A Look at Ability Grouping vs. Cooperative Learning

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A Look at Ability Grouping vs. Cooperative Learning

by
Laura Martin
May 2012

A thesis or project submitted to the
Department of Education and Human Development of the
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A Look at Ability Grouping vs. Cooperative Learning

by

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Advisor

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Director, Graduate Programs

12/14/11
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12/14/11
Date
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Pre-Calculus Unit Exams

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Algebra 2/Trigonometry Unit Exams

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Chapter 1 Introduction:

What is the ideal grouping pattern of students? Do students work better in homogeneous ability groups or in heterogeneous cooperative learning groups?

Grouping of students into homogenous or heterogeneous classes or groups has been addressed in research and in classrooms for many years. Many schools have used the practice of tracking and are now moving away from it as they realize that it does not necessarily benefit all students. As we have progressed in our thinking about education, changes have been made to better reach all students, and a lot of those changes have had to do with the grouping of students.

Another important aspect in education is a move towards differentiation as a focus for our instruction. A look at within class ability grouping can help in the practice of differentiation. If students are grouped with students of the same ability, they will more likely work at similar speeds and it allows for teachers to give enrichment or remediation where needed.

Similarly, cooperative learning, where students work together to accomplish goals, is a way in which to encourage students to work well together, become more independent and overall become better prepared for life after high school. Not only does cooperative learning benefit certain groups of students, but it helps all students to become well rounded individuals.
For this study, I have become very interested in the topic of grouping students. I have a mixture of ability levels of students within all of my class, as is the case with any teacher. I have had students working in groups, but the structure of the groups has always intrigued me. I am very interested in seeing if grouping students homogeneously or heterogeneously makes an impact; whether a positive one on low or average achieving students or a negative one on higher achieving students.

Another reason why this topic interests me is due to the structure of the school in which I work. There have had many conversations between colleagues as to possible ways to improve one of the courses that I teach, Algebra 2/Trigonometry. Currently, the district in which I work accelerates almost all of their students, which causes an issue in certain courses at the high school level. Students that are not accelerated tend to be very low achieving students, and at the Algebra 2/Trigonometry level, we are seeing huge differences between accelerated students and non-accelerated students. Also, we have many students unprepared for the course or not attempting the course due to its difficulty. Due to this issue, I am very interested in looking at the research for and against ability grouping to get a better understanding of this issue as it relates to the school district in which I teach and the courses that I teach. Also, looking at other ways, such as within class ability grouping or cooperative learning strategies, to reach these lower students in my heterogeneous classes will help me as a teacher to bring more students to a mastery level.
Overall, the grouping of students is an extremely controversial topic and throughout history the research has shown many advantages and disadvantages of ability grouping and cooperative learning. Thus, this topic is significant for students and schools everywhere. A closer look at these strategies, in comparison to each other, and with high school age students needs to be addressed further.
Chapter 2: Literature Review

Over the past few decades, the issue of grouping in schools has been a focus of educational research. Much of this research is controversial. Tracking or streaming, within-class grouping, other forms of ability-grouping and even cooperative learning all represent different forms of grouping students. Each form of grouping has its own advantages and disadvantages for student learning and achievement.

To begin with, ability-grouping "implies some means of grouping students for instruction by ability or achievement so as to reduce their heterogeneity." Ability grouping can be seen in the form of tracking or streaming, regrouping students of selected subjects, the Joplin Plan (regrouping by ability for reading), non-graded plans, special classes for high achievers, and within-class ability grouping (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987).

Tracking or streaming refers to classes consisting of students of a single ability or achievement level. In some cases, it has been reported that this form of grouping has few benefits to student achievement, with the exception of small positive gains from higher-achieving students. This form of grouping allows higher achieving students to broaden, extend and accelerate curriculum, thus increasing their achievement and opportunity within their education (Rogers, 1993). Some research has shown that tracking simply increases the gap between low-ability and high-ability
achievers; however one particular study showed that the added gap that is created in a tracking system was nearly nonexistent (Linchevski & Kutscher, 1998).

Another popular form of ability grouping is within-class grouping. These groups can vary from reading or math groups within the classroom, to a completely heterogeneous classroom with groups separated by ability. These groups can be strictly determined by achievement, or can be more flexible based on the specific topic, subject or learning style of students. A very common form of within class ability grouping involves the teacher presenting a lesson to the class as a group, then allowing students to work in their assigned ability groups while the teacher provides enrichment to the higher-ability groups and extra support to the lower-ability groups (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987).

Throughout the variety of ways that ability-grouping may be seen in schools, there are arguments for and against ability-grouping. Ability grouping can allow teacher to focus their attention and curriculum to best reach the group that they are working with (Slavin, Ability Grouping in the Middle Grades: Achievement Effects and Alternatives, 1993). However, on the other hand, this practice creates low achieving groups or classes. Often there are lower expectations for these groups which may be self-fulfilled and create a culture of low achievement (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987).
Within class ability grouping allows the teacher to participate in smaller group attention and instruction. It has been found in numerous studies that this small group attention has a significantly positive effect on student achievement over classes that did not use small group instruction (Lou, Abrami, & Spence, 2000). This focus on small group instruction has been shown to be more beneficial for high-ability students than for lower-ability students and more helpful for elementary students than for college students (Holloway, 2001).

Another important aspect of within class ability grouping is the fact that it encourages the practice of differentiation. Since students are separated by ability, teachers can more readily differentiate the assigned work for each group. This allows for enrichment activities for the higher achieving groups and extra attention and practice for the lower achieving groups (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987). This in turn, benefits all students because they are receiving work that is individualized and targeted for their strengths and weaknesses.

