^2 + 3a + 2 \)
   \( (a+2)(a+1) \)

2. \( x^2 + 8x + 7 \)
   \( (a+7)(a+1) \)

3. \( y^2 - 6y + 8 \)
   \( (y-4)(y-2) \)

4. \( y^2 - 2y - 8 \)
   \( (y-4)(y+2) \)

5. \( x^2 - 9x + 8 \)
   \( (x-8)(x-1) \)

6. \( x^2 + 11x + 24 \)
   \( (x+8)(x+3) \)

Factor using the GCF method.

7. \( 10x - 15x^3 \)
   \( 5x(2-3x^2) \)

8. \( 12y^2 - 4y \)
   \( 4y(3y-1) \)

9. \( 9b^3 - 6b^2 - 3b \)
   \( 3b(3b^2 - 2b - 1) \)

Lesson 4:
Factoring a Trinomial with Leading Coefficient > 1
(Slide and Divide and Grouping)

Factor:
1. \(4x^2 - 19x - 5\)  
   \((x-5)(4x+1)\)
2. \(9x^2 - 2x - 7\)  
   \((x-1)(9x+7)\)

3. \(12x^2 - 7x - 10\)  
   \((x^2-7x-120)\)  
   \((x-15/12)(x+8/12)\)  
   \((4x-5)(3x+2)\)
4. \(7x^2 + 53x + 28\)  
   \(x^2+53x+196\)  
   \((x+49/7)(x+4/7)\)  
   \((x+7)(7x+4)\)

5. \(4x^2 - 17x + 4\)  
   \((x^2-17x+16)\)  
   \((x-16/4)(x-1/4)\)  
   \((x-4)(4x-1)\)
6. \(5x^2 - 18x + 9\)  
   \((x^2-18x+45)\)  
   \((x-15/5)(x-3/5)\)  
   \((x-3)(5x-3)\)

7. \(4x^2 - 35x + 49\)  
   \((x^2-35x+196)\)  
   \((x-28/4)(x-7/4)\)  
   \((x-7)(4x-7)\)
8. \(6x^2 - 11x - 35\)  
   \((x^2-11x-210)\)  
   \((x-21/6)(x+10/6)\)  
   \((2x-7)(3x+5)\)

**Homework**

**Factor:**

1. \(5x^2 + 34x + 24\)  
   \(x^2+34x+120\)  
   \((x+30/5)(x+4/5)\)  
   \((x+6)(5x+4)\)
2. \(4x^2 + 38x + 70\)  
   \(x^2+38x+280\)  
   \((x+28/4)(x+10/4)\)  
   \((x+7)(2x+5)\)

3. \(2x^2 - 3x - 9\)  
   \(x^2-3x-18\)  
   \((x-6/2)(x+3/2)\)  
   \((x-3)(2x+3)\)
4. \(5x^2 + 23x + 24\)  
   \(x^2+23x+120\)  
   \((x+15/5)(x+8/5)\)  
   \((x+3)(5x+8)\)

5. \(4x^2 + 22x + 10\)  
   \(2(2x^2+11x+5)\)  
   \(2(x+5)(2x+1)\)  
   \(2(x^2+11x+10)\)  
   \(2(x+10/2)(x+1/2)\)
6. \(4x^2 - 8x - 45\)  
   \(x^2-8x-180\)  
   \((x-18/4)(x+10/4)\)  
   \((2x-9)(2x+5)\)

**Lesson 5:**

**Factoring Completely**

**THE GCF METHOD**

**REMEMBER:**

1.) Find GCF
2.) Divide the GCF from each term
1) $10x^2 - 5x$
   $5x(2x-3)$

2) $2x^4 - 6x^3 + 8x^2$
   $2x^2(x^2-3x+4)$

**FACTORIZING THE DIFFERENCE OF 2 PERFECT SQUARES**

Remember to find the square root of each term and place in parentheses. Make sure the sign in between is a subtraction sign.

**REMEMBER:**

1.) Take the square root of each term
2.) Set up (+)(-)
3.) Put square root of 1st and 2nd term in ()

3) $x^2 - 121$
   $(x-11)(x+11)$

4) $16x^4 - 9y^2$
   $(4x^2-3y)(4x^2+3y)$

**FACTORIZING TRINOMIALS and SLIP & DIVIDE**

5) $x^2 + 7x - 18$
   $(x+9)(x-2)$

6) $5x^2 - 14x - 3$
   $(x-15/3)(x+1/3)$
   $(x-5)(3x+1)$

**Putting it all together – factoring completely**

When factoring, we should always look for GCF first.

