Homogeneous versus Heterogeneous Grouping and Differentiation

Kristin Duschen

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Homogeneous versus Heterogeneous Grouping and Differentiation

by

Kristin Duschen

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Master of Science in Education
Homogeneous versus Heterogeneous Grouping and Differentiation

by

Kristin Duschen

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Introduction

Not unlike many things, no two school districts are the same. Districts across the globe vary in structure, policy, composition, and focus. Within these very different school districts, the student body is further diversified, contrasting in race, gender, ethnicity, socio-economic background, and ability levels. The one common thread weaving its way through all school districts is the fundamental role of the teacher. No matter how different the students are or what the strategies of the administration may be, all teachers must teach. Teachers must find a way to relay the necessary information to push their students to the next level of learning in order to foster intelligent, educated members of society. How this is done is the subject of debate and research across the planet.

This research focuses on one aspect of teaching and that is the question of grouping students based on academic ability. Which method of grouping is more effective, homogeneous (grouping similar academic levels together) or heterogeneous (grouping different academic levels together). Although this research focuses on a middle level mathematics classroom, it will prove important for all teachers in all subject areas and could potentially change the teacher’s role. As more districts move towards inclusive classroom settings, grouping students within these settings will take on a larger role and have greater side effects. The completion of this research hopes to provide teachers with information as to the outcome of homogeneous versus heterogeneous grouping before these methods are enacted in order to benefit student achievement and attitude. If teachers are made aware of the preconceived notions, advantages, disadvantages, and necessary accompaniments of grouping students during instruction it will grant a more appropriate plan for success.
Literature review

Over the past fifty years, schools across the world have transitioned back and forth between styles of instruction. As time goes by, society progresses, new research is constantly being conducted and theories are getting published, while educational policy is continuously scrutinized. New decades bring popular instructional strategies that reflect the various educational values and perspectives of the times.

One specific issue that has been at the forefront of educational research is the idea of grouping students by ability. Is ability grouping beneficial to the majority of students? Ability grouping has taken on many names, tracking, setting, streaming, whatever the term, the idea is the same. “Tracking is an organizational practice whose aim is to facilitate instruction and to increase learning” (Hallinan, 1994, p. 79). This definition is straightforward, it sounds efficient and relatively simple to initiate. Why then, is there so much controversy surrounding the idea? The goal of this research is to investigate tracking, its current practice, development, and consequences to decide if it is, in fact, a good idea for today’s diverse student-body. If it is not, then what types of instructional strategies can teachers use to best teach a group of mixed-ability learners in order to achieve maximum academic success?

The idea behind tracking is based on the increased efficiency of teaching groups of students who are at similar levels of understanding. Teachers should be able to tailor their instruction to meet the specific needs of each group, therefore, increasing academic success and fostering progress. However, according to research, this structure does not yield the levels of results that are intended. “There is evidence for a decreasing trend in average mathematics performance accompanied by a significant decline of students’
interest in mathematics achievement” (Schiefele & Csikszentmihalyi, 1995, p. 164).

What is the cause of this decline in ability and interest? Research tells us that the pacing of each level of instruction is not appropriate, students find it difficult to transition between sets, students in the lower tracks are being disadvantaged, and teachers use preconceived notions to establish the classroom atmosphere and norms.

An article by Dylan William and Margaret Brown focusing on the negative consequences related to ability grouping piggybacks off earlier research done by Jo Boaler. In their article, William and Brown collect qualitative research through interviews, questionnaires, and classroom observations from almost 1000 students in London. They reported, among other things, that students in every academic level lacked proper instruction because the pace was inappropriate to their level of understanding.

“Significant numbers of students experienced difficulties working at the pace of the particular set in which they were placed. For some students the pace was too slow, resulting in disaffection, while for others it was too fast, resulting in anxiety. Both responses led to lower levels of achievement” (2000, p. 634).

Finding the balance between pushing the students to excel and giving them too long to understand can make the difference between keeping students engaged and losing their interest.

The lack of flexibility amongst tracks is often times another issue. As students enter each school year, they are placed into certain sets depending on their cognitive skills and levels of development. Different districts use different categories to determine track placement. Previous grades, test scores, and teacher recommendations are among the most popular qualifiers. However, as the year goes on and students develop at varying rates, the homogeneous groups that might have started off the school year begin to
separate into distinct levels. William and Brown also discovered that, “Forty of 48 students interviewed from setted groups wanted...to change sets” (2000, p. 635). Many of the schools involved in the study explained the process that has been established to allow students to make the transition between sets if they see the need. However, the children in William and Brown’s study seemed to think there was no way for them to move up or down. The students felt stuck in their group with no hope for movement, a notion that created further feelings of despair.

The discrepancy between tracks is arguably the worst outcome of tracked, structured classrooms. “The most serious, unintended negative effect of tracking is the slower growth in achievement of students in low tracks” (Hallinan, 1994, p. 82). This is the area that most of the research is focused on within this larger topic. The students in the lower tracks are often disadvantaged for many reasons. They do not have the needed support at home to help with their studies, they have more serious matters to attend to once they do get home, they have less role models to relate to, and they may not have the resources to get extra help inside or outside of school. Hallinan further comments that, “Since academic achievement is related to students’ background, minority and low-income students are disproportionately assigned to lower tracks” (1994, p. 81). Thus, in order to save face, many administrators and districts leave more subjective methods of assignment to this lower level in order to avoid discrimination or racial concerns.

The effects of low track assignments extend much further. Research has found that students in the lowest track are not only low in the academic hierarchy but also experience negativity regarding their status and social ranking as well. Hallam and Ireson agree that structured tracks can often be detrimental to the achievement of lower track
students and add that, "It can also have a negative impact on self esteem, pupils’
experiences in school, their preferences for different grouping systems, and their attitudes
towards school" (2005, p. 4). Yair extends and broadens this idea, "Students’ alienation,
boredom, and low emotional mood while learning are inherently correlated with their
school experiences. Researchers have shown that many curricula and tests aim toward
low-level thinking skills and minimal competency standards" (2000, p. 192). Research
from Boaler agrees that many of the students in the lower tracks often felt unchallenged,
like the material was too easy, and were upset with the low level of expectations placed
on them (2006, p. 638). Even though the goal of tracking is to meet all students at their
levels of understanding and adapt instruction to help them all progress to a higher level of
comprehension, it conversely, seems to be widening the gap between the tracks.

Students from the lowest tracks, however, are not the only ones complaining.
Students in the highest tracks are also uncomfortable with the levels of academic stress
and strain placed on them. "Approximately one-third of the students in the highest ability
groups were disadvantaged by their placement in these groups because of high
expectations, fast-paced lessons and the pressure to succeed" (William & Brown, 2000, p.
633). Other research found similar ideas that the students in higher tracks are often
encouraged to work independently, to develop their own ideas, and to reflect on their
education while lower tracked students are supposed to conform, get along, cooperate,
and use specific methods and procedures (Hallam & Ireson, 2005, p. 5). The
preconceived notion of teachers is a large contributing factor to the failure of tracking in
today’s ever-changing school environments. Teachers often change the way they teach
depending on the level of their instruction. When teaching the higher tracked classes,
instruction tends to be faster, accompanied by less explanation and more concrete, less investigative class work and examples. These high achieving classes have been described by Hallam and Ireson in a study done in the UK as placing an emphasis on procedural knowledge and memorization in an effort to push through the material and progress.

What is the common opinion of teachers about tracking or streaming in our schools? Hallam and Ireson looked into this very question and interviewed over 1500 secondary teachers regarding their opinions about tracking. Teachers seemed to be in agreement that teaching is easier when classes are set. Teachers commented that they found themselves having to make more materials when the classes were composed of mixed-ability students (2005, p. 17). However, some realized that, “Teaching mixed-ability groups keep teachers on their toes. It forces them to be more creative and to maintain and develop a better range of teaching styles” (2005, p. 9). Classroom management was also agreed to be slightly less stressful when the groups are homogeneous. “All pupils make more progress when set, I find. Pupils become very frustrated when they are in mixed-ability groups. The more able pupils need to be stretched and the less able need specific attention” (2005, p. 9). It is interesting that with so much research focusing on the negative consequences of homogeneous grouping, teachers seem to believe that pupils work better during homogeneous instruction.

The gap between the experience of teachers and the findings of researchers seems to stem from the lack of ability of teachers to effectively teach mixed-ability lessons. In many traditional math classrooms, status is uncovered by the ability to discover the correct answer. If teachers can broaden their version of correct by providing more open-ended questions, valuing different learning abilities, and investigating thinking processes
than more students can feel accomplished in their math classrooms. “When there are many ways to be successful, many more students are successful” (Boaler, 2006, p. 42). Yair adds that because schools are often formal environments with structure and discipline, students can easily lose control and experience feelings of helplessness in regards to their own education (2000, p. 193). The more choices they are given and the more actively involved they are in their learning, the more engaged they will ultimately be.

The research seems to clearly point out that tracking does not accomplish the very thing that it is set out to do. It is supposed to help teachers individualize instruction to meet the needs of each track. Instead, teachers are teaching in ways according to pre-conceived notions of each level of ability. “For many students, they allege normative comparisons with others, and the public nature of whole-classroom instruction causes many students to feel they accomplish only mediocre achievements, and that school hurts one’s sense of self and one’s enjoyment from learning” (Yair, 2000, p. 194).

If homogeneous classrooms are not working and tracking is actually causing detrimental effects on students, then teaching classes with mixed-abilities is the only other option. Student with various levels of academic understanding will be in the same classroom together. This can seem like a daunting task for teachers, especially teachers that have been in the profession for many years. Is heterogeneous grouping really going to produce the positive effects and academic and social growth that is the very goal of education? Is this type of instruction going to be worth the hardships that teachers are seemingly going to face if the transition is made?
Boaler followed high school students in both the US and England to explore the effects of heterogeneous grouped classrooms. He found that,

"In both studies the schools that used mixed-ability approaches resulted in higher overall attainment and more equitable outcomes. But in both cases the mathematics departments that brought about higher and more equitable attainment employed particular methods to make the heterogeneous teaching effective" (2006, p. 41).

This finding is crucial in the effort of teachers to make mixed-ability teaching work. The key is not just to switch to mixed-ability instruction but to make that mixed-ability instruction more effective. Hallam and Ireson comply with Boaler's findings by saying,

"Successful mixed-ability teaching relies heavily on teacher skills including flexibility, the adoption of a wide variety of teaching modes in one lesson, variation in pace and style of approach, use of a range of audio-visual media and encouragement of a variety of pupil activities" (2005, p. 6).

It is not the transition from homogeneous classrooms to heterogeneous classrooms that produces positive effects. The transition has to be accompanied by effort and planning on the part of the teacher. If done correctly with support from other faculty members, access to resources, and the desire to make a positive change, a shift to heterogeneous grouping can be successful. An abundance of research on heterogeneous grouping includes statements from students expressing their enjoyment from this type of instruction. "I prefer groups when we're all mixed up...because some things the clever are good at and some things the not so educated are good at" (William & Brown, 2000, p. 643). Teachers must make connections with the students, involve them in their learning, and create lessons that are engaging and fun for the pupils. If the students can see the benefit of working together as one class then they will learn to appreciate mixed-ability instruction.
Teaching to a mixed-ability classroom is referred to as differentiated instruction. According to Rock, Gregg, Ellis, and Gable, “Differentiated instruction is the process of ensuring that what a student learns, how he/she learns it, and how the student demonstrates what he/she has learned is a match for that student’s readiness level, interests, and preferred mode of learning” (2008, p. 32). This team of researchers concentrates on two teachers trying to make the transition to heterogeneous grouping. They make a great argument by acknowledging that, “According to the 26th Annual Report to Congress on IDEA (U.S. Department of Education, 2005), roughly 96% of general education teachers have students with learning disabilities in their classrooms” (2008, p. 32). Classrooms are composed of mixed ability kids anyways. Whether or not they are actually tracked, there are students with learning difficulties in almost every classroom. Why not plan lessons with both these students and higher achieving students in mind. Many studies have found encouraging effects on differentiated instruction. One example found from Rock, Gregg, Ellis, and Gable reported on the findings of a high school teacher, “...the average student in her high school read at a 5.9 grade level. After 4 years of differentiated instruction, the average student read at an 8.2 grade level” (2008, p. 34). Another study they commented on referred to elementary teachers whose students started at an aptitude level of 80% and after years of differentiation, they progressed to approximately 95% proficiency (2008, p. 34). These are just a few of the success stories that have resulted from differentiated instruction.

If mixed-ability grouping is so successful and beneficial to student learning then why isn’t this the required style of all teachers? The answer to this question lies in the very definition of differentiation. This type of teaching will require that teachers spend
the time, obtain the resources, and step outside their comfortable zones to make lessons that are interactive while addressing multiple modalities. Yair comments that, “Such reforms call for authentic instruction and assessment of student learning; they envision a student-centered approach, where each individual is allowed freedom and autonomy to develop his or her unique talents” (2000, p. 194).

What advice is made available for teachers and districts to make this transition successfully? Rock, Gregg, Ellis, and Gable offer four guiding principles when attempting to implement differentiated instruction:

“(a) a focus on essential ideas and skills in each content area, (b) responsiveness to individual student differences, (c) integration of assessment and instruction, and (d) an ongoing adjustment of content process, and products to meet individual students’ levels of prior knowledge, critical thinking, and expression styles” (2008, p. 33).

Reed emphasizes that pre-assessment is a huge element of successful heterogeneous teaching (2004, p. 91). In order to teach to each student, teachers must know the basic understanding of each student to eliminate repetition and boredom or confusion and lack of comprehension. “Student learning style can be determined through learning styles questionnaire or inventories given early in the year. By grouping students who are kinesthetic, linguistic, and artistic into separate groups they can demonstrate three distinct ways to solve problems” (Levy, 2008, p. 162). Getting to know not only the prior knowledge of students, but their interests, how they learn, their preferences, strengths, and weaknesses will all help to differentiate. The more you know about each learner, the greater chance of reaching them during differentiated instruction. After studying the implementation of differentiation by two teachers, Rock, Gregg, Ellis, and Gable developed REACH – a system to help implement and refine differentiated instruction.
"Reflect on will and skill, Evaluate the curriculum, Analyze the learners, Craft research-based lessons, and Hone in on the data" (2008, p. 34). Keeping these five necessary ingredients in mind when starting to differentiate will help to ease the transition.

Much of the research regarding effective differentiation techniques focuses on cooperative learning. Cooperative learning is a great way to differentiate instruction. However, this also must be supported with constant monitoring, encouragement of teamwork, and respect for each others ideas and opinions. Boaler suggests that in the cooperative learning atmospheres, it may be helpful for teachers to praise an idea from a lower-ability student. This will not only increase the self-esteem of this student but it may help to change that student's perception from his peers (2006, p. 42). Teachers should continuously express the idea that hard work and effort is what creates successful mathematics students and solely ability. Lindquist comments on how small group learning when done effectively, can develop problem solving into, "...an approach rather than a content area, and thinking becomes a primary goal" (1989, p. 627). She goes on to say that group work helps develop listening skills and meaning behind the material. It has been found that students develop stronger respectful relationships when engage in cooperative problem-solving. In the right atmosphere, they learn to value each others opinions, to offer and take help when needed, and the power of multiple viewpoints.