Along with the variety of grouping practices, alternatives include a teaching strategy of cooperative learning, which involves a form of grouping. Cooperative learning is defined as “students working together in a group small enough that everyone can participate on a collective task that has been clearly assigned [where] students are expected to carry out their task without direct and immediate supervision of the teacher” (Cohen, 1994). These groups work together toward a common goal,
where students are responsible for individual learning and the overall understanding of the group. Interdependence of reward, task and materials is a significant part of this strategy – students work together toward a common goal and hold each other accountable for participation and learning progress. Cooperative learning supplements teacher instructions by giving students the ability and time to engage in meaningful conversations about the content and either practice skills presented by the teacher, or discover new concepts on their own (Slavin, Synthesis of Research on Cooperative Learning, 1991). Groups are usually heterogeneous in achievement; typically including one high-achieving student, one low-achieving student and two average-achieving students (Nattiv, 1994). Similarly to ability-grouping, there are a variety of forms of cooperative learning strategies, where group work and group goals are the focus.

There are advantages and disadvantages to the structure of cooperative learning as a teaching strategy. Within the structure of cooperative learning, a main focus of the structure is teamwork and interdependence of group members. Throughout this students demonstrate “helping behaviors” which are encouraged to make the grouping more beneficial to the group members. Higher achieving students are more responsible for giving meaningful explanations and help to their group members, which lower achieving students receive. Both types of students gain from their roles and through the results of various studies, it has been found that there is a strong relationship between giving and receiving meaningful help and achievement gains (Nattiv, 1994). However, this same strength can be considered a weakness as
well. The structure focuses on group members being responsible for each other’s learning, which can be a challenge and huge responsibility for students for the higher achieving students to take on (Randall, 1999).

Within these heterogeneous groups or teams, students receive rewards based on their achievement as an overall team. These rewards are based on individuals improving their past achievements, thus motivating them to achieve even more in the future (Slavin, Synthesis of Research on Cooperative Learning, 1991). The overall reward is based on the entire team achieving, which encourages team members to work together, since their success, in part depends on the other group members (Slavin, Synthesis of Research on Cooperative Learning, 1991).

Due to the importance of this teamwork focus of cooperative learning, some training for both teachers and students should occur to ensure the success of this strategy. This structure demands teamwork from all students; they must work together and ensure that all students in the group are learning the material. Thus, an important aspect to instruct students on is the correct way to ask for help and to give it. In one particular study that saw positive results from these “helping behaviors” students went through three weeks of instruction on giving and receiving help prior to the cooperative learning (Nattiv, 1994). Another important aspect of training includes the teachers. Teachers’ beliefs and attitudes about cooperative learning can have a significant impact on their approach and in turn success with cooperative learning strategies. Thus, teachers need to be shown the positive research, and instructed on
the correct ways to implement the various cooperative learning strategies (Antil, Jenkins, Wayne, & Vadasy, 1998).

Another aspect of both ability grouping and cooperative learning is the definition of the work that students are doing. Cooperative learning in particular focuses on encouraging students to “discuss, debate and ultimately teach one another” (Slavin, Synthesis of Research on Cooperative Learning, 1991). Therefore, when group work within a class is more discussion based or inquiry based, the implementation of a cooperative learning model is more beneficial. However, ability grouping (within class grouping) is designed to increase pace and level for higher achieving students and provide them with enrichment, while allowing the teacher to give more attention to lower achieving students with repetition and review (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987). Thus, when group work is more based on practicing skills, extensions of learning or remediation, within class ability grouping is more beneficial for students. Also, a structure of within class ability grouping encourages an atmosphere of differentiation for students, which means that the work is a more appropriate level for each individual student; thus helping all students at all levels reach success.

Overall these previously discussed strategies either benefit or hinder each of the ability groups; low-achieving students, average-achieving students and high-achieving students. Any of the grouping options that create lower ability classes or
groups can be detrimental to lower ability groups. Due to the perceived low expectations of these students, the result can be a slower pace and a lower achievement. However, these students can benefit from extra support or remediation that is offered to them through within class ability grouping (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987). In one study of college students homogenous grouping of students within the class did not harm the lower achieving students in any way (Baer, 2003). In terms of cooperative learning, lower ability students can benefit from the peer help that they receive based on the structure of the groups or teams (Nattiv, 1994). Cooperative learning allows for all students to demonstrate their strengths at various times. When this occurs, praising the lower students for their accomplishments can increase their achievement and self-esteem (Cohen, 1994). In one particular study, with a focus on cooperative learning and computer use, low ability students’ mathematics anxiety decreased, as compared to those receiving individual instruction (Bracey, 1992).

Throughout the research, we can conclude that homogenous ability grouping is more beneficial for higher ability students (Baer, 2003). This is especially evident in higher education. A study of college students in both homogenous and heterogeneous groups was completed. Higher ability students in the homogenous had statistically significantly higher scores on their midterm and final exams (Baer, 2003). Through homogenous grouping, whether it is demonstrated through tracking and high ability students are in a self-contained class, or it is evident through within class grouping, higher ability students are given work that is more at their level and given
much desired enrichment. Often times higher-ability students get a similar advantage in cooperative grouping as well. The higher ability students are partially responsible for teaching or helping weaker students. This teaching can reinforce concepts and in turn increase these students achievement. However, this same relationship can have a negative impact on higher achieving students. They often feel overwhelmed with the responsibility of “teaching” other students and in turn have a very negative view of group work (Baer, 2003).

Throughout much of the research, there is little recorded information about the effects of both ability grouping and cooperative learning on the average achieving students. Most average achieving students did not show significant differences from homogenous grouping to heterogeneous grouping (Baer, 2003). In the few studies that did record results on average achieving students, through the use of cooperative learning strategies slight positive differences were found for these students (Slavin, Synthesis of Research on Cooperative Learning, 1991).