Sometimes, we can factor more than once. Continuing to factor until we can’t factor anymore is called factoring completely

**Practice**: Factor Completely – LOOK FOR A GCF FIRST!!!

7) $5x^2 - 45$
   $5(x-3)(x+3)$

8) $3x^2 - 6x - 24$
   $3(x^2-2x-4)$
   $3(x-4)(x+2)$

9) $ax^2 + 3ax + 2a$
   $a(x^2+3x+2)$
   $a(x+2)(x+1)$

10) $s^{t^2} - s$
    $s(t^2-1)$
    $s(t-1)(t+1)$

11) $x^3 - 8x^2 + 12x$
    $x(x^2-8x+12)$
    $x(x-6)(x-2)$

12) $x^4 - 16$
    $(x^2-4)(x^2+4)$
    $(x+2)(x-2)(x^2+4)$

13) $2x^2 - 32$
    $2(x^2-16)$

14) $16x^2 - 16x + 4$
    $4(4x^2-4x+1)$

15) $ax^2 + 3ax$
    $ax(x+3)$
FACTORING MAZE

See if you can find your way to the end of the maze!

You can move between squares (diagonally too) if they share a common factor.

Lesson 5 Homework:
Factor completely.

1. \(2a^2 - 2b^2\)
   \[2(a+b)(a-b)\]

2. \(ax^2 - ay^2\)
   \[a(x-y)(x+y)\]

3. \(3x^2 + 6x + 3\)
   \[3(x^2+2x+1)\]
   \[3(x+1)(x+1)\]

4. \(x^3 + 7x^2 + 10x\)
   \[x(x^2+7x+10)\]
   \[x(x+5)(x+2)\]

5. \(6y^2 - 14y + 4\)
   \[2(3y^2-7y+2)\]
   \[2(y^2-7y+6)\]
   \[2(y-6/3)(y-1/3)\]
   \[2(y-2)(3y-1)\]

6. \(18a^2 - 3a - 6\)
   \[3(6a^2-a-2)\]
   \[3(a^2-a-6)\]
   \[3(a-3/6)(a+2/6)\]
   \[3(2a-1)(3a+2)\]

Algebra 2

Lesson 1:

New Method! Sum and Difference of Perfect Cubes

First, what are our perfect cubes?
1, 8, 27, 64, 125, 216, etc

Steps for Sum of Different Cubes:
1.) To get the binomial, take the cube root or the original two terms

2.) Square the first term, multiply the two terms, square the last term.

3.) The signs will always be \((+)(-)+\)

In general:

\[a^3+b^3=(a+b)(a^2-ab+b^2)\]

Ex: \(x^3 + 27\)

\[(x+3)(x^2-3x+9)\]
Ex: $125x^3 + y^6$

$(5x+y^2)(25x^2-5xy^2+y^4)$

**Difference of Perfect Cubes:**

Note, the only difference from the Sum of Perfect Cubes is the signs.

In general: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Ex: $x^3 − 64$

$(x-4)(x^2+4x+16)$

Ex: $8r^3 − 343s^2$

$(2r-7)(4r^2+14r+49)$

So, let's practice. Factor completely $y^6 + 3y^3 − 40$

$(y^3+8)(y^3-5)$

$(y+2)(y^2-2y+4)(y^3-5)$

**Lesson 2:**

**SLIDE AND DIVIDE!!!**

Steps: 1.) Multiply $a$ and $c$ to create a new trinomial

2.) Factor as normal

3.) Divide each factor by $c$.

Ex: $5x^2 − 14x + 8$

$x^2-14x+40$

$(x-10/5)(x-4/5)$

$(x-2)(5x-4)$
Lesson 2 Homework

For questions 1-8, write each of the following trinomials in factored form.

1) \( x^2 - 7x - 18 \)
   \((x-9)(x+2)\)

2) \( 18x^2 - 25x + 8 \)
   \(x^2-25x+144\)
   \((x-9/18)(x-16/18)\)
   \((2x-1)(9x-8)\)

3) \( 5x^2 - 41x + 8 \)
   \(x^2-41x+40\)
   \((x-40/5)(x-1/5)\)
   \((x-8)(5x-1)\)

4) \( 3x^2 + 4x - 20 \)
   \(x^2+4x-60\)
   \((x+10/3)(x-6/3)\)
   \((3x+10)(x-2)\)

5) \( 2x^2 - 29x - 15 \)
6) \( x^2 - 17x + 30 \)
9) Consider the trinomial $12x^2 + 7x - 10$.