There are many other ways to differentiate instruction that does not involve cooperative learning in groups. Allowing students to preview the material before actually teaching the lesson is a great way to activate prior knowledge for the struggling students. Incorporating some type of graphic organizer can help to engage the visual learners in the classroom. An organizer will also help the weaker students by providing a framework for
where they need to be by the end of the lesson or the unit. Granting students a choice of
daily tasks will show students that their opinions are valued and that their ideas matter.

“The more students feel in command of their learning and feel active and excited by it,
the more they fulfill their learning potential” (Yair, 2000, p. 193). Throughout the lesson,
teachers can use open-ended questions to help support the progression of student thought.
By adding open-ended questions to investigations and discussions teachers are permitting
multiple answers and approaches. Choral responses give students a feeling of
camaraderie in the classroom that will foster teamwork and the development of a
community atmosphere. This technique will also help musical and artist learners by
connecting the material to rhythms and sounds. To address the kinesthetic learner, the use
of manipulatives is a direct and effective tool. Research has also found positive outcomes
from peer-assisted learning strategies (PALS). This activity benefits both the weaker and
stronger student by requiring reciprocal teaching. Most importantly, teachers need to take
advantage of these differentiation techniques as constant assessments. Due to the varied
levels of student thought and progress that will inevitably exist in mixed-ability
classrooms, teachers need to keep close eye on student achievement by using a variety of
formative assessments throughout each unit of instruction.

Despite this extra attention on student understanding through consistent
assessment, it can still be easy to plan the majority of the lesson for the average student.
Reed spent time in a high school geometry differentiated class to witness the types of
advanced problems and techniques that the teacher set into place for the gifted students.
She defined mathematically gifted students as those who, “…are able to do mathematics
typically accomplished by older students or engage in qualitatively different
mathematical thinking than their classmates or chronological peers" (2004, p. 90). The teacher that Reed observed had great ideas for constantly challenging this group without interfering with regular class time. Once it was clear that a group of students had finished and mastered the lesson and material designated for that class period, the teacher had extensions ready for investigation. She gave them ideas for higher level topics to research and present to the class, she developed an inquiry wall for the students to ask and answer higher leveled thinking questions, or she had them generalize findings and develop their own theories. Thinking outside the box allowed this group of gifted students in a heterogeneous classroom to challenge themselves, ponder open-ended questions, and investigate the material at a higher complexity led by their own explorations.

It is clear that students like the idea of heterogeneous grouping and statistics have shown how academic progress has made as a product of this mixed-ability instruction. Research has found many other positive effects relating to social, emotional, and attitudinal changes in student behavior. Schiefele and Csikszentmihalyi were interested in how interest and motivation relate to academic success. They analyzed data from over one hundred high school freshman and sophomores from questionnaires and aptitude tests. They found that, "...intrinsic motivation can only be maintained as long as learning activities lead to a certain level of positive emotional experience" (1995, p. 177). By differentiating instruction and guiding lessons to meet the needs of all learners, then positive classroom attitudes will persever. "Positive feelings contribute to students’ creativity, problem-solving capacity, and deep comprehension. Furthermore, the quality of experience during learning is a crucial factor for future motivation to learn" (Schiefele & Csikszentmihalyi, 2005, p. 164). Once these positive results are established, then
students gain more motivation and start down a path of academic success. Finally, this
team of researchers realized that, "Specifically, interest showed significant relations to
potency, intrinsic motivation, self-esteem, importance, and the perception of skill" (1995, p. 173). Ultimately, interest, motivation, and academic success become a cycle of
achievement and a great way to set this cycle into motion is through heterogeneous
grouping and interactive lessons.

At this current state of educational policy, heterogeneous grouping seems to be a
great fit for producing the academic and social/emotional success that students, teachers,
parents, administrators, and policymakers are craving. However, it will only be a good fit
if teachers and students are utilizing this method of instruction the way it is intended.
There is no doubt that a transition to mixed-ability grouping will not come without
hardships and struggles. The secret to success is to stay positive while keeping long-term
goals and objectives in mind; if that can be done, then heterogeneous grouping may hold
the key to an overall improvement in student success.
Experimental Design

The first step in this research study was to complete a preliminary review of literature. After researching grouping from more than ten different sources, it is feasible to draw the conclusion that heterogeneous grouping and differentiated instruction is most beneficial grouping method for the largest number of students in a mathematics classroom.

Taking these findings into account, the next step in this research is to further investigate heterogeneous grouping and differentiation and either agree with the research (heterogeneous grouping and differentiation is the most beneficial method of grouping) or disagree with the findings (shows that homogeneous grouping will be more successful) and discuss further implications. The investigation in this case has been done through the use of a differentiated unit plan focusing on Elementary Set Theory that is based on the research and suggested differentiated strategies for effective heterogeneous grouping. This unit plan is included in the subsequent section. Set Theory, a unit included in the NYS Integrated Algebra Standards has literal elements and straightforward components that can be pushed and progressed to a much more theoretical level. The unit includes multiple strategies to address all students in an inclusive classroom, various modalities of instruction to help all types of learners, and the basic essentials as well as advanced extensions in each lesson which can be tailored individually to keep students motivated and engaged.

Upon completion of the research-based differentiated unit plan, the next step is to use this unit to test the initial research conclusions. This has been done by distributing it to experienced teachers to read and reflect. Because I do not yet have my own classroom
for action research, I will rely on the expertise of other teachers to comment on the deliverance of the unit plan. Teachers have read over the unit and all its materials and drawn their own conclusions regarding various aspects of student performance as if they had conveyed this unit to a full classroom themselves.

After these teachers read the unit they reflected on what responses they think students will have after participating in this unit. These reflections took place on a Teacher Questionnaire that is included in Appendix A. The Questionnaire format portrays a Likert scale and asked teachers to respond on a range from strongly disagree (1) to strongly agree (5) with an option of undecided (3). A quantitative analysis of their opinions is discussed below. Under each question, space was provided to further explain their rating or give additional thoughts if deemed necessary. A sampling of these teacher responses occurs later in the study. All questions have been asked in a positive manner to maintain consistency throughout the questionnaire. The positive format of the questions was done to minimize confusion and to allow for representative results.
Unit Plan

Unit Introduction

Lesson 1 – Set Theory Introduction

Lesson 2 – Set Notation

Lesson 3 – Types of Sets

Lesson 4 – Set Relationships

Lesson 5 – Set Operations

Lesson 6 – RAFT Project

Unit Conclusion – Review and Exams
Unit Introduction

Classroom Factors

Lesson Structure

Overall Goals

Pre Assessment

Dessert Project

Unit Calendar

Unit Graphic Organizer
Classroom Factors

This unit is designed for an integrated algebra inclusive eighth grade classroom. Students with special needs, IEPs, or many other classifications should be able to successfully complete this unit. While it is an introduction to algebra, it also demonstrates increased depth and difficulty. I have intended this unit for a class of 25 students, but the materials could very easily be adjusted. Small group learning, reciprocal teaching, partner work, and whole class discussions are all utilized throughout this unit in conjunction with direct instruction. A daily instructional change is provided in order to help maintain student interest and classroom energy.

Lesson Structure

Every class is structured the same to sustain order and efficiency in the calculated 40 minute class time. The structure is based on food, a topic that everyone can relate to. Appetizers (warm-ups) start each class to ensure that no time is wasted and to activate students' prior knowledge. This is followed by the Salad Course, a jumping off point for the rest of the lesson. The Salad Course may include direct instruction and note taking or may involve student discovery; either way the salad provides the building blocks for the lesson. The essence of the lesson is captured in the Main Course. This is the part of the lesson where the essentials are tackled, the standards are met, and the material is formalized while guided practice usually follows along. Whether notes are taken during the salad or the main course, there are always three versions provided. The versions differ in the amount the students are required to fill in and complete. Struggling students or
students with special needs can take advantage of the more completed versions of the notes so they can focus on the words themselves and not on writing them down.

After the students have the Meat and Potatoes, they have the option of choosing Dessert. These dessert assignments are great extensions to the daily lesson. Like real dessert, it is not a necessary part of the meal (lesson) however, it is nice to have once in a while and usually encouraged by others. Before the students leave their dining experience, they must “Pay the Bill” (ticket to leave). Paying the Bill is a necessary part of each lesson because it provides formative assessment for the teacher as well as a great source of self-reflection for the students.

Homework is assigned almost everyday and should be stressed to help practice, increase comprehension, and help students to be ready to eat the next day! There are two different homework assignments for almost every day during the unit, they are entitled, “Still Hungry” and “Full”. Still Hungry assignments are those more focused on the essential elements in the lesson. The questions are straightforward and literal emphasizing knowledge and comprehension. Students are still hungry if they need a little more practice before they have mastered the material for the day. Full assignments are for those students who have had all they want to eat; they fully understand the material and are ready for a little challenge. Questions in the Full assignments include knowledge and comprehension levels as well as application and analysis. There may be more difficult mathematical concepts embedded or diverse question formats. Students have the option of choosing whichever version of the homework they think is best for them or teachers can challenge students to pick up the Full version to help push student thinking and test their understanding.
The food analogy is especially great for a middle school, but could easily be removed for a ninth grade high school classroom. Not only is it easy for students to grasp on to but it provides them with a daily timeline for each class period so they know where they still need to go at any point in time. This structure also has incredible organizational implications as students should know that the Main Courses provide the essential pieces of the unit while the salad, appetizers, and desserts offer guidance and additional assistance.

**Overall Goals**

1) Set Theory Concepts and Standards – First and foremost, students will be familiar and comfortable working with sets in all representations. They will be able to visualize sets, write about them, read and understand their composition, relate sets to the real world, and properly reference sets through speech using correct vocabulary.

2) Students will demonstrate an ability to successfully reciprocal teach and learn from each other in an effort to increase the levels of understanding and the progression of thinking done by each student. Students will successfully work together in pairs, small groups of three, four, and five, and as a whole class to ask questions, take risks, respond, and collaborate to increase overall comprehension.

3) Students will be given the chance to feel successful in their learning through the use of choice, reciprocal teaching, formative assessment, multiple modalities, and various instructional techniques. Students will increase their academic confidence as their
questions and responses are supported in the classroom. Students of various learning styles will be able to connect with the unit activities leading to an increase in mathematical interest and enjoyment.

4) Students who are struggling through this unit will be given daily support not only by the teacher but by other students. Students who are ready for higher level skills will be given the chance to delve deeper into this material on a daily basis. All students should feel challenged but supported throughout the studying of this unit in an effort to transform the everyday classroom into a community of learners.
Integrated Algebra
Set Theory - Pre Assessment

Answer the following questions to the best of your ability:

Can you match the following vocabulary words to their appropriate symbols?

- Intersection ~
- Empty Set U
- Universal Set { }
- Equivalent Sets ∅
- Union ∩

Given: \( F = \{\text{factors of twenty four}\} \quad \text{and} \quad O = \{\text{positive odd numbers less than 10}\} \)

Draw a Venn Diagram that represents Sets \( F \) and \( O \) and fill it in with the appropriate elements.

Represent the elements in the following sets:

\[ F \cup O \]
\[ F \cap O \]
\[ F^c \]

Create a set that is equivalent to \( F \cap O \):
Dessert Project

Option #1:
The Biography of Georg Ferdinand Ludwig Philipp Cantor

a) Three Page Paper including:
   1) Mathematical contributions
   2) Birthplace and personal life
   3) Mathematicians worked with
   4) Areas of influence

b) Poster with details from paper

Option #2:
The History of Venn Diagrams

a) Three Page Paper including:
   1) Origins
   2) Simple Examples
   3) Complex Examples
   4) Classroom use

b) Poster with details and examples
## Integrated Algebra Grade 8: Set Theory Unit Calendar

<table>
<thead>
<tr>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Test</td>
<td>Lesson 1 - Introduction</td>
<td>Lesson 2 – Set Notation</td>
<td>Lesson 3 – Types of Sets</td>
<td>Lesson 4 – Set</td>
</tr>
<tr>
<td>Grade</td>
<td>Whole Class Activity</td>
<td>Partner Work (2)</td>
<td>Group Activity (5)</td>
<td>Relationships</td>
</tr>
<tr>
<td>Organizer</td>
<td>Partner Work (2)</td>
<td>Group Activity (4)</td>
<td>Homework 3</td>
<td>Whole Class Activity</td>
</tr>
<tr>
<td>Unit Calendar</td>
<td>Homework 1</td>
<td>Homework 2</td>
<td></td>
<td>Homework</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lesson 5 – Set</td>
<td>Lesson 6 – RAFT</td>
<td>Continue work on RAFT</td>
<td>Finish RAFT</td>
<td>RAFT presentations</td>
</tr>
<tr>
<td>Operations</td>
<td>RAFT Groups (4)</td>
<td>Write rough drafts</td>
<td>Write final copies</td>
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<tr>
<td>Small Groups (3)</td>
<td>Brainstorm RAFT</td>
<td>Start visual aid</td>
<td>Finish visual aid</td>
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<tr>
<td>Whole Class</td>
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<td>Practice</td>
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<td>Activity</td>
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<td>Homework 5</td>
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<tr>
<td>Jigsaw</td>
<td>Unit Exam</td>
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<td>Unit Review</td>
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<td>Group Activity</td>
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</table>

### Unit Facts:
- Twelve day unit
- Daily formative assessment
- Two summative assessments: RAFT project and unit exam
- Daily activities
- Daily group or partner work
- Auditory, visual, and kinesthetic Support
- Multiple graphic organizers
- Daily extension opportunities
- Unit extension opportunities
- Daily opportunities for student choice
- Support for weaker students
- Multiple versions of each note sheet
SET THEORY

The Basics

Set Notation

Types of Sets

Set Relations

Set Operations
Lesson 1 – Set Theory Introduction

Formal Lesson Plan

Appetizer

Salad Course: blank notes

Salad Course: modified notes

Salad Course: completed notes

Main Course

Pay the Bill

Dessert Course

Homework: still hungry

Homework: full
I. Lesson Objectives

➢ Students will demonstrate an ability to identify sets of objects or numbers and the elements in a set.

➢ Students will demonstrate an ability to describe sets using a listing and descriptive representation.

➢ Students will demonstrate an ability to read and write about sets using the term inclusively.

Standards/Curriculum:

A.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

A.CN.6 Recognize and apply mathematics to situations in the outside world

A.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

A.R.2 Recognize, compare, and use an array of representational forms

Possible Strategies/Materials:

➢ Cooperative Organization
➢ Questioning Techniques
➢ Compare/Contrast Organizer
➢ Literacy Extension Activity
➢ Pay the Bill
➢ Homework

II. Assessment

Pre-Assessment:
This was addressed through the use of the pre-test.

Formative Assessment:
During the lesson, student understanding will be assessed through the use of informal questioning, the compare/contrast organizer, the extended literacy activity, and more specifically through the ‘pay the bill’ activity.
Post-Assessment:
The accurate completion of the homework assignment, a jigsaw, and the unit exam will serve this purpose.