Overall, from the research on both ability grouping and cooperative learning, there are advantages and disadvantages for all level students in each teaching strategy. In general, higher ability students benefit most positively from ability grouping, whether by tracking or within-class homogenous grouping. On the other hand, most studies show that lower ability students benefit most from a cooperative learning set up which entails mixed-ability grouping. The average ability students seem to benefit a bit more in mixed ability grouping, but they are not affected by
grouping differences as much as the other two groups. From this information, using only one method of grouping may not be ideal for all students.

Throughout much of the research on both ability grouping and cooperative learning, there are some aspects that are missing. First of all, there is little information in the research as to the way in which teachers led class. It was not clear how teachers taught differently in classes with tracking, within class ability grouping, cooperative learning based classes or even in control classes. The way that teachers structure the class, not just the groups, makes a difference and is an important aspect to consider. Also, many of the studies, especially the ones on within class grouping, do not last very long. Most of the studies last less than a school year, so they may not show the extent to which certain grouping helps or hinders students (Slavin, Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis, 1987). Lastly, there are many studies on ability grouping versus a control and cooperative learning versus a control, but there are few, if any, studies that compare within class ability grouping to cooperative learning strategies. Lastly, most of the research has been conducted in elementary schools and middle or junior high schools. Very little data has been collected in upper middle school, and even less has been collected at the high school level.
Chapter 3: Applications and Evaluations

Throughout my research, I have found that different types of grouping strategies can benefit certain types of students over other methods. Previous research has been done on both cooperative learning strategies (mixed ability grouping) and ability grouping (homogenous grouping) with different outcomes for low, average and high achieving students. It has been found that high achieving students benefit more from ability groups, and lower achieving students benefit more from cooperative learning strategies. Average students show limited changes from the various grouping patterns.

Based on the information that I have found from previous studies, I decided to conduct a study in which I look at both types of grouping patterns for a class of students. Even though most of these studies have been conducted with younger students, I believe that I will find similar results as the previous research with my study at the high school level. Thus, I hypothesize that when I compare high achieving students’ test scores, the ones in ability groups will be higher than those in cooperative learning groups. Also, I believe that when I compare low achieving students’ test scores, the ones in cooperative learning groups will show more positive results than those in ability groups. Lastly, I believe that the average achieving students will show limited difference from one grouping strategy to another.

Outcomes will be measured based on unit exam scores. Unit exam scores will be used to first determine if a student is considered to be a low, average or high
achieving student in order to correctly group them. Then, unit exam scores will be compared to track progress after being placed in the various grouping strategies.

For this research, a group of high school students from a suburban school in Rochester, NY were used. A total of 61 mathematics students in 9th, 10th, 11th and 12th grade from two different courses were analyzed. The two courses, Algebra 2/Trigonometry and Pre-Calculus, were taught by me, the researcher. Out of the total students, 32 came from two different Pre-Calculus classes and 29 came from two different Algebra 2/Trigonometry classes. Each class lasts 80 minutes; Pre-Calculus meets every other day and Algebra 2/Trigonometry meets every other day with an extra 40 minutes every 6 days. In both of these classes, students were put into groups of three or four using the two different grouping techniques. In the Pre-Calculus classes less than 50% of the class-time was spent teacher led, with any additional time allotted for students to work in their groups on practice problems. In Algebra 2/Trigonometry, about 60% of the time was spent with teacher led instruction, with any additional time for students to work in their groups. During this student work time, students were encouraged to work with their group members while also using the teacher to assist in problem solving. Throughout each unit, for both courses, there was at least one work day (review) per class which was completely group work with no teacher led instruction and in Pre-Calculus most units had at least two completely group work days. During all instructional and work times, students were seated in groups.
For the data collection and analysis, I have chosen to compare unit exam scores of the students because I am interested in testing the effects of various grouping strategies on performance data. Within the Pre-Calculus classes, two unit exams were recorded to determine a baseline for the students to categorize them as low, average, or high achieving. The two exam scores were averaged together, and then within each class, the students were ranked and divided into the three ability groups. Similarly, for the Algebra 2/Trigonometry classes, four unit exams were examined to determine if a student was a low, average, or high achieving student. The four exam scores were averaged together, and then, within each class, the students were ranked and divided into three ability groups. One aspect to note though is that for various reasons, not every student in each class is accounted for in my data reporting and analysis. However, students as an entire class were ranked to determine their achievement level, so in some cases, there may seem to be more or less of one type of achievement category within a class, when in reality, that is not the case. All of the specific unit exam scores and achievement categories for every student included in this study can be found in Table 9, 10, 11, and 12 in the Appendix.

For the study, I looked at the average test scores for each of the Pre-Calculus students and after completing a two-tailed equal variance t-test of the data, determined that the two classes were considered to be equivalent as recorded on Table 1.
Table 1

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<th>Ability Group</th>
<th>Cooperative Learning Group</th>
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This was concluded based on the fact that the calculated p value is greater than the designated alpha value of .05, which means there is no statistically significant difference between the two Pre-Calculus classes. The same process was completed with the two Algebra 2/Trigonometry classes with the same result, that there is no statistically significant difference between the two different Algebra 2/Trigonometry
classes as noted in Table 2. Again, from this we can conclude that the two Algebra 2/Trigonometry classes are considered equivalent.