(a) Does this trinomial have a greatest common factor that could be “factored out?”
No, there is nothing in common between 12, 7 and 10.

---

**Lesson 3:**
Name: ____________________________________________ Date:_____

**FACTORING COMPLETELY**

When an expression is **completely factored**, it is written as a product of polynomials, none of which can be factored further.

Ex: Consider the trinomial $2x^2 - 4x - 6$.

(a) Verify through multiplication that the following are factors of the given trinomial.

$$2x^2 - 4x - 6 = (2x + 2)(x - 3)$$

(b) Why would neither of the expressions from part (a) be considered to be **completely factored**?

There is a GCF in each of the factors. A GCF can be pulled out first.

$$2(x - 3)(x + 1) = 2(x + 1)(x - 3)$$
(c) Note that the two solutions you obtained from part (c) are equivalent. Show that you could also obtain this result by factoring out a GCF from \( 2x^2 - 4x - 6 \) to begin with.

\[
\begin{align*}
2(x^2-2x-3) \\
2(x-3)(x+1)
\end{align*}
\]

** When factoring, it's best to always look for a GCF first!!! **

**Try:** Factor each of the following expressions completely.

(a) \[4x^2 + 12x - 40\]

\[
\begin{align*}
4(x^2+3x-10) \\
4(x+5)(x-2)
\end{align*}
\]

(b) \[75 - 3x^2\]

\[
\begin{align*}
3(25-x^2) \\
3(5-x)(5+x)
\end{align*}
\]

(c) \[10x^2 + 55x - 105\]

\[
\begin{align*}
5(2x^2+11x-21) \\
2(x^2+11x-42) \\
2(x-3/2)(x+14/2) \\
2(2x-3)(x+7)
\end{align*}
\]

(d) \[12x^2 + 57x - 15\]

\[
\begin{align*}
3(4x^2+19x-5) \\
3(x^2+19x-20) \\
3(x+20/12)(x-1/12) \\
3(3x+5)(12x-1)
\end{align*}
\]

Factor each of the following expressions:

(a) \[6x^2 - 13x + 6\]

\[
\begin{align*}
(x^2-13x+36) \\
(x-9/6)(x-4/6) \\
(2x-3)(3x-2)
\end{align*}
\]

(b) \[12x^2 + 29x - 8\]

\[
\begin{align*}
(x^2+29x-96) \\
(x-3/12)(x+32/12) \\
(4x-1)(3x+8)
\end{align*}
\]

**Lesson 3 Homework:**

Completely factor each of the following expressions:

1. \[2x^2 - 14x - 36\]

\[
\begin{align*}
2(x^2-7x-18) \\
2(x-9)(x+2)
\end{align*}
\]

2. \[3x^2 - 192\]

\[
\begin{align*}
3(x^2-64) \\
3(x-8)(x+8)
\end{align*}
\]

3. \[8x^2 + 12x - 8\]

\[
\begin{align*}
4(2x^2+3x-2) \\
4(x^2+3x-4) \\
4(x+4/2)(x-1/2) \\
4(x+2)(2x-1)
\end{align*}
\]

4. \[5x^2 + 70x + 245\]

\[
\begin{align*}
5(x^2+14x+49) \\
5(x+7)(x+7)
\end{align*}
\]

5. \[10x^3 - 26x^2 - 12x\]

6. \[28x - 7x^3\]
2x(5x^2-13x-6) 
2x(x^2-13x-30) 
2x(x-15/5)(x+2/5) 
2x(x-3)(5x+2) 

7x(4-x^2) 
7x(2-x)(2+x) 

7. 15x^2 - 110x + 120 
5(3x^2-22x+24) 
5(x^2-22x+72) 
5(x-18/3)(x-4/3) 

8. 12x^2 - 20x + 3 
x^2-20x+36 
(x-18/12)(x-2/12) 
(2x-3)(6x-1) 

9. 45x - 20x^3 
5x(9-4x^2) 
5x(3-2x)(3+2x) 

10. 8x^2+67x + 24 
x^2+67x+192 
(x+3/8)(x+64/8) 
(8x+3)(x+8) 

11. 20x^2 + 112x - 48 
4(5x^2+28x-12) 
4(x^2+28x-60) 
4(x+30/5)(x-2/5) 
4(x+6)(5x-2) 