Self-Assessment:
The students will be able to self-assess during the ‘pay the bill’ activity before they leave class for the day. Some of the students will also be able to develop their own sets, write about them, and represent them in multiple ways.

III. Task Analysis

Appetizer: Kinesthetic Activity – Having the students develop their own groups (sets) to introduce the activity will help them to activate prior knowledge, discover the idea of sets, and start on a level ground. Following this with informal questioning will help to extend their thought process and clarify some ideas. Mixing the students randomly will help to pair weaker and stronger students together for this lesson.

Salad Course: Formal Notes – Notetaking will help to formalize the class discussion while providing students with a note sheet to use throughout the unit. Students that need help with note taking organization can be given the outlined notes and students with further difficulties including poor handwriting skills or auditory disabilities can be given the completed note sheet.

Main Course: Compare/Contrast Organizer – The completion of this organizer in pairs will help to differentiate instruction amongst the students. This activity will allow for reciprocal teaching while encouraging all students to think mathematically at a higher level. This will also help to increase the students’ mathematical literacy as they compare and contrast different representations. We will come together as a whole class to discuss their paired findings. As a whole class we will discover any student misconceptions.

Dessert Course: Literacy Activity – The dessert course asks students to reflect upon sets in the real world. This activity is designed for any student that is ready for critical thinking with this topic. This activity is not a necessary part of the unit, but a great extension.

Pay the Bill – This short activity is to serve as a formative assessment for the teacher along with a self-assessment for the student.

Homework: The homework includes both straightforward comprehension questions which help to assess basic student understanding as well as application questions asking the students to develop their own problems depending on which assignment the students’ choose.
IV. Differentiated Elements:

- The above activities address multiple modalities. There are kinesthetic activities, visual organizers, and auditory support to help all types of learners.

- This lesson has a variety of instructional strategies: students start by being grouped randomly, this is followed by class discussion to help formalize vocabulary, then a partner activity that offers reciprocal teaching, and closes with an independent reflection.

- There is an opportunity for student choice in the homework assignment.

- Support is provided for weaker learners while extension opportunities are provided for advanced learners.
- Without talking, organize yourselves in whatever manner you choose using the items you were given as you entered the classroom.

- Once you have organized into groups, sit together and wait for the rest of the class to finish.

- You will be given three minutes to do this.
Salad Course

Notes: Introduction to Set Theory

Set: ____________________________________________________________

Set Label: _____________________________________________________

Ex: ___________________________________________________________

Element: _______________________________________________________

Ex: ___________________________________________________________

Notation: _____________________________________________________

Listing Representation: _________________________________________

_______________________________________________________________

Ex: ___________________________________________________________

Inclusively: ___________________________________________________

Descriptive Representation: _____________________________________

_______________________________________________________________

Ex: ___________________________________________________________

_______________________________________________________________
Notes: Introduction to Set Theory

**Set**: a clearly defined group of objects

**Set Label**: a capital letter that stands for or marks a certain set

Ex: __________________________

**Element**: individual items in a set

Ex: __________________________

Notation: __________________________

**Listing Representation**: when we actually write down or name every individual element in a set

Ex: __________________________

**Inclusively**: __________________________

**Descriptive Representation**: when we explain the types of elements in a set or we refer to what makes up a set

Ex: __________________________
Salad Course

Notes: Introduction to Set Theory

Set: a clearly defined group of objects

Set Label: a capital letter that stands for or marks a certain set

Ex: $S = \{\text{school supplies}\}$ or $S = \{\text{tape, stapler, hole punch}\}$

Element: individual items in a set

Ex: the tape is an element of the set of school supplies

Notation: element $= \in$ not an element $= \notin$

Listing Representation: when we actually write down or name every individual element in a set

Ex: $A = \{\text{April, August}\}$ the set of months that start with A

Inclusively: including

Descriptive Representation: when we explain the types of elements in a set or we refer to what makes up a set

Ex: $C = \{\text{the set of all continents}\}$

Ex: $O = \{\text{the set of all Olympic gymnasts}\}$
Let's organize our thoughts about set representations!

**Compare/Contrast Organizer**

**Listing vs. Descriptive Representations**

<table>
<thead>
<tr>
<th></th>
<th>Pros</th>
<th>Cons</th>
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<tbody>
<tr>
<td><strong>Listing</strong></td>
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<tr>
<td>Representation</td>
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<td>Ex:</td>
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<tr>
<td><strong>Descriptive</strong></td>
<td></td>
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</tr>
<tr>
<td>Representation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ex:</td>
<td></td>
<td>Ex:</td>
</tr>
</tbody>
</table>
List three things you learned about set theory today.

1) 

2) 

3)
Describe two places or two examples in the real world of where you recognize sets and elements of these sets. Write your response in complete sentences and use the terms set and element.

Ex: When I hear people talk about the Yankees, they are referring to the set of all players that are currently playing for the Yankees baseball team. Derek Jeter is an element of this set.
In all of the following problems, make sure to use \( \{ \} \in \mathcal{E} \) and set labels as needed.

- Given: \( \mathcal{W} = \{\text{whole numbers between 2 and 10 inclusively}\} \)

1) Describe the elements in set \( \mathcal{W} \) using a listing representation.

2) True or False: \( 2 \in \mathcal{W} \)?

- Given: \( \mathcal{E} = \{\text{even numbers between 5 and 15}\} \)

3) Describe the elements in set \( \mathcal{E} \) using a listing representation.

4) True or False: \( 16 \in \mathcal{E} \)?

- Given: \( \mathcal{O} = \{1, 3, 5, 7, 11, 13, 15, 17, 19\} \)

5) Describe the elements in set \( \mathcal{O} \) using a descriptive representation.

6) True or False: \( 21 \in \mathcal{O} \)?

- Given: \( \mathcal{J} = \{\text{January, June, July}\} \)

7) Describe the elements in set \( \mathcal{J} \) using a descriptive representation.

8) Write down an element that is not in \( \mathcal{J} \).
In all of the following problems, make sure to use \{\} ∈ ∈ and set labels as needed.

- Given: \( F = \{\text{multiples of four between 4 and 28 inclusively}\} \)

1) Describe the elements in set \( F \) using a listing representation.

2) True or False: \( 4 \in F? \)

- Given: \( P = \{\text{the perfect squares between 1 and 100}\} \)

3) Describe the elements in set \( P \) using a listing representation.

4) True or False: \( 100 \in P? \)

- Given: \( N = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \)

7) Describe the elements in set \( N \) using a descriptive representation.

8) True or False: \( 15 \in N? \)
Given: $J = \{\text{January, June, July}\}$

9) Describe the elements in set $J$ using a descriptive representation.

10) Write down an element that is not in $J$.

Using your own set and set label:

11) Describe the elements in your set using a descriptive representation.

12) Describe the elements in your set using a listing representation.
Lesson 2 – Set Notation

Formal Lesson Plan

Appetizer

Salad Course

Main Course: blank notes

Main Course: modified notes

Main Course: completed notes

Pay the Bill

Dessert Course

Homework: still hungry

Homework: full
I. Lesson Objectives

➢ Students will demonstrate an ability to independently discover properties of set notation.

➢ Students will demonstrate an ability to graph sets using a number line.

➢ Students will demonstrate an ability to identify, describe, and create sets using set builder and interval notation.

➢ Students will demonstrate an ability to work together to create and represent their own sets.

Standards/Curriculum:

A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

A.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

A.CM.11 Represent word problems using standard mathematical notation

A.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A.CN.6 Recognize and apply mathematics to situations in the outside world

A.A.29 Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form

Possible Strategies/Materials:

➢ Notation Exploration
➢ Whole Class Discussion
➢ Formalized Notes
➢ Partner Activity
➢ Word Problem Extension
➢ Pay the Bill
➢ Homework
II. Assessment

Pre-Assessment:
This was addressed through the use of the pre-test.

Formative Assessment:
During the lesson, student understanding will be assessed during the whole class discussion and formalized note taking, the partner activity, the word problem extension, and more specifically through the ‘pay the bill’ activity.

Post-Assessment:
The accurate completion of the homework assignment, a jigsaw, and the unit exam will serve this purpose.

Self-Assessment:
The students will be able to self-assess throughout the partner activity and during the ‘pay the bill’ activity before they leave class for the day. All of the students will also be able to develop their own sets, write about them, and represent them in multiple ways as well.

III. Task Analysis

Appetizer: The Match Game – Requiring the students to recall the previous day’s material in order to organize themselves into pairs is a great way to activate prior knowledge while randomly mixing the students. It should be a quick activity to help get the students started as they walk in the classroom door.

Salad Course: Notation Exploration – This exploration activity will allow students to develop their own conjectures and generalizations by looking at specific examples. This is a great activity to for different leveled students to work together to think critically and form conclusions.

Main Course: Formalized Notes and Partner Activity – The formalized note sheet will help all students to learn from each other as we discuss the set notation generalizations they discovered in their pairs and requires some guided practice. Formal note taking will also provide students with a source they can look to for help as we delve further into the unit. The partner activity helps to supply some independent practice for the students before they leave class for the day. Allowing the pairs of students to create their own sets also extends their levels of thinking while giving them the opportunity for reciprocal teaching if that need is present.

Dessert Course: Word Problem Extension – The dessert course requires students to use set notation given real world word problems. This activity is designed for any student that is ready for higher level thinking with this topic. This activity is not a necessary part of the unit, but a great extension.
Pay the Bill - This short activity is to serve as a formative assessment for the teacher along with a self-assessment for the student. This activity helps to focus the students on the essential pieces of this lesson.

Homework - The homework includes both straightforward comprehension questions which help to assess basic student understanding as well as application questions asking the students to develop their own problems depending on which assignment the students’ choose. This assignment is meant to constantly shift student thinking from one set notation to another in an effort to familiarize them with multiple representations.

IV. Differentiated Elements:

➢ This lesson has a variety of instructional strategies: students start by being paired randomly, this is followed by whole class discussion to help formalize vocabulary, then a partner and group activity that offers reciprocal teaching, and closes with an independent reflection.

➢ There is an opportunity for student choice in the homework assignment.

➢ Support is provided for weaker learners while extension opportunities are provided for advanced learners.

➢ Notes are provided for the varying levels of note taking ability. Teachers can determine which variation students’ require and can help students to gradually move to more independent note taking.
Appetizer

Match Game:

- Find the person who has the same set as you do; find your match.

- If you are given a set in a listed format then your partner will have the same set in a descriptive format or vice versa.

- Once you have found your partner, take a seat next to them.

- If your set contains numbers, please assume that only whole numbers are being included.
the set of all even numbers between 2 and 20 inclusively

\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}

the set of all days in a week

\{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday\}

the set of all positive numbers less than 15

\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}

the set of all months in a year


the set of all whole numbers between 4 and 12 inclusively

\{4, 5, 6, 7, 8, 9, 10, 11, 12\}

the set of all four core classes

(math, science, social studies, english)

the set of all primary colors
the set of all even numbers between 2 and 20 inclusively

\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}

the set of all days in a week

\{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday\}

the set of all positive numbers less than 15

\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}

the set of all months in a year


the set of all whole numbers between 4 and 12 inclusively

\{4, 5, 6, 7, 8, 9, 10, 11, 12\}

the set of all four core classes

(math, science, social studies, english)

the set of all primary colors
{the set of all even numbers between 2 and 20 inclusively}
\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}

{the set of all days in a week}
[Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday]

{the set of all positive numbers less than 15}
\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}

{the set of all months in a year}

{the set of all whole numbers between 4 and 12 inclusively}
\{4, 5, 6, 7, 8, 9, 10, 11, 12\}

{the set of all four core classes}
(math, science, social studies, english)

{the set of all primary colors}
{red, blue, yellow}

{the set of all possible digits in a telephone number}

\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}

{the set of all colors in the American flag}

{red, white, blue}

{the set of all multiples of 10 less than 100}

\{10, 20, 30, 40, 50, 60, 70, 80, 90\}

{the set of all factors of 24}

\{1, 2, 3, 4, 6, 8, 12, 24\}

{the set of all states that start with O}

\{Ohio, Oklahoma, Oregon\}

{the set of all even numbers between 1 and 15}

\{2, 4, 6, 8, 10, 12, 14\}
\{\text{the set of all colors on a traffic light}\}

\{\text{red, yellow, green}\}

\{\text{the set of all numbers on a clock}\}

\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}
Salad Course
Notation Exploration

By looking at the following examples, draw some conclusions about the two different ways to describe and write sets. Once again, assume that we are only using whole numbers.

Set Builder Notation

a) \( \{x \mid x < 2\} \)
   = \{...-3, -2, -1, 0, 1\}

b) \( \{n \mid 1 < n < 9\} \)
   = \{2, 3, 4, 5, 6, 7, 8\}

c) \( \{2y \mid y > 3\} \)
   = \{8, 10, 12, 14, 16...\}

d) \( \{x^2 \mid x > 0\} \)
   = \{1, 4, 9, 16, 25, 36, 49...\}

Interval Notation

a) \([-4, 3]\)
   = \{-4, -3, -2, -1, 0, 1, 2, 3\}

b) \((1, 11]\)
   = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}

c) \([-2, 7)\)
   = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}

d) \((-21, -14)\)
   = \{-20, -19, -18, -17, -16, -15\}

List three things that all the above examples have in common:

1) __________________________
2) __________________________
3) __________________________

Take an educated guess at the following questions:

1) What do you think [ means?

2) What do you think ( means?

______________________________
______________________________
Main Course
Notes: Set Notation

There are two formal notations used for describing and writing sets:
Set Builder Notation and Interval Notation

1) Set Builder Notation
   A) General Form: \{formula for elements \mid restrictions\}
   B) Example: \{x \mid x < 5\} = \{-1, 0, 1, 2, 3, 4\} (use only whole numbers)
      This is read as: "the set of all x such that x is less than 5"
      \{= \ldots x = \ldots \mid = \ldots \}
   C) Guided Practice:
      1) Convert into set builder notation: "the set of all x such that x is less than 7"

      \__________

      2) Convert into set builder notation: "the set of all 5y - 1 such that y is between 2 and 8 inclusively"

      \__________

      3) List the elements of the following set: \{n \mid -2 < n < 5\}

      \__________

      4) How would \{m \mid m > 10\} be read?