Table 2

t-Test: Two-Sample Assuming Equal Variances

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</table>

Since we can conclude that both Pre-Calculus classes are equivalent and that both Algebra 2/Trigonometry classes are equivalent, I looked at each course as one group of students. I chose at random one class to break into ability groups (homogeneous groups) and another class to rearrange students into cooperative learning groups
The ability groups consisted of either three or four students (depending on the class size) of the same ability. The cooperative learning groups had one high achieving student, one low achieving student and either one or two average achieving students (depending on the class size).

Within the mixed ability groups, I encouraged students to direct questions towards a certain group member (the higher achieving student). For instance, as students worked through problems I would check the higher achieving student’s answer first. Once I determined his or her work was correct, other group members were instructed to seek help from that particular student. Also, at times when a lower achieving student would ask me a question, I would refer him or her to the high achiever in the group as long as that student was comfortable handling the question. These practices helped to encourage group work amongst the members.

I also used the grouping patterns in the ability groups to the students’ advantage. In those classes, all of the lower students were grouped together, so I could give them more attention and have re-teaching opportunities while other students were able to work independently. Also, at times I was able to give enrichment activities to the higher achieving groups while lower groups were still working on specific, more basic problems. High achievers also had opportunities to try more challenging problems.
Chapter 4: Results

For this study, as previously stated, I used unit exams to track changes in student achievement. All students in Pre-Calculus received common assessments for each unit as did all students in Algebra 2/Trigonometry. Each test was graded fairly by me in order to keep the process consistent. Similar mistakes were marked off in the same way; therefore there was no difference in grades due to the examination or grading process.

Overall, throughout all units that were included in this study, the intended learning outcome was mastery of the topics included in the unit. Different grouping patterns were used to encourage and increase the mastery of all students. Previous research has found that different grouping patterns, as compared to only direct instruction, have benefitted students. Also, research has found that different forms of grouping may benefit certain achieving students more than others. Using both types of grouping patterns, ability grouping and cooperative learning strategies, along with a limited amount of teacher-led instruction, the goal of this methodology was to reach students in a different way and increase achievement on unit exams.

Another issue that sparkled this methodology has been past experience with students. I have observed how students work independently and with others. I have noticed that students tend to group themselves naturally both ways – heterogeneously and homogeneously. Students seemed to have benefitted from their own grouping methods. In seeing these patterns naturally occur, it inspired me to be deliberate in
creating these grouping patterns in order to look at the specific data to see if there were significant differences for the entire class in one grouping pattern over another.

In terms of this specific study, once students spent a unit in their assigned grouping patterns, I recorded the unit exam scores for each type of grouping pattern and compared the data from one type of grouping method to the other using a two tailed, equal variance t-test. In doing this, for the Pre-Calculus classes, there was no statistically significant difference between the two grouping methods. This was determined due to the fact that the calculated p value for this set of data is greater than our alpha at the .05 level which can be seen in Table 3. Similar results were found with the unit exam scores in the Algebra 2/Trigonometry classes. There were no statistically significant differences from the unit exam scores in the ability grouped class to those of the cooperative learning grouped class. This is represented below in Table 4.
Table 3:

**t-Test: Two-Sample Assuming Equal Variances**

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<thead>
<tr>
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<th>Cooperative Learning Groups</th>
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After determining that there was no significant difference from one grouping pattern to the other, I decided to take a closer look at individual students' progress from the units prior to the grouping method to after they had been assigned to a specific grouping method for a unit. The charts below show the differences in each individual student's unit exams scores. I compared the average of their first units (the ones used to determine which achievement category they would be part of) to their unit exam score after the specific grouping method they were a part of. The places
where the bar graph is above the x-axis shows that a student’s score increased after the grouping method; while the places where bar graph is below the x-axis shows that a student’s score decreased after the grouping method.

Table 5

![Bar Chart]

Table 6

![Bar Chart]
As you can see from Table 5, 6, 7 and 8 above, both Pre-Calculus classes and both Algebra 2/Trigonometry classes had students whose grades made significant increases, significant decreases and changes that were minimal. My original research
found that ability grouping tends to benefit higher achieving students more than other students. However, in the specific Pre-Calculus class that was arranged by ability grouping, out of the six students who were considered to be high achieving students, only one showed to have a unit exam score that was more than 10 points different after the grouping method, and it actually decreased by more than 10 points. Similarly, for the high achieving students in the cooperative learning grouping method, out of the six high achieving students, only one had a unit exam score that was more than 10 points different after the grouping method, and again the score had decreased by more than 10 points. In the specific Algebra 2/Trigonometry class that was arranged by ability grouping, out of the five students who were considered to be high achieving students, none of them made changes in their unit exam scores by 10 or more points. However in the Algebra 2/Trigonometry class that was arranged using cooperative learning strategies, out of the six students who were considered to be high achieving students, two of them had changes in their test grades by more than 10 points, but both were decreases by more than 10 points. Overall, these results are not consistent with past research on the effects of grouping patterns on high achieving students.

Looking at the data in a similar way for low achieving students, my previous research concluded that low achieving students benefit most from cooperative learning methods. Out of the two students who were considered low achieving in the cooperative learning Pre-Calculus class, only one had a unit exam score that was more than 10 points different after the grouping method and it had decreased by more
than 10 points. However, in the Pre-Calculus class that was set up with ability groups, out of the four students who were categorized as low achieving students, two of them had unit exam scores that differed by more than 10 points after the grouping method, and in both cases there was an increase by more than 10 points. In the Algebra 2/Trigonometry class that was grouped using cooperative learning strategies, out of the four students who were considered to be low achieving students, every student made positive improvements in their unit test scores, one of which was by more than 10 points. However, in the Algebra 2/Trigonometry class that was separated into ability groups, out of the three students who were considered to be low achieving students, all three had decreases in their unit exam grades, one of which being by more than 10 points. In this small sample of students, the Algebra 2/Trigonometry results were consistent with previous research, however the results of lower achievers in the Pre-Calculus classes were not consistent.