12. 90x^3 - 90x^2 + 20x 
10x(9x^2-9x+2) 
10x(x^3-9x+18) 
10x(x-6/9)(x-3/9) 
10x(3x-2)(3x-1) 

13. 8x^2 + 30x + 28 
2(4x^2+15x+14) 
2(x^2+15x+56) 
2(x+8/4)(x+7/4) 
2(x+2)(4x+7) 

14. 27x^2 - 3 
3(9x-1) 
3(3x-1)(3x+1) 

15. 18x^2 - 39x - 15 
3(6x^2-13x-5) 
3(x^2-13x-30) 
3(x-15/6)(x+2/6) 
3(2x-5)(3x+1) 

Lesson 4: 

**Factoring by Grouping** 

When we are given a polynomial to factor, and we have more than 3 terms with no GCF, we try factoring by grouping. 

Steps:
1.) Group
2.) Take GCF of both binomials
3.) Factor the common result out of the entire expression
4.) Factor completely

Ex: $5a^3 - a^2 + 5a - 1$

$a^2(5a-1)+1(5a-1)$

$(a^2+1)(5a-1)$

Ex: $2x^3 + 10x^2 + 7x + 21$

$2x^2(x+5)+7(x+3)$

**Can not be factored because what is in the parenthesis is not the same.

Ex: $x^3 + x^2 - 4x - 4$

$x^2(x+1)-1(x+1)$

$(x^2-1)(x+1)$

$(x+1)(x-1)(x+1)$

Ex: $8x^4 + 24x^3 + 125x + 375$

$8x^3(x+3) + 125(x+3)$

$(x+3)(8x^3+125)$

$(x+3)(2x+5)(4x^2+10x+25)$

Ex: $x^2 - ax - bx + ab$

$x(x-a)-b(x-a)$

$(x-b)(x-a)$

Ex: $4x^3 - 8x^2 - x + 2$

$4x^2(x-2) -1(x-2)$

$(4x^2-1)(x-2)$

$(2x-1)(2x+1)(x-2)$

Ex: $x^4 + 5x^3 + 8x + 40$

$x^3(x+5)+8(x+5)$

$(x^3+8)(x+5)$

$(x+2)(x^2+2x+4)(x+5)$
<table>
<thead>
<tr>
<th>Method</th>
<th># of Terms</th>
<th>Pattern</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GCF</td>
<td>Any</td>
<td>Write the GCF out front and divide each term by the GCF to fill in the parenthesis</td>
<td>$3x^2y^4z+9xz$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$3xz(xy^4+3)$</td>
</tr>
<tr>
<td>2 Trinomial</td>
<td>3</td>
<td>Multiply to the ending term, adding to the middle term.</td>
<td>$1.) \ x^2-6x+9$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(x-3)(x-3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$2.) \ 5x^2 - 41x + 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x^2-41x+40$</td>
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<td></td>
<td>$(x-40/5)(x-1/5)$</td>
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<td></td>
<td></td>
<td>$(x-8)(5x-1)$</td>
</tr>
<tr>
<td>3 Difference of Squares</td>
<td>2</td>
<td>Take the square root of the first term and the square root of the second term with opposite signs in the parenthesis</td>
<td>$x^2y^4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(x-y^2)(x+y^2)$</td>
</tr>
<tr>
<td>4 Sum of Cubes</td>
<td>2</td>
<td>Take the cube root of the first term and second term. Square the first term of the binomial, multiply the two terms in the binomial, and square the second term in the binomial</td>
<td>$8x^3+343$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(2x+7)(4x^2+14x+49)$</td>
</tr>
<tr>
<td>5 Difference of Cubes</td>
<td>2</td>
<td>Take the cube root of the first term and second term. Square the first term of the binomial, multiply the two terms in the binomial, and square the second term in the binomial</td>
<td>$8x^3-343$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(2x-7)(4x^2+14x+49)$</td>
</tr>
<tr>
<td>6 Grouping</td>
<td>4</td>
<td>Take the first two terms as a group, find the GCF of them, take the second two terms as a group and find the GCF, factor out the common factor</td>
<td>$4x^3 - 8x^2 - x + 2$</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>$4x^2(x-2)-1(x-2)$</td>
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<td></td>
<td></td>
<td>$(4x^2-1)(x-2)$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$(2x-1)(2x+1)(x-2)$</td>
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