      \__________
2) Interval Notation

A) General Form:  bracket #, #  bracket

B) Example: [-1,5)

This includes the numbers: -1, 0, 1, 2, 3, 4

And looks as follows on a number line:

( = ________________________

[ = ________________________

C) Guided Practice:

1) Rewrite the following set using interval notation:
   \( E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \)

2) Rewrite the set shown on the number line using interval notation:
   \( O \quad \bullet \quad \rightarrow \)

3) Rewrite this set \( O = \{1, 2, 3, 4, 5, 6, 7, 8\} \) using interval notation:
There are two formal notations used for describing and writing sets: **Set Builder Notation** and **Interval Notation**

1) **Set Builder Notation**
   A) **General Form:** \{formula for elements \mid restrictions\}

   B) Example: \{x \mid x < 5\} = \{-1, 0, 1, 2, 3, 4\} (use only whole number)

   This is read as: "the set of all \(x\) such that \(x\) is less than 5"

   \{ = "the set of" \quad x = "all \(x\)" \quad \mid = "such that"

   C) **Guided Practice:**
      1) Convert into set builder notation: "the set of all \(x\) such that \(x\) is less than 7"

   2) Convert into set builder notation: "the set of all \(5y - 1\) such that \(y\) is between 2 and 8 inclusively"

   \{ \_ \mid \_ \}

   3) List the elements of the following set: \{n \mid -2 < n < 5\}

   \{-1, 0, \_ \_ \_ \_ \_ \_ \}

   4) How would \{m \mid m > 10\} be read?

   "the set of all \(m\) such that \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_\_ \_ \_ \_"
2) Interval Notation

A) General Form: bracket #, # bracket

B) Example: [-1,5)

This includes the numbers: -1, 0, 1, 2, 3, 4

And looks as follows on a number line:

I I I I I I I I I I

( = open bracket, [ = closed bracket,

C) Guided Practice:

1) Rewrite the following set using interval notation:
   \[ E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \]

   \([1, 10]\) or \([1, 11]\) or \((0, 10]\) or \((0, 11)\)

2) Rewrite the set shown on the number line using interval notation:

   \[ E \]

   \[ (\), (\), \]

3) Rewrite this set \[ O = \{1, 2, 3, 4, 5, 6, 7, 8\} \] using interval notation:
1) Set Builder Notation
   A) General Form: \{formula \text{ for elements} \mid \text{restrictions}\}
   
   B) Example: \{x \mid x < 5\} = \{-1, 0, 1, 2, 3, 4\} \text{ (use only whole numbers)}
   
   This is read as: “the set of all \(x\) such that \(x\) is less than 5”
   
   \(\{ = \text{“the set of”} \quad x = \text{“all \(x\)”} \quad | = \text{“such that”}\)

C) Guided Practice:
   1) Convert into set builder notation: “the set of all \(x\) such that \(x\) is less than 7”
      
      \{x \mid x < 7\}
   
   2) Convert into set builder notation: “the set of all \(5y - 1\) such that \(y\) is between 2 and 8 inclusively”
      
      \{5y - 1 \mid 2 \leq y \leq 8\}
   
   3) List the elements of the following set: \{n \mid -2 < n < 5\}
      
      \{-1, 0, 1, 2, 3, 4\}
   
   4) How would \{m \mid m > 10\} be read?
      
      “the set of all \(m\) such that \(m\) is greater than 10"
2) Interval Notation

A) General Form: bracket #, # bracket

B) Example: [-1,5)

This includes the numbers: -1, 0, 1, 2, 3, 4

And looks as follows on a number line:

( = open bracket, does not include

[ = closed bracket, does include, inclusively

C) Guided Practice:

1) Rewrite the following set using interval notation:
   \( E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \)

   \([1, 10]\) or \([1, 11]\) or \((0, 10]\) or \((0, 11)\)

2) Rewrite the set shown on the number line using interval notation:

3) Rewrite this set \( O = \{1, 2, 3, 4, 5, 6, 7, 8\} \) using interval notation:

   \([1, 8]\) or \((0, 8]\) or \([1, 9]\) or \((0, 9)\)
1) List three things you need to write a set in proper set builder notation:

a) 

b) 

c) 

2) When using Interval Notation:

a) What does [ or ] mean?

b) What does ( or ) mean
1) Your basketball team decides to go out to dinner to celebrate winning the sectional title. Your coach asks you to call and make reservations. He says that ten players are definitely coming but all twenty of them might come.

   a) List the elements in set $P = \{\text{the } \# \text{ of possible players going to dinner}\}$. 

   b) Using set builder notation, represent set $P$: 

   c) Represent set $P$ in interval notation: 

   d) Graph set $P$ on a number line: 

2) Your brother needs to buy a new bedroom set. However, because he is a struggling college student, the most he can afford to spend is $500.00.

   a) If $B = \{\text{the amount your brother can spend on bedroom furniture}\}$, represent set $B$ in set builder notation: 

   b) Now write set $B$ using interval notation:
Note: Assume you are using only whole numbers unless otherwise noted

Model Problem: Given \( A = \{2, 3, 4, 5, 6, 7, 8, 9\} \):

\( a) \) Represent set \( A \) in set builder notation:

\[
A = \{a \mid 2 \leq a \leq 9\} \quad \text{or} \quad A = \{a \mid 1 < a < 10\}
\]

\( b) \) Represent set \( A \) in interval notation:

\[
A = [2, 9] \quad \text{or} \quad A = (1, 10) \quad \text{or} \quad A = (1, 9] \quad \text{or} \quad A = [2, 10)
\]

c) Graph set \( A \) on a number line:

---

Use the model problem to help answer the following questions. Make sure to use \{ \} and set labels as needed.

1) Given \( B = \{x \mid 3 < x < 12\} \):

\( a) \) List the elements in set \( B \):

\( b) \) Rewrite set \( B \) using interval notation:

\( c) \) Graph set \( B \) on a number line:

2) Given \( C = [7, 15) \):

\( a) \) List the elements in set \( C \):

\( b) \) Graph set \( C \) on a number line:
3) Given \( D = \{-12, -11, -10, -9, -8, -7, -6\} \):

a) Represent set \( D \) using set builder notation:

b) Represent set \( D \) using interval notation:

c) Graph set \( D \) on a number line:

4) Set \( E \) is shown on the accompanying number line:

a) List the elements included in set \( E \):

b) Represent set \( E \) in interval notation:

c) Rewrite set \( E \) using set builder notation:

5) Given \( F = [101, 116) \):

a) List the elements in set \( F \):

b) Graph set \( F \) on a number line:

c) Rewrite set \( F \) using set builder notation:
Note: Assume only whole numbers unless otherwise noted.
Make sure to use { } and set labels as needed to answer the following questions.

1) Given \( B = \{x \mid 3 < x < 12\} \):
   a) List the elements in set \( B \):
   b) Rewrite set \( B \) using interval notation:
   c) Graph set \( B \) on a number line:

2) Given \( C = [7, 15) \):
   a) List the elements in set \( C \):
   b) Graph set \( C \) on a number line:

3) Given \( D = \{-12, -11, -10, -9, -8, -7, -6\} \):
   a) Represent set \( D \) using set builder notation:
   b) Represent set \( D \) using interval notation:
   c) Graph set \( D \) on a number line:
4) Set E is shown on the accompanying number line:

   a) List the elements included in set E:

   b) Represent set E in interval notation in four different ways:

   c) Rewrite set E using set builder notation in four different ways:

5) List the elements in set F = \{2f \mid 3 \leq f \leq 7\}:

6) Your volleyball coach can take a minimum of 8 and a maximum of 14 new athletes this year.

   a) If \( V = \{\text{the number of new volleyball players}\} \), represent set \( V \) in interval notation:

   b) Graph set \( V \) on a number line:

7) Create your own real world set example: ________________________________________

   a) Represent your set in set builder notation:

   b) Represent your set using interval notation:
Lesson 3 – Types of Sets

Formal Lesson Plan

Appetizer

Salad Course

Main Course: blank notes

Main Course: modified notes

Practice in Groups

Pay the Bill

Dessert Course

Homework: still hungry

Homework: full
I. Lesson Objective

➢ Students will demonstrate an ability to discover commonalities between sets in an effort to better understand their makeup.

➢ Students will demonstrate an ability to identify and create sets as finite, infinite, or empty.

➢ Students will demonstrate an ability to identify a universal set and acknowledge how many elements are in these sets.

Standards/Curriculum:

A.A.29 Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form

A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

A.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

Possible Strategies/Materials:

➢ What Do We Have in Common Activity
➢ Whole Class Discussion/Jigsaw
➢ Guided and Independent Practice
➢ Literacy Extension Activity
➢ Pay the Bill
➢ Homework

II. Assessment

Pre-Assessment:
The level of the student’s prior knowledge will be assessed during the review in the beginning of the lesson. Adjustments to add more detail surrounding the concept of infinite will be made as necessary.

Formative Assessment:
During the lesson, student understanding will be assessed through the use of the “What Do We Have in Common” Activity, informal questioning, practice throughout the class, and more specifically through the “Pay the Bill” activity.
Post-Assessment:
The accurate completion of the homework assignment, a jigsaw, and the unit exam will serve this purpose.

Self-Assessment:
The students will be able to self-assess during the group activity and in-class practice. The homework assignment also requires the students to develop their own sets while the dessert course will push them to think critically about different types of sets.

III. Task Analysis

Appetizer – This warm-up is a straightforward activity to activate students’ prior knowledge and to recall the previous day’s lesson.

Salad Course: What Do We Have in Common? – Participation in this activity will help to differentiate instruction amongst students. Teachers can determine which sets are given to which students. Struggling student can be given sets that are straightforward while other students can be given the more obscure infinite sets. Teachers can also determine who will be the Head Chefs. Head Chefs can be chosen for leadership qualities, public speaking skills, or to increase self-confidence. This activity will allow for reciprocal teaching and group work.

Whole Class Discussion – This discussion is important to discover any student misconceptions, play devil’s advocate to get students thinking in different directions, or to re-group students if their groups are slightly incorrect. Once groups are correct, the students will be jigsawed so that one member from each group represents a different type of set.

Main Course: Guided and Independent Practice – Note taking will help to formalize the class discussion while providing students with a note sheet to use throughout the unit. This will be started amongst their groups and then additional notes will be provided by the teacher. Students that need help with note taking organization can be given the outlined notes and students with further difficulties including poor handwriting skills or auditory disabilities can be given the completed note sheet.

Dessert Course: Literacy Activity – The dessert course asks students to reflect upon the relation of universality and finite sets; it requires advanced thinking skills. This activity is designed for any student that is ready for critical thinking with types of sets. This activity is not a necessary part of the unit, but a great extension.

Pay the Bill – This short activity is to serve as a formative assessment for the teacher along with a self-assessment for the student; it is focused on the essential elements of the lesson.

Homework: The homework includes both straightforward comprehension questions which help to assess basic student understanding as well as application questions...
asking the students to develop their own problems. Depending on which assignment
the students' choose they may be faced with a more summative assignment which
requires them to use all of the new topics they have learned in the unit thus far.

IV. Differentiated Elements:

➢ The above activities address multiple modalities. There are kinesthetic activities,
visually chunked notes, and auditory support to help all types of learners.

➢ This lesson has a variety of instructional strategies: the lesson begins with
independent work, then students are grouped depending on teacher choice, this is
followed by a whole class discussion to help formalize vocabulary, then
regrouping jigsaw activity, and ends with an independent reflection.

➢ There is an opportunity for student choice in the homework assignment.

➢ Support is provided for weaker learners while extension opportunities are
provided for advanced learners.
Appetizer

1) List the elements in the following set:

\[ E = \{2x \mid 3 \leq x \leq 9\} \]

2) Given \( H = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} \):

   a) Rewrite set \( H \) using set builder notation:

   b) Rewrite set \( H \) using interval notation:

   c) Graph set \( H \) on a number line:

3) Given \( O = \{1, 3, 5, 7, 9\} \), how many elements are in set \( O \)?

4) Given \( I = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 \ldots\} \), how many elements are in set \( I \)?
Salad Course
WHAT DO WE HAVE IN COMMON?

Chefs - you will be given a slip of paper with a set on it. You will be given three minutes to organize yourselves into groups of your choosing.

Head Chefs - Once the groups have been formed, you will each be assigned to one group. Then, it is your job to work with your group to complete your worksheet describing your set.

Once all groups have completed their worksheets, you will present to the rest of the class, what you and your group members “Have in Common.”
\[ E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20 \ldots \} \]

\[ A = \{x \mid x > 10\} \]

\[ B = (-\infty, \infty) \]

\[
\begin{array}{cc}
J = & 0 \\
& \ldots -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 \ldots
\end{array}
\]

\[ B = \{\text{the set of all atoms that will ever exist in the universe}\} \]

\[ K = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\} \]

\[ M = \{n \mid 8 < n < 22\} \]

\[ P = (-4, 5] \]

\[
\begin{array}{cc}
R = & 0 \\
& \ldots -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 \ldots
\end{array}
\]
T = \{the set of all students in this class\}

C = \{the set of all lions that have 25 legs\}

D = \{the set of all retired people that are 2 yrs. old\}

Z = \{the set of all walking goldfish\}

Y = \{the set of all cars with 1000 tires\}

U = \{the set of all talking elephants\}

S = \{the set of all 50 states\}

f = \{the set of all possible test scores\}

x = \{the set of all the letters in the English alphabet\}

w = \{the set of all negative one-digit numbers\}

Q = \{the set of all continents on the earth\}
Salad Course
WHAT DO WE HAVE IN COMMON?

By looking at your group members' sets, see if you can determine what you all have in common. Once you have decided what you all have in common, try and come up with an appropriate title to describe your group.

1) Does it have to do with the types of elements that are in your sets?

2) Does it have to do with the set labels?

3) Does it have to do with the number of elements in your sets?

Group Title: ______________________________________
Main Course
Notes: Types of Sets

Set Title: ________________________________

Example (from paper): ________________________________

Definition: ______________________________________

Example (from class): ________________________________

Note: ____________________________________________

Set Title: ________________________________

Example (from paper): ________________________________

Definition: ______________________________________

Example (from class): ________________________________

Note: ____________________________________________
<table>
<thead>
<tr>
<th>Set Title:</th>
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<tbody>
<tr>
<td>Example (from paper):</td>
</tr>
<tr>
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<td>Example (from paper):</td>
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<tr>
<td>Definition:</td>
</tr>
<tr>
<td>Example (from class):</td>
</tr>
<tr>
<td>Note:</td>
</tr>
</tbody>
</table>
Main Course
Notes: Types of Sets

Set Title: Infinite sets

Example (from paper): ________________________________

Definition: Infinite sets contain infinitely many elements

Example (from class): ______________________________

Note: $\infty = \text{infinite}$  $\ldots = \text{indicates that it goes on forever}$

Set Title: Finite Sets

Example (from paper): ________________________________

Definition: Finite sets have a definite number of elements or we can count how many elements there are

Example (from class): ______________________________

Note: We can still use $\ldots$ but it will be stopped by an element on each side.
Set Title: Empty or Null Sets

Example (from paper): 

Definition: Sets are empty if they contain no elements.

Example (from class): 

Note: Empty sets are indicated by \{ \} or \emptyset.

Set Title: Universal Sets

Example (from paper): 

Definition: Universal sets contain all the possible elements in a given scenario.