In looking at the previous research, there is often a slight advantage for average achieving students with cooperative learning strategies; however this achieving group usually is not affected much by different grouping patterns. In the Pre-Calculus class which was grouped using cooperative learning strategies out of the seven students who were considered average achieving students, four showed changes of more than 10 points on their unit exam after the grouping method, however two of those students had increased by more than 10 points on their unit exam while two had decreased by more than 10 points. In the Pre-Calculus class that was arranged by ability grouping, out of the seven students who were considered
average achieving students, three of them had changes in their unit exam scores by more than 10 points after the grouping methods, however, one increased by more than 10 points and the other two decreased by more than 10 points on their unit exams. In the ability grouped Algebra 2/Trigonometry class, out of the four average achieving students, two made changes in their unit exam grades by more than 10 points, however one was an increase while the other was a decrease. In the cooperative learning grouped Algebra 2/Trigonometry class, out of the seven average achieving students, four of these students saw changes in their unit exam grades of more than 10 points, however all of them were decreases. This is somewhat consistent with previous research, in that average achieving students did not seem to be more influenced by one grouping pattern over another.

Overall, this analysis shows that some individual students were successful in their grouping structure, while others were not. Some of this data is consistent with previous research about grouping patterns and their effects on different achieving student and some was not. However there are many factors that could be affecting my results. First of all, there is a small sample of students used for the overall study and even more so when looking at the results for individuals in each of the three categories of achievement levels. Do to this fact, my results may not be able to be repeated in other studies and the results should not be generalized for all students.

Another issue to consider with the recorded data is specific to the school at which the data was collected. This particular school allows students the ability to
retest any and every mathematics test that they take. Due to this, there are some students who may not put forth 100% effort on their first attempt at a test because they know that they can retest. From this, looking at students’ original unit exam scores to determine if a student is a high, average or low achiever may not be completely accurate. Also, their results on the unit exam score after the grouping method may have similar effects due to the mentality of students that they can always retest if they do not succeed the first time.

Also, throughout the units in which students were in their specific grouping patterns, a few minor issues came up that may have affected the validity of the data. First of all, periodically students decided to work in a different group than the one that they were originally assigned to. I did my best to watch for this and make students move back to their assigned group, however there may have been days that I was not aware of student changes in groups at various times. Also, some classes had one day with a substitute teacher, who may or may not have enforced the assigned seats. In the cooperative learning grouped Pre-Calculus class, this was the case on their unit exam review day, which is completely student led and intended to be a day devoted to group work. Unfortunately, with a substitute that day, there is no way to know for sure if students stayed in their assigned groups, which could have affected their results (or lack thereof) on the unit exam after the grouping method.

The last issue which may have affected the results outside of the grouping method was the specific topics dealt with during these units. In the case of Pre-
Calculus, the first two units that were used to determine a baseline and categorize students to a specific achievement group are considered to be easier units, with a significant amount of information that is repetitive from their previous course of Algebra 2/Trigonometry. However, the unit that was taught during my study had many more new and difficult topics for students. In terms of Algebra 2/Trigonometry, the first four units that were used to determine a baseline had a mixture of both easier topics/units and more challenging units. The specific unit that was taught during the study is considered to be of average difficulty, and uses a lot of information from previous units. Thus, students who excelled earlier in the course, the average and high achieving students may have had an easier time with this unit. The lower achieving students, who may not have mastered previous topics reoccurring in this unit, were at more of a disadvantage.
Chapter 5: Conclusions and Recommendations

Throughout this study, I specifically looked at unit test scores to make data driven conclusions about various grouping patterns within my classroom. However, I also made observations of my students throughout group work time, had conversations with students, listened to student comments throughout the process and even got feedback about the process from a few parents. All of these aspects are important to record and consider in addition to the specific data from the unit exam scores.

To begin with, I will discuss some interesting things that I observed happening during group work time in my classes. In my ability grouped Pre-Calculus class, some groups worked very well together and were constantly having conversations with each other, helping each other with problems, initiating great discussions surrounding problem solving and using me as a resource when they needed it. In particular, this occurred in one of the high groups, one of the average groups and one of the low groups, which showed that at least one of each type of achievement group used the resource of their peers, whether it affected their unit exam scores or not. However, within the same class, some groups, no matter how much group work was encouraged did not use the resource of their group members and had limited to no conversations with each other. Thus, for these students, the fact that they were grouped may have had no impact on them due to the fact that the grouping was not being utilized.
In the cooperative learning Algebra 2/Trigonometry class specifically, there were two groups that really used their high achieving student as a resource, especially for the low achieving student. This class had a great deal of peer teaching occurring throughout group work time. Again, this may or may not have affected student unit exam scores, but it is a good practice to be occurring throughout a class. However, this was the class in which all low achieving students made improvements and the one with the largest improvement came from one of these two groups were a great deal of cooperation between members occurred.

One of the biggest issues throughout this study, which occurred in all four classes, was students complaining about their groups. Many students wanted to and in some instances tried to switch groups. Some days it was a struggle to get students to stay in their assigned group. The biggest complaint was the fact that students wanted to work with their friends or with people they were comfortable with. Some students work better with people they are comfortable with. On the other hand, groups consisting of friends may end up socializing too much and even though they think they are accomplishing a lot, in reality they are not. In one particular case, a Pre-Calculus student admitted his negative feelings about his “new” group which caused him to become lazy. He felt that laziness caused him to get a lower unit exam grade than he was capable of. He blamed it on the fact that he was not able to work with classmates that he was comfortable with.
Similarly, I had a meeting with a parent of an average achieving student in my ability grouped Pre-Calculus class. In the random grouping for the first two units, he had been arbitrarily placed with mainly high achieving students. After the grouping pattern changed to ability groups, his unit exam grade decreased by 8 points. His mother expressed to me that when her son works with students he feels are academically beneath him, he does not work to his full potential. In her opinion, he benefits from being surrounded by students who pull him up academically. She proposed that this lower exam grade may have been due to the grouping pattern.

I believe that the issue of compliance may have affected unit exam scores in the grouping patterns. Again, many students expressed that they did not like the members of their group or preferred to work with their friends. From this information, I wonder if the data would have been different if student choice in group members was considered, in addition to achievement level or instead of achievement level.

Overall, there were some benefits for individual students from the various groups in which they were assigned, but I do not feel as though I can make any concrete conclusions from my research. To increase the validity of the results of this study, I would make some significant changes in the future. First of all, ideally I would have used more students to increase the population being observed. In doing so, the possibility of looking at additional mathematics courses or teachers may affect student results. However, the length of time over which the data was collected may have resulted in more consistency with precious research. In this case, students were
grouped in only one specific pattern for only one unit. In the future, I would like to conduct the same study but with students in a specific grouping pattern for at least two or more units. I would also like to extend the study to allow students experience both grouping methods.

Even though there was no statistically significant difference shown in the data collected in this study, I feel the issue of grouping students is still an important one that needs more consideration. I agree with findings shared in my research that students benefit more from working together than working independently. The question remains as to whether there is one grouping method ideal for all students.
References:


Appendix:

Table 9

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1. For each of the functions, determine the following using interval notations: [1 pt each]

(a)

(i) domain ________________

(ii) range ________________

(iii) x-intercept(s)__________

(iv) y-intercept ________________

(b)

(i) domain ________________

(ii) range ________________

(iii) x-intercept(s)__________

(iv) y-intercept ________________
2. Use the figure to solve each equation or inequality. [2 pts each]

\[ f(x) = g(x) \]

\[ f(x) < g(x) \]

\[ f(x) \geq g(x) \]

\[ y_1 - y_2 = 0 \]

3. Use the screen to solve the equation or inequality. Here the function \( y_1 = f(x) \) is a linear function defined over the domain of real numbers. Express answers in interval notation. [1 pt each]

\[ y_1 = 0 \]

\[ y_1 < 0 \]

\[ y_1 > 0 \]

\[ y_1 \leq 0 \]
4. Consider the linear function \( f(x) = 2(x - 1) + 7 \) and \( g(x) = x + 3 - 3(x - 2) \).

(a) Solve \( f(x) = g(x) \) analytically, showing all steps. Also, check analytically. [2]

(b) Graph \( y_1 = f(x) \) and \( y_2 = g(x) \). [2]

(c) Based on the graph above, determine the interval that satisfies \( f(x) < g(x) \). [1]

(d) Based on the graph above, determine the interval that satisfies \( f(x) > g(x) \). [1]
5. Consider the linear function \( f(x) = 3(x-1)-\frac{1}{3}(6x-9) \).

(a) Solve the equation \( f(x) = 0 \) analytically. [2]

(b) Solve the inequality \( f(x) \leq 0 \) analytically. [2]

(c) Graph \( y = f(x) \) in an appropriate viewing window and explain how the graph supports your answers in parts (a) and (b). [2]
6. Find the equation of the line passing through the point (3, -2) and
(a) parallel to the line with equation \( y = 2x + 3 \). [2]

(b) perpendicular to the line with equation \( 3x + y = 0 \). [2]

7. For the line whose standard form is \( 3x + 7y = -9 \), find: [1 pt each]
(a) the \( x \)-intercept
(b) the \( y \)-intercept
(c) the slope

8. Give the equation of both (a) the horizontal and (b) the vertical lines passing through the point \( (4, -7) \). [1 pt each]
(a) ________________
(b) ________________


<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Debt</td>
<td>5478</td>
<td>5606</td>
<td>5629</td>
<td>5770</td>
<td>6198</td>
</tr>
</tbody>
</table>

(a) Find the least-squares regression line for the data. Give the correlation coefficient and round it to the nearest ten thousandth. [2]

(b) Use the above equation to predict the federal debt in 2010. [1]
10. Suppose that an empty circular wading pool has a radius of 7 feet. During a storm, rain falling at a rate of 1 inch per hour begins to fill the pool. A small drain at the bottom of the pool is capable of draining 35 gallons of water per hour.

(a) Determine the number of cubic inches of water falling into the pool in one hour. 
(Hint: Each hour a layer of water 1 inch thick falls into the pool.) [2]

(b) One gallon of water equal about $231$ cubic inches. Write a formula for a function $g$ that computes the gallons of water landing in the pool in $x$ hours. [2]

(c) How many gallons of water land in the pool during a 3 hour storm? [1]

(d) Will the drain be able to keep up with the rainfall? If not, how many such drains would be needed? [1]
Chapter 2 Test Form A

Name: ______________________

Date: ___________________  Block: ____________________

Directions: Answer each of the questions on this paper. To receive full credit you must show all necessary calculations. A graphing calculator may be used.