Example (from class): 

Note: Universal sets are indicated by \( U \).
State whether the following sets are Finite or Infinite:

1) \{1, 2, 3, 4, 5...\}  
2) \{2, 4, 6, 8...200\}  
3) \{The set of all students that attend this school\}

State whether the following sets are Finite, Infinite, or Empty:

4) \{The set of all men that weigh 15,000 lbs.\}  
5) \{The number of points on a line\}  
6) \{The number of months in ten years\}

How many elements are in the following universal sets:

7) \(U = \{\text{the set of all the desks in this room}\}\)  
8) \(U = \{\text{the set of all the letters in the alphabet}\}\)  
9) \(U = \{\text{the set of all positive one-digit numbers}\}\)
List the four types of sets and give one detail about each one (definition, example, description):

1) Detail

2) Detail

3) Detail

4) Detail
Are all universal sets finite?

Think about this question and answer it in complete sentences. You must use the terms universal, finite, and element. You must also include an example to support your thoughts.
State whether the following sets are finite or infinite:

1) \{\text{the set of all integers}\}
2) \{\text{the set of all prime numbers less than 25,000}\}
3) \{\text{the set of all current United States citizens}\}
4) \{\text{the set of all negative numbers}\}

Identify the following sets as empty, finite, or infinite.

5) \{\text{the set of all people with two hundred legs}\}
6) \{\text{the set of all prime numbers}\}
7) \{\text{the set of all faculty at this school}\}
8) \{\text{the set of all cats that bark}\}

How many elements are in the following universal sets?

9) \(U = \{\text{the set of all possible report card grades}\}\)
10) \(U = \{\text{the set of all positive two-digit numbers}\}\)

Create your own sets (include set labels and be sure to use \{ \}):

11) Create you own finite set:

12) Create you own infinite set:

13) Create you own empty set:

14) Create your own Universal set and tell how many elements are in it?
State whether the following sets are finite or infinite:

1) \{ the set of all negative numbers \}

2) \{ x^2 \mid -4 < x < 4 \}

3) \{ the set of all current United States citizens \}

4) (0, \infty)

Identify the following sets as empty, finite, or infinite.

5) \{ the set of all people with two hundred legs \}

6) \{ 0 \}

7) \{ 3m \mid m < -1 \}

8) (-\infty, -1,000,000)

How many elements are in the following universal sets?

9) \( U = \{ \text{the set of all possible report card grades} \} \)

10) \( U = \{ \text{the set of all positive two-digit numbers} \} \)

Create your own sets (include set labels and be sure to use \{ and \}):

11) Create you own finite set:


12) Create you own infinite set:


13) Create you own empty set:


14) Create your own Universal set and tell how many elements are in it?
Lesson 4 – Set Relationships

Formal Lesson Plan

Appetizer

Salad Course: pizza activity

Main Course: blank notes

Main Course: modified notes

Practice in Groups

Pay the Bill

Dessert Course

Homework: still hungry

Homework: full
I. Lesson Objective

➢ Students will demonstrate an ability to identify equal and equivalent sets.
➢ Students will demonstrate an ability to identify any and all subsets from a given set.
➢ Students will demonstrate an ability to critically think about the relationships between sets of numbers or objects.

Standards/Curriculum:

A.RP.11 Use a Venn diagram to support a logical argument
A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form
A.R.2 Recognize, compare, and use an array of representational forms

Possible Strategies/Materials:

➢ Pizza Activity/Informal Questioning Techniques
➢ Formal Notes and Practice
➢ Pay the Bill
➢ Critical Thinking Extension
➢ Homework

II. Assessment

Pre-Assessment:
The level of the student’s prior knowledge will be assessed during the review in the beginning of the lesson. Adjustments to add more detail surrounding the previous lesson’s vocabulary will be made as necessary.

Formative Assessment:
During the lesson, student understanding will be assessed through the use of the Pizza Activity, informal questioning, active participation, practice throughout the class, and more specifically through the “Pay the Bill” activity.

Post-Assessment:
The accurate completion of the homework assignment, a jigsaw, and the unit exam will serve this purpose.

Self-Assessment:
The students will be able to self-assess during the group activity and in-class practice. The homework assignment also requires the students to develop their own sets while the dessert course will push them to think critically about set relationships.
II. Task Analysis

Appetizer – This warm-up is a straightforward activity to activate students’ prior knowledge and to recall the previous day’s lesson.

Salad Course: Pizza Activity – Participation in this activity will help to involve all students in the class discussion by relating sets to a food that all students are familiar. This activity should be led by the teacher but can be steered by student participation. By talking about the relationships between different kinds of pizza combinations, the students are discovering relationships between sets.

Main Course: Formal Notes and Practice – Note taking will help to formalize the class discussion while providing students with a note sheet to use throughout the unit. The use of Venn Diagrams is introduced during the subset portion of the notes which will provide a great transition for the next lesson in which Venn Diagrams are heavily utilized. Students that need help with note taking organization can be given the outlined notes and students with further difficulties including poor handwriting skills or auditory disabilities can be given the completed note sheet.

Dessert Course: Critical Thinking Extension – The dessert course asks students to think at a higher level about the relationships among sets, empty sets, and subsets. This activity is designed for any student that is ready for critical thinking with types of sets. This activity is not a necessary part of the unit, but a great extension.

Pay the Bill – This short activity is to serve as a formative assessment for the teacher along with a self-assessment for the student; it is focused on the essential elements of the lesson.

Homework: The homework includes both straightforward comprehension questions which help to assess basic student understanding as well as application questions asking the students to develop their own problems. Depending on which assignment the students’ choose they may be faced with the task of finding the relationship between subsets and the number of elements in each set.

V. Differentiated Elements:

➢ The above activities address multiple modalities. There are kinesthetic activities, visually chunked notes, visual elements, and auditory support to help all types of learners.

➢ This lesson has a variety of instructional strategies: the lesson begins with independent work, then students are moved around the room during a whole class discussion, followed by formalized notes taking and guided practice, and ends with an independent reflection.

➢ There is an opportunity for student choice in the homework assignment. Student will be enthusiastic about participating during the pizza activity.

➢ Support is provided for weaker learners while extension opportunities are provided for advanced learners.
III. Task Analysis

*Appetizer* – This warm-up is a straightforward activity to activate students’ prior knowledge and to recall the previous day’s lesson.

*Salad Course: Pizza Activity* – Participation in this activity will help to involve all students in the class discussion by relating sets to a food that all students are familiar. This activity should be led by the teacher but can be steered by student participation. By talking about the relationships between different kinds of pizza combinations, the students are discovering relationships between sets.

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*Homework:* The homework includes both straightforward comprehension questions which help to assess basic student understanding as well as application questions asking the students to develop their own problems. Depending on which assignment the students’ choose they may be faced with the task of finding the relationship between subsets and the number of elements in each set.

IV. Differentiated Elements:

- The above activities address multiple modalities. There are kinesthetic activities, visually chunked notes, visual elements, and auditory support to help all types of learners.

- This lesson has a variety of instructional strategies: the lesson begins with independent work, then students are moved around the room during a whole class discussion, followed by formalized notes taking and guided practice, and ends with an independent reflection.

- There is an opportunity for student choice in the homework assignment. Student will be enthusiastic about participating during the pizza activity.

- Support is provided for weaker learners while extension opportunities are provided for advanced learners.
## Appetizer

Match each vocabulary word with **two** appropriate examples:

<table>
<thead>
<tr>
<th>Vocabulary Word</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Set</td>
<td>( { x \mid 0 &lt; x &lt; 10 } )</td>
</tr>
<tr>
<td></td>
<td>{the set of all 5,000 foot long pencils}</td>
</tr>
<tr>
<td>Finite Set</td>
<td>{the set of all fifty states}</td>
</tr>
<tr>
<td></td>
<td>{m \mid m &gt; 2}</td>
</tr>
<tr>
<td>Infinite Set</td>
<td>([0, \infty))</td>
</tr>
<tr>
<td></td>
<td>{-4, -3, -2, -1, 0, 1, 2, 3, 4}</td>
</tr>
<tr>
<td>Universal Set</td>
<td>{the set of all purple ladybugs}</td>
</tr>
<tr>
<td></td>
<td>{the set of all negative numbers}</td>
</tr>
</tbody>
</table>
Equal versus Equivalent Sets
Pizza Activity
Teacher Directions

Set-Up:
1) Give each student a pizza ingredient to wear around his/her neck.
2) Assemble students into possible pizza combinations around the room, according to either:
   a) Teacher choice or b) Student favorite pizzas

Note: Make sure to:
   a) Line up two of the same exact pizza combinations (equal sets)
   b) Line up two combinations with the same number but different ingredients (equivalent sets)

Discussion: This can look very different based on student responses, here are some guiding questions and ideas.
1) Ask students to look around the room and make their own conclusions about the pizzas.
   a) Similarities/differences
   b) Plain pizzas/Complex combinations
   c) The number of ingredients
2) Are any two pizzas they same? (Develop this into an equal set discussion)
3) Do any two pizzas have the same number of ingredients? (Develop into an equivalent set discussion and how this is different than equal)
4) Start with a cheese pizza and then add one ingredient at a time. Reference the transformation from simple to complex and these things relate. (Develop into a subset discussion)

Extension: If students are ready, integrate: a) sets being subsets of themselves and b) the empty set being a subset of every set.

Guided Practice:
1) Line the ingredients (students) up in the back of the room and ask them to assemble themselves into equal and equivalent pizzas (sets).
2) Pull out the ingredients for a supreme pizza and ask students to assemble different subsets of this pizza.
3) Create any combinations you think will help. Let the students run with this.
Main Course: Notes and Practice
Relationships between Sets

1) Equal Sets

Ex 1: \( T = \{10, 20, 30, 40, 50\} \)
\( N = \{\text{positive even numbers less than 10}\} \)
\( E = \{2, 4, 6, 8\} \)

Which sets are equal?

Create a set that is equal to Set \( T \)?

Ex 2: Is there an equal relationship between:

\{the set of positive even numbers less than 100\} and
\{the set of multiples of 2 less than 100\}?

How many elements are in these sets?

Ex 3: Is there an equal relationship between:

\{men 29 feet tall\} and
\{women with hair over one hundred feet long\}?

How many elements are in these sets?
2) Equivalent/Matching Sets = __________________________

______________________________

Note: __________________________________________

Remember: ______________________________________

Ex 1: 

E = \{2, 4, 6, 8, 10\}
F = \{1, 2, 3, 4\}
G = \{\text{science, social studies, math, English}\}
H = \{4, 8, 12, 16, 20\}

Which sets are equivalent? \_______ \ and \_______

Create a set that is equivalent to set G.

______________________________

Ex 2: Is there an equivalent relationship between:

{\text{the colors in a rainbow}} \ and \ {\text{the set of seasons in a year}}?

\_______

How many elements are in these sets? \_______

Ex 4: Is there an equivalent relationship between:

{\text{a set of one dozen eggs}} \ and \ {\text{the set of months in a year}}?

\_______

How many elements are in these sets? \_______
3) Subset = __________________________________________

__________________________________________________

Note: ______________________________________________

Ex 1: Set A = {the set of all students in this class period}
Set B = {the set of all girls in this class}

Ex 2: \( S = \{ \text{the set of all of the United States} \} \)
\( N = \{ \text{the set of all the United States that start with the letter N} \} \)

Ex 3: \( E = \{ \text{the set of all even numbers between 1 and 9} \} \)
\( P = \{ 2, 4, 6, 8, 10 \} \)

Is \( P \) a subset of \( E \)? ____________________________

Special Case #1: ________________________________

Special Case #2: ________________________________
1) Equal Sets

Note: We indicate this by using an equal sign between the set labels.

Ex 1: \( T = \{10, 20, 30, 40, 50\} \)
\( N = \{\text{positive even numbers less than 10}\} \)
\( E = \{2, 4, 6, 8\} \)

Which sets are equal?

Create a set that is equal to Set \( T \)?

\( \{\text{the set of multiples of 10 between 10 and 50 inclusively}\} \)

Ex 2: Is there an equal relationship between:

\( \{\text{the set of positive even numbers less than 100}\} \) and
\( \{\text{the set of multiples of 2 less than 100}\} \)?

How many elements are in these sets?

Ex 3: Is there an equal relationship between:

\( \{\text{men 29 feet tall}\} \) and
\( \{\text{women with hair over one hundred feet long}\} \)?

How many elements are in these sets?
2) Equivalent/Matching Sets = 

Note: We indicate this by using ~ sign between the set labels.

Remember: The elements in the sets do not have to be the same.

Ex 1: 
E = \{2, 4, 6, 8, 10\}  
F = \{1, 2, 3, 4\}  
G = \{\text{science, social studies, math, English}\}  
H = \{4, 8, 12, 16, 20\}

Which sets are equivalent? _______ and _______

Create a set that is equivalent to set G.

\{x \mid 0 < x < 5\}

Ex 2: Is there an equivalent relationship between:

{\text{the colors in a rainbow}} and  
{\text{the set of seasons in a year}}?  

How many elements are in these sets?  

Ex 4: Is there an equivalent relationship between:

{\text{a set of one dozen eggs}} and  
{\text{the set of months in a year}}?  

How many elements are in these sets?  

3) Subset =

Note: We indicate this by writing $A \subseteq B$.

Ex 1: $A = \{\text{the set of all students in this class period}\}$
$B = \{\text{the set of all girls in this class}\}$

Then, $B \subseteq A$.

Ex 2: $S = \{\text{the set of all of the United States}\}$
$N = \{\text{the set of all the United States that start with the letter N}\}$

Ex 3: $E = \{\text{the set of all even numbers between 1 and 9}\}$
$P = \{2, 4, 6, 8, 10\}$

Is $P$ a subset of $E$? ________________

Special Case #1: The empty set is a subset of everything.

Special Case #2: Every set is a subset of itself.
Main Course: Notes and Practice
Relationships between Sets

1) Equal Sets - Set A is equal to Set B if all of the elements in Set A are “exactly the same” as all of the elements in Set B.

**Note:** We indicate this by using an equal sign between the set labels.

**Ex 1:**

T = \{10, 20, 30, 40, 50\}
N = \{positive even numbers less than 10\}
E = \{2, 4, 6, 8\}

Which sets are equal?  \(N = E\)

Create a set that is equal to Set T?

\{the set of multiples of 10 between 10 and 50 inclusively\}

**Ex 2:** Is there an equal relationship between:

\{the set of positive even numbers less than 100\} and \{the set of multiples of 2 less than 100\}?

Yes

How many elements are in these sets? 49

**Ex 3:** Is there an equal relationship between:

\{the set of men 29 feet tall\} and \{the set of women with hair over one hundred feet long\}?

Yes

How many elements are in these sets? None

Empty Sets
2) Equivalent/Matching Sets = Two sets are equivalent if they have the same number of elements in them.

*Note:* We indicate this by using ~ sign between the set labels.

*Remember:* The elements in the sets do not have to be the same.

**Ex 1:**

- \(E = \{2, 4, 6, 8, 10\}\)
- \(F = \{1, 2, 3, 4\}\)
- \(G = \{\text{science, social studies, math, English}\}\)
- \(H = \{4, 8, 12, 16, 20\}\)

Which sets are equivalent? \(E \sim H\) and \(F \sim G\)

Create a set that is equivalent to set \(G\). \(\{x \mid 0 < x < 5\}\)

**Ex 2:** Is there an equivalent relationship between:

- \{the colors in a rainbow\} and \{the set of seasons in a year\}? 
  - No

How many elements are in these sets? 7 and 4

**Ex 4:** Is there an equivalent relationship between:

- \{a set of one dozen eggs\} and \{the set of months in a year\}? 
  - Yes

How many elements are in these sets? 12 in both
3) Subset = Set A is a subset of Set B if every element in A is also an element of B.