1. For each of the following express your answer in interval notation. [1 pt each]

   a. domain of \( f(x) = x^2 + 3 \) ______________________
   
   b. range of \( f(x) = x^2 + 3 \) ______________________
   
   c. domain of \( f(x) = \sqrt{x} + 3 \) ______________________
   
   d. range of \( f(x) = \sqrt{x} - 3 \) ______________________
   
   e. domain of \( f(x) = \sqrt[3]{x} + 3 \) ______________________
   
   f. range of \( f(x) = \sqrt[3]{x} - 3 \) ______________________
   
   g. domain of \( x = y^2 - 3 \) ______________________
   
   h. range of \( x = y^2 - 3 \) ______________________
   
   i. domain of \( f(x) = |x-3| \) ______________________
   
   j. range of \( f(x) = |x| + 3 \) ______________________
2. The graph of \( y = f(x) \) is shown here. Sketch the graph of each of the following. Use ordered pairs to indicate 3 points on the graph. [1 pt each]

a. \( y = f(x) - 3 \)

b. \( y = f(x - 3) \)

c. \( y = -f(x) \)

d. \( y = f(-x) \)

e. \( y = 3f(x) \)

f. \( y = |f(x)| \)
3. a) Use transformation of graphs to sketch the graph of \( y = -2|x + 2| - 3 \), starting with the parent function. [1 pt]

b) State the domain. [1 pt]

c) State the range. [1 pt]
4. Write a description that explains how the graph of \( y = 3\sqrt{x - 2} - 6 \) can be obtained by transforming the graph of \( y = \sqrt{x} \). [2 pts]
5. Observe the coordinates displayed on the graph of $y = f(x)$. Answer each of the following based on your observation. [1 pt each]

a. If the graph is symmetric with respect to the y-axis, what are the coordinates of another point on the graph?

b. If the graph is symmetric with respect to the origin what are the coordinates of another point on the graph?

c. Suppose the graph is symmetric with respect to the y-axis. Draw the graph you would expect to see in this window.
6. Consider the graph of the function shown below. [8 pts total]

Use interval notations and state the interval(s) over which the function is:

a. increasing

b. decreasing

c. constant

d. continuous

e. What is the domain of the function?

f. What is the range of this function?
7. (i) Solve each of the following **analytically**, showing all steps. Express your answer in interval notation when possible. [2 pts each]

(ii) Next graph $y_1 = |3x - 6|$ and $y_2 = 3$ on your calculator and sketch the graphs. Then state how the graphs help support your solution in each case. [2 pts]

a. $|3x - 6| = 3$

b. $|3x - 6| < 3$

Explain:

c. $|3x - 6| > 3$
8. Given \( f(x) = 2x^2 + x - 6 \) and \( g(x) = 5x + 3 \), find each of the following. Simplify the expression when possible.

a. \((f - g)(x)\) [1 pt]

b. \(\frac{f}{g}(x)\) [1 pt]

c. the domain of \( \frac{f}{g}(x) \) [2 pts]

d. \((f \circ g)(x)\) [2 pts]

e. \(\frac{f(x + h) - f(x)}{h}(h \neq 0)\) [5 pts]
9. a) Graph the piecewise-defined function defined by \( f(x) = \begin{cases} x^2 - 8 & \text{if } x < 4 \\ -\sqrt{x} - 4 & \text{if } x \geq 4 \end{cases} \). [4 pts]

b) Evaluate \( f(x) \) at the following values: [1 pt each]

\[ f(4) = \quad \]

\[ f(9) = \quad \]

\[ f(-\sqrt{3}) = \quad \]
10. In Here or There one can go to a coffee shop and pay to use their internet service. If \( x \) represents the number of minutes one is online, where \( x > 0 \), then the function defined by \( f(x) = 0.50\lfloor x \rfloor + 1.50 \) gives the total cost in dollars.

a. Sketch the graph of \( f(x) \). Label the axes. [4 pts]

b. Use the graph to find the cost of being online for 6.5 minutes. [1 pt]
11. Bennie Hopschnu’s band, Dedicated Indifference, wants to record a CD. The cost to record a CD is $900 for studio fees plus $5.50 for each CD produced. [1 pt each]

a. Write a cost function \( C \), where \( x \) represents the number of CDs produced.

b. Find the revenue function \( R \), if each CD in part (a) sells for $10.00.

c. Give the profit function \( P \).

d. How many CDs must be produced and sold before the band earns a profit?

e. Support the results of part (d) graphically. Label the axes.
Chapter 3 Test Form A

Part I: Circle the correct answer. No partial credit will be given. [2 pts each]

1. Use Descartes' Rule of Signs to determine the possible number of negative real zeros for the function. \( P(x) = 4x^6 - 8x^4 - 7x^3 + 2x^2 - 4x \)
   a) 2 or 0
   b) only 2
   c) 3 or 1
   d) only 3

Given the following function: \( f(x) = -x^5 + 16x^4 - 69x^3 - 14x^2 + 392x \) answer questions 2 and 3

2. Identify the end behavior
   a) \( \infty \)
   b) \( -\infty \)
   c) \( \infty \)
   d) \( -\infty \)

3. Identify the type of extrema
   a) 1 abs max, 1 abs min, 1 local max, 1 local min
   b) 1 abs max, 1 local max, 1 local min
   c) 2 local max, 2 local min
   d) 1 local max, 1 local min

4. What is the interval for the quadratic inequality \( 2x^2 < -5x + 12 \)?
   a) \( \left( -4, \frac{3}{2} \right) \)
   b) \( (-\infty, -4) \cup \left( \frac{3}{2}, \infty \right) \)
   c) \( (-\infty, \infty) \)
   d) None of the above
Part II

5. Put the following quadratic function into the form $f(x) = a(x - h)^2 + k$ by completing the square. [4 pts]

$$f(x) = 2x^2 - 12x + 7$$

6. A piece of cardboard is 3 times as long as it is wide. Equal sized squares measuring 5 inches on each side are to be cut off from the corners of the piece of cardboard, and the flaps are to be folded up to form a box with an open top.