*Note:* We indicate this by writing $A \subset B$.

![Subset Diagram]

**Ex 1:** Set $A = \{\text{the set of all students in this class period}\}$
Set $B = \{\text{the set of all girls in this class}\}$

Then, $B \subset A$.

**Ex 2:**
$S = \{\text{the set of all of the United States}\}$
$N = \{\text{the set of all the United States that start with the letter N}\}$

Then, $N \subset S$.

**Ex 3:**
$E = \{\text{the set of all even numbers between 1 and 9}\}$
$P = \{2, 4, 6, 8, 10\}$

Is $P$ a subset of $E$? No - Then, $P \not\subset E$.

**Special Case #1:** The empty set is a subset of everything.

**Special Case #2:** Every set is a subset of itself.
1) Create a set that is equivalent to:
\[ C = \{\text{red, orange, yellow, green, blue, purple}\} \]

2) \[ A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \quad B = \{x \mid 0 < x < 10\} \]
What relationship exists between set \( A \) and set \( B \)?

3) List the elements in a subset of: \[ E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \]

4) \[ F = \{\text{apples, bananas, grapes}\} \quad V = \{\text{carrots, celery, cucumbers}\} \]
What relationship exists between set \( F \) and set \( V \)?
Dessert Course

Answer the following questions in complete sentences, use vocabulary words when appropriate.

1) Can sets be both equivalent and equal?
   Give an example to support your claim.

2) Can a set be a subset of itself?
   Draw a Venn Diagram to support your answer.

3) The empty set is a subset of what?
Integrated Algebra – Set Theory 4
To Go Food (homework) – Still Hungry

A) True or False:

1) \{2, 4, 6\} ⊂ \{2, 4, 6, 8, 10\}

2) \{1, 4, 7, 9\} ⊂ \{9, 1, 7, 4\}

3) \{1, 3, 5, 7, 9\} = \{the set of positive odd numbers less than 20\}

4) \{1, 2, 3, 4\} = \{the number of seasons\}

5) \{Anne, Sue, Molly\} ~ \{Matt, Mike, Dave\}

6) \{4, 5, 6, 7\} ~ \{10, 21, 35, 65, 79\}

B) Given: A = \{1, 2, 3, 4, 5\}
C = \{1, 3, 5, 7\}

B = \{2, 4, 6, 8, 10\}
D = \{multiples of 2 less than 12\}

7) Which sets are equal?

8) Which sets are equivalent?

9) Create a set that is equal to Set C?

10) Create a set that is equivalent to Set C?

11) List the elements in a subset of D.

12) List the elements in a subset of B.
A) True or False:

1) \{2, 4, 6\} \subseteq \{2, 4, 6, 8, 10\}  

2) \{1, 4, 7, 9\} \subseteq \{9, 1, 7, 4\}

3) \{\} \subseteq \{\text{positive numbers}\}

4) \{1, 2, 3, 4\} = \{\text{the number of seasons}\}

5) \{\text{Anne, Sue, Molly}\} \sim \{\text{Matt, Mike, Dave}\}

6) \{4, 5, 6, 7\} \sim \{10, 21, 35, 65, 79\}

B) Given:  

A = \{1, 2, 3, 4, 5\}  
B = \{2, 4, 6, 8, 10\}

C = \{1, 3, 5, 7\}  
D = \{\text{multiples of 2 less than 12}\}

7) Which sets are equal?

8) Which sets are equivalent?

9) Create a set that is equal to Set C?

10) Create a set that is equivalent to Set D?

11) Create a set that set B could be a subset of.

12) List the elements in a subset of D.
C) Subset/Element Correlation:

13) List all the subsets of the following set: \( T = \{3, 6\} \)

How many elements are in Set \( T \)?

14) List all the subsets of the following set: \( M = \{10, 20, 30\} \)

How many elements are in Set \( M \)?

15) List all the subsets of the following set: \( F = \{4, 8, 12, 16\} \)

How many elements are in Set \( F \)?

16) What is the relationship between the number of subsets and the number of elements in any given set?
Lesson 5 – Set Operations

Formal Lesson Plan

Appetizer

Salad Course: blank notes

Salad Course: modified notes

Salad Course: completed notes

Main Course

Pay the Bill

Dessert Course

Homework
I. Lesson Objective

➢ Students will demonstrate an ability to identify the intersection, union, and complement of sets.

➢ Students will demonstrate an ability to critically think about the operations between sets of numbers or objects.

Standards/Curriculum:

A.A.30 Find the complement of a subset of a given set, within a given universe

A.A.31 Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)

A.A.29 Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form

Possible Strategies/Materials:

➢ Questioning Techniques
➢ Small group work
➢ Venn Diagrams
➢ Kinesthetic Activity
➢ Pay the Bill
➢ Critical Thinking Extension

II. Assessment

Pre-Assessment:
The level of the student’s prior knowledge will be assessed during the review in the beginning of the lesson. Adjustments to add more detail surrounding the use of Venn Diagrams will be made as necessary.

Formative Assessment:
During the lesson, student understanding will be assessed through the use of the small group activity and notes, the kinesthetic activity, and more specifically through the “Pay the Bill” assessment before the students leave class.

Post-Assessment:
The accurate completion of the homework assignment, a jigsaw, and the unit exam will serve this purpose.
Self-Assessment:
The students will be able to self-assess during the group activities and in-class practice. The homework assignment also requires the students to develop their own sets while the dessert course will push them to think critically about set operations.

III. Task Analysis

**Appetizer** – This warm-up is designed to recall information about Venn Diagrams and to get the students thinking about mathematical operations. Both of these intentions will provide a great introduction to the lesson.

**Salad Course: Small Group Activity** – Participation in this activity will allow for reciprocal teaching if needed amongst a small group. This activity will also be accompanied by whole class discussion and note taking to support group work. Students that need help with note taking organization can be given the outlined notes and students with further difficulties including poor handwriting skills or auditory disabilities can be given the completed note sheet.

**Main Course: Kinesthetic Activity** – This activity will turn students into sets while allowing them to move around the room and take an active role in their learning.

**Dessert Course: Critical Thinking Extension** – The dessert course asks students to think at a higher level about the operations between sets and the use of Venn Diagrams. This activity is designed for any student that is ready for critical thinking with types of sets. This activity is not a necessary part of the unit, but a great extension.

**Pay the Bill** – This short activity is to serve as a formative assessment for the teacher along with a self-assessment for the student; it is focused on the essential elements of the lesson including a Venn Diagram and set operations.

**Homework:** There is only one homework assignment for this lesson. It is wonderful graphic organizer to support visual learners and all students to compare and contrast set operations among all representations.

IV. Differentiated Elements:

- The above activities address multiple modalities. There are kinesthetic activities, visually chunked notes, graphic elements, and auditory support to help all types of learners.

- This lesson has a variety of instructional strategies: the lesson begins with independent work, then students are required to work in small groups supported by whole class discussion, then they move around the room during a whole class activity which serves as guided practice, and ends with an independent reflection.
➢ Teachers can assign small group heterogeneously to help the students who need help and further progress the students with a solid understanding of this topic.

➢ Support is provided for weaker learners while extension opportunities are provided for advanced learners.
Appetizer

Use a Venn Diagram to show the following relationships:

1) In a class of 20 students, 12 play sports, 6 play an instrument, and 3 play a sport and an instrument. How many students in the class do not play a sport or an instrument?

2) In a survey of 5000 residents in Henrietta, 1500 people shop at Tops for groceries, 3400 shop at Wegmans, and 800 people sometimes shop at both to get the lowest prices. How many people shop at neither Tops nor Wegmans?

3) List as many mathematical operations as you can think of.
Given the three sets below, fill in the Venn Diagram with the appropriate elements.

A: \{\text{multiples of three less than 20}\} = \{3, 6, 9, 12, 15, 18\}
B: \{\text{even numbers less than 20}\} = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}
C: \{\text{odd numbers less than 20}\} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}

1) Intersection = 

Ex 1: What elements are in $A \cap B$? 

Ex 2: What elements are in $A \cap C$? 

Ex 3: What elements are in $B \cap C$?
2) Union

Ex 1: What elements are in $A \cup B$?

Ex 2: What elements are in $A \cup C$?

Ex 3: What elements are in the universal set made up of Set $A$, $B$, and $C$?

Ex 4: Describe the universal set in Example 3?

3) Complement

Ex 1: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4, 5\}$

What are the elements in $A'$? $A' = \text{______________}$

Ex 2: Given the same universal set, what is $A'$ if $A = \{1, 3, 5, 7, 9\}$?

$A' = \text{______________}$
Given the three sets below, fill in the Venn Diagram with the appropriate elements.

A: \{multiples of three less than 20\} = \{3, 6, 9, 12, 15, 18\}
B: \{even numbers less than 20\} = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}
C: \{odd numbers less than 20\} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}

1) Intersection = 

Ex 1: What elements are in A \(\cap\) B? \{6, 12, 18\}
Ex 2: What elements are in A \(\cap\) C? 
Ex 3: What elements are in B \(\cap\) C? 

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2) Union =

Ex 1: What elements are in $A \cup B$?

$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

Ex 2: What elements are in $A \cup C$?

Ex 3: What elements are in the universal set made up of Set $A$, $B$, and $C$?

Ex 4: Describe the universal set in Example 3?

$U = \{\text{the set of natural numbers less than 20}\}$

3) Complement =

Ex 1: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4, 5\}$

What are the elements in $A'$?

$A' = \{6, 7, 8, 9, 10\}$

Ex 2: Given the same universal set, what is $A'$ if $A = \{1, 3, 5, 7, 9\}$?

$A' =$
Salad Course
Operations with Sets

Given the three sets below, fill in the Venn Diagram with the appropriate elements.

A: \{\text{multiples of three less than 20}\} = \{3, 6, 9, 12, 15, 18\}
B: \{\text{even numbers less than 20}\} = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}
C: \{\text{odd numbers less than 20}\} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}

1) **Intersection** = the intersection of Set X and Set Y contains all of the elements that are common to both Set X and Set Y. We denote this as \(X \cap Y\).

**Ex 1:** What elements are in \(A \cap B\)? \{6, 12, 18\}

**Ex 2:** What elements are in \(A \cap C\)? \{3, 9, 15\}

**Ex 3:** What elements are in \(B \cap C\)? \{\}
2) **Union** = the union of two or more sets represents all of the elements in these sets joined together. We denote this as $A \cup B$.

**Ex 1:** What elements are in $A \cup B$?

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$$

**Ex 2:** What elements are in $A \cup C$?

$$A \cup C = \{1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19\}$$

**Ex 3:** What elements are in the universal set made up of Set A, B, and C?

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$$

**Ex 4:** Describe the universal set in Example 3?

$$U = \{\text{the set of natural numbers less than 20}\}$$

3) **Complement** = the complement of Set $X$ is everything in the universal set ($U$) that is not in Set $X$. We denote this by either $X'$ or $X'$.

**Ex 1:** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4, 5\}$

What are the elements in $A'$?

$$A' = \{6, 7, 8, 9, 10\}$$

**Ex 2:** Given the same universal set, what is $A'$ if $A = \{1, 3, 5, 7, 9\}$?

$$A' = \{2, 4, 6, 8, 10\}.$$
Main Course: Kinesthetic Activity
Teacher Directions

Set-Up:
1) Give each student a number card
2) Have students stand in circle holding their number card for everyone to see.

Practice:
1) The teacher will put the following descriptions on the board.
   - \( A = \{ \text{the set of all prime numbers} \} \)
   - \( B = \{ \text{the set of all perfect squares} \} \)
   - \( C = \{ \text{the set of all multiples of three} \} \)
   - \( D = \{ \text{the set of all positive odd numbers} \} \)
   - \( E = \{ \text{the set of all positive even numbers} \} \)

   Note: Students may need help determining what elements are in the above sets.

2) Teacher writes set operations on the board. If students (their number cards) fall within the given operation, they must move to the middle of the circle.

   Note: The amount of time given for students to enter the circle is determined by the teacher.

3) Suggested Operations:
   - 1) \( A \cap B \)  
   - 2) \( A \cup B \)  
   - 3) \( C \cap D \)  
   - 4) \( D \cap E \)  
   - 5) \( B \cap C \)  
   - 6) \( A \cap E \)  
   - 7) \( A \cap C \)  
   - 8) \( B \cup E \)  

4) Discuss student misconceptions as needed, have fun!
Pay the Bill

Given: \( R = \{\text{red, orange, yellow, green, blue, purple}\} \)
\( F = \{\text{green, pink, blue, black, white}\} \)
\( G = \{\text{red, blue, green, black, white}\} \)

1) Draw a Venn Diagram that represents that given sets.

2) List the elements in \( R \cup G \).

3) List the elements in \( F \cup G \).

4) List the elements in \( R \cap F \).

5) List the elements in \( R \cap G \cap F \).

6) List the elements in \( R' \).
Dessert Course

Take some time to think about the following questions. Use complete sentences in your explanations.

1) Do you think the use of Venn Diagrams helped you in learning about operations of sets? Why or why not?

2) \( A \cup A' = \) ___________ when \( A \) is any set? You must use the terms set, complement, universal, and element in your explanation.
In all of the following problems, make sure to use \( \{ \} \cup \cap \) and set labels as needed.

Fill in the table below:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Definition</th>
<th>Example</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intersection</td>
<td></td>
<td></td>
<td><img src="" alt="Intersection Diagram" /></td>
</tr>
<tr>
<td>Union</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complement</td>
<td></td>
<td></td>
<td><img src="" alt="Complement Diagram" /></td>
</tr>
</tbody>
</table>

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In all of the following problems, make sure to use \{ \} \cup \cap and set labels as needed.

Fill in the table below:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Definition</th>
<th>Example</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Union</td>
<td></td>
<td></td>
<td><img src="image" alt="Union" /></td>
</tr>
<tr>
<td>Complement</td>
<td></td>
<td></td>
<td><img src="image" alt="Complement" /></td>
</tr>
</tbody>
</table>
Lesson 6 – RAFT Project

Formal Lesson Plan

Appetizer

RAFT Requirements

RAFT Rubric

Sample RAFT

Teacher Comment Sheet

Peer RAFT Reflections

Homework
I. Lesson Objective

- Students will demonstrate an ability to think critically about set theory by using its elements in multiple representations.

- Students will demonstrate an ability to work collaboratively in small group to create a story, interview, or rap song using set theory ideas and vocabulary.

- Students will demonstrate an ability to create visual organizers to accompany their story.

- Students will demonstrate an ability to speak in front of their peers as they present their RAFT.

Standards/Curriculum:

A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

A.RP.11 Use a Venn diagram to support a logical argument

A.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A.CN.6 Recognize and apply mathematics to situations in the outside world

A.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

A.A.30 Find the complement of a subset of a given set, within a given universe

A.A.31 Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)

Possible Strategies/Materials:

- Roundtable Activity
- RAFT
- Visual Aids
- Group Presentations
II. Assessment

**Pre-Assessment:** The class will be started will review to activate the students prior knowledge and remind them of the material they are learning. This will be done using a Roundtable Activity.

**Formative Assessment:** The students are required to show me their ideas before they start putting their presentations together. I planned this to make sure that all of the students are working together and so their RAFT’s will include all of the necessary information.

**Post-Assessment:** The accurate completion of the project requirements, a jigsaw activity, and a unit exam will serve this purpose.

III. Task Analysis

*Appetizer: Roundtable Activity* – This warm-up is a straightforward activity to activate students’ prior knowledge and to recall the material from the entire unit. Students should be given time after the activity to check their answers with their peers to ensure they have the correct answers.

*RAFT Project* – This project is a great way to push student thinking to a higher level. Students must work together to create stories or songs or newspaper articles that exemplify sets in the real world. The group orientation of this project will require reciprocal teaching for any students having difficulty. Students should be placed in heterogeneous groups to help this process.

*Visual Aids* – This will help visual learners to better understand their group’s story and set details. It is also important that students become comfortable using Venn Diagrams to represent various mathematical ideas.

*Group Presentations* – Presenting the RAFT to the rest of the class will help to encourage students to put more work in their projects. This element of the project will also help to give the students’ public speaking and character development skills some practice.

IV. Differentiated Elements:

- The above activities address multiple modalities. Elements in this project include visual support, auditory aid, and possible music development to help all types of learners.

- This lesson involves heterogeneous grouping which can provide reciprocal teaching accompanied by required teacher confirmation.

- Students are not only required to choose the components of their RAFT but they must also choose the role in which they are going to play in its presentation. This will allow all students to play a small or large role in this project depending on their levels of knowledge during this unit.
Appetizer
Answer Sheet for Roundtable

Question #1:

E = \{the set of all even numbers\} \quad O = \{the set of all odd numbers\}

a) \_______________

b) \_______________

c) \_______________

d) \_______________

e) \_______________

Question #2:

U = \{the set of all four seasons\} \quad S = \{seasons that start with S\}

a) \_______________

b) \_______________

c) \_______________

d) \_______________

e) \_______________

f) \_______________
Question #3:

Describe the following sets using proper set notation:

a) ____________________________________________________________
b) ____________________________________________________________
c) ____________________________________________________________
d) ____________________________________________________________

Question #4:

List the elements in the following sets:

a) ____________________________________________________________
b) ____________________________________________________________
c) ____________________________________________________________
d) ____________________________________________________________
e) ____________________________________________________________
f) ____________________________________________________________
Question #1:

E = \{the set of all even numbers\}

a) List the elements in set E?

b) List the elements in set O?

c) Is set E finite or infinite?

d) Is set O finite or infinite?

e) What are the elements in E ∪ O?

f) What are the elements in E ∩ O?

Question #2:

U = \{the set of all four seasons\}

a) List the elements in U?

b) List the elements in S?

c) List the elements in S’?

d) What are the elements in S ∩ S’?

e) True or False: U ⊆ S?

f) True or False: Autumn ∈ S?
Question #3:

Describe the following sets using proper set notation:

a) \( A = \{a, b, c, d, e, f, g, h, i, j, k, l\} \)

b) \( N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)

c) Is \( A \sim N? \)
d) Is \( A = N? \)

Question #4:

List the elements in the following sets:

a) \( P = \{\text{prime numbers greater than 2 and less than 20}\} \)
b) \( O = \{\text{odd numbers less than 20}\} \)

c) List the elements in \( P \cap O? \)
d) Is \( (P \cap O) = P? \)

e) Is \( (P \cup O) \sim P? \)
f) Is \( (P \cup O) \subset P? \)
RAFT Requirements:

1) Select a role, audience, and format from the chart below:

<table>
<thead>
<tr>
<th>Role</th>
<th>Audience</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journalist</td>
<td>America</td>
<td>Article</td>
</tr>
<tr>
<td>Talk show host</td>
<td>TV watchers</td>
<td>Interview</td>
</tr>
<tr>
<td>Rap Artist</td>
<td>MTV watchers</td>
<td>Rap Song</td>
</tr>
</tbody>
</table>

The basic topic for your RAFT is of course...Set Theory!

2) Throughout the creation of your RAFT, your group needs to make sure that you are including at least five vocabulary words from the word bank below:

set, element, infinite, finite, empty, universal, equal, equivalent, intersection, union, complement

3) Assign the following tasks to your group members: Recorder, Artist, Presenter #1, Presenter #2.

4) Check your ideas with the teacher before you start writing your rough draft.

5) Create a visual representation of your RAFT that includes a Venn Diagram.

6) Present your RAFT and visual representation to the class.
# RUBRIC FOR RAFT

<table>
<thead>
<tr>
<th>Accuracy of Information</th>
<th>Creativity</th>
<th>Visual Aid</th>
<th>Presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Excellent 8 - 12</strong></td>
<td>At least five vocabulary words are included and clearly defined.</td>
<td>Ideas are presented in a creative and effective way. Elements of the RAFT are clear.</td>
<td>Shows creativity while displaying all necessary pieces of information in a Venn Diagram</td>
</tr>
<tr>
<td><strong>Satisfactory 4 - 8</strong></td>
<td>Less than four vocabulary words are included or vocabulary words are presented in a confusing way.</td>
<td>Ideas are presented in a way that meets requirements but with average creativity and effectiveness. Elements of the RAFT are foggy at times.</td>
<td>Shows limited creativity while displaying most of the necessary information in a Venn Diagram</td>
</tr>
<tr>
<td><strong>Unsatisfactory 0 - 4</strong></td>
<td>Less than four vocabulary words are included and are presented incorrectly or in a confusing way.</td>
<td>Ideas are presented in a way that does not meet requirements, is ineffective, and lacks creativity. Elements of the RAFT are vague.</td>
<td>Lacks creativity and is missing necessary pieces of information or information is not presented in a Venn Diagram</td>
</tr>
</tbody>
</table>

Score: ________ + Score:__________ + Score:_______ + Score:_______ = Total: _______
Set Theory Skit

This story is about a sixth grade boy named Matt. He just came home from school and is having a hard time.

Mom: Hi honey, I’m glad you’re home.

Matt (sad): Hey mom.

Mom (concerned): What’s wrong, you sound upset?

Matt: I’m just having a hard time at school that’s all.

Mom: What happened is everything alright?

Matt: I guess so. It’s just really hard for me to get adjusted to being involved in so many activities. Fifth grade wasn’t this hard.

Mom: Well, aren’t there other kids in a similar situation?

Matt: No mom, I am the only kid in the intersection between the set of musicians at school and the set of soccer players. Even though I am an element in both set I don’t really feel connected to either one.

Mom: Okay, I was worried that this might happen when you entered middle school. What can we do to change the situation, to make things better?

Matt: I don’t know but something has to happen quickly. Otherwise, I might end up quitting one of the activities and I really don’t want to do that.

Mom: I think we should try to have a fun activity that joins both teams together. Maybe some of the soccer players will become interested in music or some of the musicians will start to appreciate soccer.

Matt (excited): I think your idea might work Mom. I think we should have a union between the sets!

Mom: Exactly. Do you have any ideas?

Matt: What about a pizza party? Everyone loves pizza.

Mom: Perfect, we’ll have it at 3:00 pm right after music practice but before soccer practice.

Matt: Thanks Mom, I think this might work!
Comment Sheet – RAFT assignment

Group ______:

Member Tasks: Recorder ____________________________________________
    Artist ____________________________________________
    Presenter #1 ____________________________________________
    Presenter #2 ____________________________________________

Accuracy of Information:

    vocabulary word #1 -
    vocabulary word #2 -
    vocabulary word #3 -
    vocabulary word #4 -
    vocabulary word #5 -

Creativity:


Visual Aid:


Presentation:
Peer RAFT Reflections

This reflection sheet is for you to fill out as you listen to the presentations of your peers.

**Group #1:**

Vocabulary Words Used:

_________________  __________________  __________________

_________________  __________________

Creativity: (scale from 1 to 5)  

Character Development: (scale from 1 to 5)  

**Group #2:**

Vocabulary Words Used:

_________________  __________________  __________________

_________________  __________________

Creativity: (scale from 1 to 5)  

Character Development: (scale from 1 to 5)
Group #3: ____________________________________________

Vocabulary Words Used:

________________________

Creativity: (scale from 1 to 5) ______

Character Development: (scale from 1 to 5) ______

Group #4: ____________________________________________

Vocabulary Words Used:

________________________

Creativity: (scale from 1 to 5) ______

Character Development: (scale from 1 to 5) ______
Group #5: __________________________________________

Vocabulary Words Used:

_________    _________    _________

_________    _________

Creativity: (scale from 1 to 5) ______

Character Development: (scale from 1 to 5) ______

Group #6: __________________________________________

Vocabulary Words Used:

_________    _________    _________

_________    _________

Creativity: (scale from 1 to 5) ______

Character Development: (scale from 1 to 5) ______
Integrated Algebra

Given: \( P = \{ \text{perfect squares less than 30} \} \)
\( F = \{ \text{multiples of five less than 20 inclusively} \} \)
\( H = \{ \text{positive numbers less than 10} \} \)

1) Is \( P \) a finite or infinite set?

2) Is \( F \) a finite or infinite set?

3) What are the elements in \( P \cup F \)?

4) What are the elements in \( H \cap F \)?

5) List the elements of a subset of set \( P \).

6) Rewrite set \( H \) in set builder notation.

9) Rewrite set \( H \) using interval notation.

10) Graph set \( H \) on a number line.

11) Draw a Venn Diagram representing sets \( P, F, \) and \( H \) and list all appropriate elements.

12) List the elements in \( P' \).

13) List the elements in \( U \).
Unit Conclusion

Jigsaw Review Activity

Unit Exam: modified version

Unit Exam: version 2
Integrated Algebra
Jigsaw Review Activity

Complete only one question in your “Expert Groups” and then work through the other questions in your “Home Groups”.

1) $E = \{\text{positive even numbers less than 20}\} \quad O = \{\text{positive odd numbers}\}$

A) Represent Set E using set builder notation. 

B) Represent Set O in a listing format.

C) Is Set E finite or infinite?

D) Is Set O finite or infinite?

E) List the elements in $E \cup O$?

F) List the elements in $E \cap O$?

2) $U = \{\text{the set of all four seasons}\} \quad S = \{\text{seasons that start with S}\}$

A) Represent the elements in this universal set in a listing format.

B) List the elements in Set S.

C) List the elements in Set $S^{c}$.

D) What are the elements in $S \cap S^{c}$?

E) True or False: $U \subseteq S$?

F) True or False: Autumn $\in S$?
3) \( A = \{a, b, c, d, e, f, g, h, i, j, k, l\} \) \( N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)

A) Represent Set \( A \) in a descriptive format. 

B) Represent Set \( N \) using interval notation.

C) True or False: \( A \sim N? \)

D) True or False: \( A = N? \)

E) Explain your answer to part (C).

F) Explain your answer to part (D).

4) \( F = \{\text{factors of twenty four}\} \) \( O = \{\text{positive odd numbers less than } 10 \text{ inclusively}\} \)

A) Draw a Venn Diagram that represents Sets \( F \) and \( O \) and fill it in with the appropriate elements.

Represent the elements in the following sets in a listing format.

B) \( F \cup O? \)

C) \( F \cap O? \)

D) Represent set \( O \) using interval notation.

E) \( F^c? \)
Integrated Algebra
Set Theory – Unit Exam 1

**Match the correct symbol in Column A to its term in Column B to its definition in Column C**

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ∈</td>
<td>Intersection</td>
<td>A set which contains no elements</td>
</tr>
<tr>
<td>2) ∪</td>
<td>Empty Set</td>
<td>When two sets have the same amount of elements</td>
</tr>
<tr>
<td>3) ∩</td>
<td>Universal Set</td>
<td>Individual items in a set</td>
</tr>
<tr>
<td>4) { }</td>
<td>Equivalent Sets</td>
<td>This operation contains elements that are common to both sets</td>
</tr>
<tr>
<td>5) U</td>
<td>Union</td>
<td>The entire set of elements in a given situation</td>
</tr>
<tr>
<td>6) ~</td>
<td>Element</td>
<td>This operation represents all elements in both sets joined together</td>
</tr>
</tbody>
</table>

S = *{the set of all x such that x is between 2 and 8 inclusively}*  

7) Rewrite set S using set builder notation:

---

8) a) Convert set S into interval notation:  
   b) Graph set S on a number line:

---

I = {-7, -6, -5, -4, -3, -2, -1, 0, 1, 2}  

9) Rewrite set I using set builder notation:

---

10) a) Convert set I into interval notation:  
    b) Graph set I on a number line:
Given: \( E = \{\text{positive odd numbers}\} \)

15) Represent set \( O \) in a descriptive format.

16) Is \( E \) finite or infinite?

17) Is \( O \) finite or infinite?

18) List the elements in \( E \cap O \).

19) True or False: Is \( E \sim O \)?

20) True or False: Is \( O \subseteq E \)?

21) List one possible subset of set \( E \).

22) Create a set that is equivalent to \( O \). Include a list of its elements and a set label.
Given: \( A = \{ \text{colors in the American flag} \} \) \quad P = \{ \text{red, yellow, blue} \}

23) List the elements in A.

Draw a Venn Diagram that represents sets A and P and fill it in with the appropriate elements.

24) List the elements in \( A \cap P \).

25) List the elements in \( A \cup P \).

26) True or False: \( A \sim P \)?

27) True or False: \( A = P \)?

28) True or False: \( P \subseteq A \)?

29) True or False: White \( \in P \)?

30) List the elements in \( A^c \).
Integrated Algebra
Set Theory – Unit Exam 2

Match the correct symbol in Column A to its term in Column B to its definition in Column C

<table>
<thead>
<tr>
<th></th>
<th>Intersection</th>
<th>A set which contains no elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>∈</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>∪</td>
<td>Empty Set</td>
</tr>
<tr>
<td>3)</td>
<td>∩</td>
<td>Universal Set</td>
</tr>
<tr>
<td>4)</td>
<td>{ }</td>
<td>Equivalent Sets</td>
</tr>
<tr>
<td>5)</td>
<td>U</td>
<td>Union</td>
</tr>
<tr>
<td>6)</td>
<td>~</td>
<td>Element</td>
</tr>
</tbody>
</table>

A set which contains no elements
When two sets have the same amount of elements
Individual items in a set
This operation contains elements that are common to both sets
The entire set of elements in a given situation
This operation represents all elements in both sets joined together

F = {multiples of four between 4 and 28 inclusively}

7) Rewrite set F using set builder notation:

F = \{n \in \mathbb{Z} \mid 4 \leq n \leq 28 \text{ and } n \equiv 0 \pmod{4}\}

8) Rewrite set E using set builder notation:

E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}

9) Convert set I into interval notation:

I = [-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4]

10) Graph set I on a number line:
A = \{Alabama, Alaska, Arizona, Arkansas\}  \quad C = \{California, Colorado, Connecticut\}

11) True or False: A = C?

12) True or False: A \sim C?

13) List the elements in A \cup C.

14) List the elements in A \cap C.

Given: E = \{positive odd numbers\}  \quad O = \{1, 3, 5, 7, 9, 11, 13, 15\}

15) Represent Set O in a descriptive format.

16) Is E finite or infinite?

17) Is O finite or infinite?

18) True or False: Is E \sim O?

19) True or False: Is O \subset E?

20) Given G = \{1, 2, 3, 4\}, List all possible subsets of set G.

21) The empty set is a subset of other set/s?
Given: \( A = \{\text{colors in the American flag}\} \quad \text{P} = \{\text{red, orange, yellow, green, blue, purple}\} \)

22) List the elements in \( A \).

\[
\begin{align*}
\text{List the elements in } A. \\
\end{align*}
\]

Draw a Venn Diagram that represents sets \( A \) and \( P \) and fill it in with the appropriate elements.

23) List the elements in \( A \cap P \).

\[
\begin{align*}
\text{List the elements in } A \cap P. \\
\end{align*}
\]

24) List the elements in \( A \cup P \).

\[
\begin{align*}
\text{List the elements in } A \cup P. \\
\end{align*}
\]

25) True or False: \( A \sim P \)

\[
\begin{align*}
\text{True or False: } A \sim P. \\
\end{align*}
\]

26) True or False: \( A = P \)

\[
\begin{align*}
\text{True or False: } A = P. \\
\end{align*}
\]

27) True or False: \( P \subseteq A \)

\[
\begin{align*}
\text{True or False: } P \subseteq A. \\
\end{align*}
\]

28) True or False: White \( \in P \)

\[
\begin{align*}
\text{True or False: } \text{White} \in P. \\
\end{align*}
\]

29) List the elements in \( A^c \).

\[
\begin{align*}
\text{List the elements in } A^c. \\
\end{align*}
\]

30) List the elements in \( P^c \).

\[
\begin{align*}
\text{List the elements in } P^c. \\
\end{align*}
\]
Data Participants and Collection

The only subjects in this research are the teachers that have reflected on the unit plan. There are fifteen teacher participants from various districts, grade levels, backgrounds, subject areas, and states (participant details provided in accompanying table). The selected teachers embody a wide range of personalities, teaching styles, and years of experience.

Given the varied backgrounds of the teacher sample, the analysis of their thoughts and ideas has resulted in data and reflections that are representative of a larger population. No other pre-existing questionnaires were used as sources for this instrument. The development of the Teacher Questionnaire was specifically designed to test the topics of interest from the initial research and focuses of the unit plan. These topics are all possible consequences of implementing this unit which concentrates on heterogeneous grouping and differentiated instructional strategies. After receiving the completed Teacher Questionnaires, a qualitative and quantitative analysis was performed using the participant responses. Tables and charts have been used to better draw conclusions from the years of teacher expertise. Specific elements of the questionnaires were evaluated regarding the actual methods of differentiation, changes or additions to the unit, and any perceived alterations in certain specific student behaviors which are later discussed.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Grade Level</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>4 - 8</td>
<td>Special Education</td>
</tr>
<tr>
<td>#2</td>
<td>8</td>
<td>Math - Algebra</td>
</tr>
<tr>
<td>#3</td>
<td>7</td>
<td>Math and Science</td>
</tr>
<tr>
<td>#4</td>
<td>10</td>
<td>Math - Geometry, Pre-Calculus</td>
</tr>
<tr>
<td>#5</td>
<td>7</td>
<td>Math</td>
</tr>
<tr>
<td>#6</td>
<td>4</td>
<td>All Subjects</td>
</tr>
<tr>
<td>#7</td>
<td>8 - 12</td>
<td>Math - all subjects</td>
</tr>
<tr>
<td>#8</td>
<td>7</td>
<td>Math</td>
</tr>
<tr>
<td>#9</td>
<td>8</td>
<td>Math - Algebra</td>
</tr>
<tr>
<td>#10</td>
<td>12</td>
<td>Math - AP Calculus</td>
</tr>
<tr>
<td>#11</td>
<td>K - 12</td>
<td>English/Administration</td>
</tr>
<tr>
<td>#12</td>
<td>9 - 12</td>
<td>Math - all areas</td>
</tr>
<tr>
<td>#13</td>
<td>8</td>
<td>Math - Algebra</td>
</tr>
<tr>
<td>#14</td>
<td>4 - 6</td>
<td>All subjects</td>
</tr>
<tr>
<td>#15</td>
<td>11</td>
<td>English</td>
</tr>
</tbody>
</table>
Analysis

Although the Teacher Questionnaire (provided in Appendix A) examined many aspects of student attitude and behavior, this study specifically focuses on individual student variables. This research concentrates on perceived changes in mathematical interest, motivation, risk taking, class participation, engagement, critical thinking, and effort. Teacher comments on the specific elements of differentiation and its effects on stronger and weaker students will also be discussed. A dual faceted approach to data analysis will be provided containing both quantitative and qualitative feedback.

The mean, range, and modal response of teacher opinions to certain questions measuring the above student variables as a product of delivering the proposed unit on Elementary Set Theory are listed below in Table 1. These descriptive statistical measures provide a basic yet appropriate analysis of the teacher responses.

<table>
<thead>
<tr>
<th>Student Variables</th>
<th>Mean response</th>
<th>Maximum response</th>
<th>Minimum response</th>
<th>Modal Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Interest</td>
<td>4.53</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Student Motivation</td>
<td>4.73</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Academic Risk Taking</td>
<td>4.33</td>
<td>5</td>
<td>3</td>
<td>4, 5</td>
</tr>
<tr>
<td>Class Participation</td>
<td>4.60</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Levels of Student Engagement</td>
<td>4.60</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Critical Thinking Skills</td>
<td>4.60</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Student Effort</td>
<td>4.40</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Total Teacher Responses = 15
The percent of teacher responses (1 – 5) to the student variables listed above in Table 1 is given in the graph below. This graph should provide a more detailed representation of how the teacher responses were distributed. As you can see from the pie chart, no responses of 1 or 2 were given in the Teacher Questionnaires.

A portion of the qualitative data provided from the Teacher Questionnaires is provided below in Table 2. The quotes included below represent multiple teacher responses from various backgrounds including special education, history, and mathematics from elementary, middle level, and advanced grades. The selected responses are specific to the student variable in question. Names of the teacher participants whose thoughts were used in this table and throughout the study have been kept confidential to obtain anonymity.
<table>
<thead>
<tr>
<th>Student Variables</th>
<th>Specific Teacher Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Interest</td>
<td>“The integration of kinesthetic activities for middle school combined with the meaningful translation of abstract math concepts into concrete examples is exceptional.”</td>
</tr>
</tbody>
</table>
| Student Motivation       | “The group work and structure are highly motivating, typically I don’t think students engage as much in this unit, but I have a real positive feel that these lessons will move them to more engagement and positive experiences.”  
                           | “I believe that students will feel like they have complete control in this unit. They even become part of the problem and the solution in one of the activities.”                                                              |
| Academic Risk Taking     | “When students feel comfortable and learning at their level, they will take risks.”                                                                                                                                           |
| Class Participation      | “This is a unit that everyone will be able to understand at the conceptual, verbal level. Participation and class discussions will be of a high quality.”                                                                               |
| Levels of Student Engagement | “The high interest activities, varied response opportunities, scaffolding, and the clearly developed progression of the task analysis should encourage engagement.”                                                    |
| Critical Thinking Skills | “What is nice about this unit is that it is done so all students can raise their level of thinking, not just those who typically get conceptual math.”                                                                               । "With the several open ended questions, the students will become more creative and critical in their responses as the unit progresses.” |
| Student Effort           | “If this unit cannot increase effort, then I don’t know what will!”                                                                                                                                                           |
Discussion

Looking at the tables of data and comments above, it is clear that the differentiated unit plan is expected to yield overall positive student responses. All student variables are expected to move in a positive direction with no teacher responses disagreeing to any extent. This is further shown by the pie chart which shows that all teacher responses were either 3: undecided, 4: agree, or 5: strongly agree. Showing that an overwhelming 58% of teacher responses strongly agreed with the positive response of all students, the success of this unit plan is anticipated. Student motivation is the variable that is expected to deliver the most dramatic results while academic risk taking will provide the least dramatic but still positive result.

The Teacher Comments clearly indicate the encouraging and hopeful results that are expected to come from this unit. One teacher went on to comment, “I think you’ve given all students the ability to learn in a manner appropriate for them.” It seems as if all teachers from this wide range of participants are in agreement on multiple fronts, that this unit will successfully differentiate instruction and lead to exceptional exhibitions of student attitude and behavior throughout this unit.

More specific comments were made to address the individual learning anticipated from weaker students. One teacher suggested, “Make the final assessment and the pre-assessment start out simple enough to not discourage the learners who are not quite there yet. You will get more effort and less of a defeatist attitude.” This is an important idea to allow all students to feel capable and competent as they start this new unit. Another teacher added, “This unit is set up in a way that relates to students lives’ while taking a hard concept and breaking it into more manageable pieces.” This feedback is gratifying and helps to build a case for the struggling students’ ability to relate to this unit and to feel confident throughout.
Other teachers pointed out the benefits of this unit and of differentiation for the stronger students. "The culminating activities for the unit are excellent and should provide advanced students with the opportunity to use their creativity to integrate multiple areas of mathematical understandings with their background skills and knowledge." Addressing the advanced students is a function that can be very difficult for every day instruction. It is encouraging to hear that this unit goal will be met to some degree. A final comment summed up one of the overall objectives of this unit, "I believe you will get a different group of students than the usual willing to participate because this unit uses both the mathematical and linguistic portions of the brain." By using multiple modalities, more students will become engaged and interested.

Schiefele and Csikszentmihalyi are researchers mentioned above who are focused on interest and motivation levels relating to academic success. They found that motivation can be sustained by providing positive learning experiences (p. 177). This team of researchers goes on to comment on the positive effects of student interest and its direct relationship with further motivation and self-esteem (p. 173). Taking their findings into account along with the responses of teachers in this current study, it is fair to conclude that this differentiated unit plan will lead to a certain degree of academic success. Furthermore, it has been clearly shown that all teachers who participated in the current study believe that student interest will positively respond to this set theory unit.

"Typically this is a tough unit for the diehard computation loving students, but I think the level of peer engagement along with the opportunity for real world exploration and connections will help them stay engaged. In addition, it can be extremely confusing for the student who lacks conceptual background but with the built in group support and multiple tasks, they too, should be able to hang in and enjoy the unit" (anonymous teacher participant).

It is reasonable to conclude through this research as well the research of others that interest and motivation are linked and definite contributors to academic achievement. This study has
strengthened the notion that heterogeneous grouping and interactive differentiated lessons will help provide academic success for all students.

**Summary**

After completing an initial review of literature, developing a unit plan based on the research, distributing this unit to area teachers, and analyzing their reflections, it is acceptable for this research to agree with the initial research findings and state that heterogeneous grouping and differentiation is ultimately the most beneficial grouping method in a mathematics classroom. However, there is most certainly room for further investigation into this topic. Now that a more definite conclusion has been drawn in regards to grouping, more research is needed to dive into the repercussions of this finding on teacher preparation and attitude. As found in the literature review, many teachers still feel intimidated and perplexed at the idea of actually implementing differentiation into their classrooms.

Until teachers wholeheartedly believe in the power of differentiation and heterogeneous grouping and learn to fully encompass this mode of instruction in their classrooms, its effects will not be seen, its full potential will not be reached, and student success will never rise to the height of its capability. Professional development opportunities and true teaming environments are necessary in all schools where heterogeneous grouping resides in order to make teachers feel more comfortable with this type of instruction.
Bibliography


Appendix A – Teacher Questionnaire

Kristin Duschen
Homogeneous versus Heterogeneous Grouping and Differentiation

Teacher Reflection Questionnaire

I have completed this questionnaire based on my own opinions, thoughts, and ideas to the best of my ability. I am aware that my personal identification will be kept confidential, but my thoughts will be compared, analyzed, and used to make conclusions.

Date ____________

Please fill out questions A – F to the best of your ability:

A) Yrs. of Teaching Experience ______________________________

B) Current Level of Instruction ______________________________

C) Area of Certification ______________________________________

D) Preferred Teaching Style (Explain) __________________________

E) Do you have any experience grouping students homogeneously or heterogeneously? (Explain)

________________________________________________________________________________________

________________________________________________________________________________________

F) Do you have any prior experience with differentiation? (Explain) __________________________

________________________________________________________________________________________

________________________________________________________________________________________
Now that you have read the unit plan developed by Kristin Duschen pertaining to Grouping and Differentiation, please complete the following response questions.

A) Rating Scale:

Rate how strongly you agree or disagree with a statement by circling a specific number. Space is provided under each statement to further explain your responses if you choose to do so.

1 – strongly disagree  2 – disagree  3 – undecided  4 – agree  5 – strongly agree

Student Attitude:

1) Students’ interest in math content will respond positively to this unit.

2) Students’ interest in math class will respond positively to this unit.

3) Students will respond positively to accepting a mathematical challenge.

4) Students’ will be willing to take risks through their oral and/or written responses in this unit.

5) Student motivation will respond positively to this unit.
<table>
<thead>
<tr>
<th></th>
<th>1 – strongly disagree</th>
<th>2 – disagree</th>
<th>3 – undecided</th>
<th>4 – agree</th>
<th>5 – strongly agree</th>
</tr>
</thead>
</table>

**Student Behavior:**

6) Students’ will be motivated to help other students throughout this unit.  
   1 2 3 4 5

7) Students’ levels of engagement during class activities will respond positively throughout this unit.  
   1 2 3 4 5

8) Students’ levels of engagement during class discussions will respond positively during this unit.  
   1 2 3 4 5

9) Students’ frequency of class participation will respond positively during this unit.  
   1 2 3 4 5

10) Students’ levels of self-advocacy will respond positively to this unit of curriculum.  
    1 2 3 4 5

11) Students’ levels of critical thinking will respond positively during this unit.  
    1 2 3 4 5
1) strongly disagree  2) disagree  3) undecided  4) agree  5) strongly agree

12) Students’ overall levels of effort will respond positively during this unit.

13) Students will be motivated to complete homework during this unit.

B) Short Answer/Free Response:

The following questions require short answers and explanations. Please answer to the best of your ability; write as much or as little as you see fit. Responses should be your opinions based on your previous teaching experiences.

14) Do you feel that all students would be actively engaged throughout this unit?

15) Are there opportunities for advanced students to be challenged?

16) Are there opportunities for weaker students to contribute and shine?
17) Are there opportunities for continual formative assessment?


18) Are there opportunities for student feedback and reflection?


18) Do you foresee any potential problems or shortcomings within the unit?


19) Do you have any additions or changes to the differentiated unit?


C) Final, additional thoughts:

Please use the remaining space for any additional comments, thoughts, or concerns you have with this unit, its activities, content, or outcomes.