a) Picture [1 pt]

b) Determine the function $V(x)$ in standard form that best describes the volume of the box as a function of $x$, where $x$ is the width of the original metal sheet in inches. [3 pts]

c) Determine the restrictions on $x$. (Theoretical Domain) [1 pt]

d) Find the dimensions of the box if its volume is 1435 cubic inches graphically. [2 pts]
7. Perform the following for the function defined by $f(x) = x^4 - 6x^2 - 40$

   a) Find all zeros analytically. [4 pts]

b) Use the graph and the results from part (a) to find the solution set in interval notation for the following inequality. [2 pts]

   $f(x) \geq 0$

8. The table gives the high school dropout rate (as a percent of enrollment) in the United States for the years 1994 to 2001. Let $x = 4$ represent 1994, $x = 5$ represent 1995, and so on.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dropout rate</td>
<td>5.0</td>
<td>5.4</td>
<td>4.7</td>
<td>4.3</td>
<td>4.4</td>
<td>4.7</td>
<td>4.5</td>
<td>4.7</td>
</tr>
</tbody>
</table>

   a) Find the correlation coefficient after a Quadratic, Cubic, and Quartic regression to the nearest ten thousandth. [1 pt each]

   Quadratic ________________                  Cubic ________________
   Quartic ________________

b) Which one of these regressions is the best model for the given data? Justify your answer. [1 pt]
9. Given that $f(x) = x^5 + 2x^4 + 3x^3 - 8x^2 - 22x - 12$ has $-1$ as a zero of multiplicity 2, and 2 as a single zero, find all other zeros of $f$. [6 pts]
10. Perform the following for the function defined by \( f(x) = 2x^4 + x^3 - 13x^2 - 5x + 15 \)

a) List all possible rational zeros. [2 pts]

b) Use the calculator to identify the actual rational zeros from the list in part a). [1 pt]

c) Use the intermediate value theorem to show that there must be a real zero between 2 and 3. Explain! [2 pts]
11. Find a polynomial function $P(x)$ in standard form of least possible degree, having real coefficients, with the given zeros. [6 pts]

2 – 5i, and 3 with a multiplicity of 2
Multiple Choice (2 pts each)
Identify the choice that best completes the statement or answers the question.

1. Solve the equation and check the solution.

\[ |2x - 2| = 8 \]

a. \( x = 5 \) or \( x = -3 \)
b. \( x = 3 \) or \( x = -3 \)
c. \( x = 3 \) or \( x = 5 \)
d. \( x = 3 \) or \( x = 10 \)

2. Which graph represents the following inequality?

\[ |2x + 6| < 16 \]

a. \( x < -5 \) or \( x > 5 \)
b. \( -11 < x < 5 \)
c. \( -22 < x < 10 \)
d. \( -11 > x > 5 \)

3. A furniture maker uses the specification \( 21.88 \leq w \leq 22.12 \) for the width \( w \) in inches of a desk drawer. Write the specification as an inequality.

a. \( |w - 0.24| \leq 22.12 \)
b. \( |w - 22| \leq 0.12 \)
c. \( |w - 22| \leq 0.24 \)
d. \( |w - 0.12| \leq 22 \)
4. Which scatterplot diagram shows the strongest positive correlation?

4. Which scatterplot diagram shows the strongest positive correlation?

a. 

b. 

c. 

d. 

5. Use the vertical-line test to determine which graph represents a function.

5. Use the vertical-line test to determine which graph represents a function.

a. 

b. 

c. 

d. 

A2.S.08

A2.A.52
6. Write the ordered pairs for the relation. Find the domain and range. [A2.A.51]

a. \{(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)\}; domain: \{-2, -1, 0, 1, 2\}; range: \{1, 2, 5\}
b. \{(5, -2), (2, -1), (1, 0), (2, 1), (5, 2)\}; domain: \{-2, -1, 0, 1, 2\}; range: \{1, 2, 5\}
c. \{(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)\}; domain: \{1, 2, 5\}; range: \{-2, -1, 0, 1, 2\}
d. \{(5, -2), (2, -1), (1, 0), (2, 1), (5, 2)\}; domain: \{1, 2, 5\}; range: \{-2, -1, 0, 1, 2\}

Short Answer

Solve the equation algebraically. Check for extraneous solutions.

7. \(|4x + 3| = 9x - 2\) (4 pts) [A2.A.01]
8. A) State the domain of the relation below: (2pts) [A2.A.38, A2.A.51]

B) State the range of the relation below: (2pts)

C) Is this relation a function? Explain why. (2pts)

9. Solve the equation graphically. Show appropriate work for full credit. (4 pts) [A2.A.01]

\[ |x - 3| = 5 \]
10. Solve the following inequality and graph your solutions.
\[-3|6-x| < -15\] (4 pts) [A2.A.01]
11. The accompanying table illustrates the number of movie theaters showing a popular film and the film’s weekly gross earnings, in millions of dollars.

<table>
<thead>
<tr>
<th>Number of Theaters (x)</th>
<th>Gross Earnings (y) (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>443</td>
<td>2.57</td>
</tr>
<tr>
<td>455</td>
<td>2.65</td>
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<tr>
<td>493</td>
<td>3.73</td>
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<tr>
<td>530</td>
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<td>569</td>
<td>4.76</td>
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<tr>
<td>657</td>
<td>4.76</td>
</tr>
<tr>
<td>723</td>
<td>5.15</td>
</tr>
<tr>
<td>1,064</td>
<td>9.35</td>
</tr>
</tbody>
</table>

A) Write the linear regression equation for this set of data, rounding values to five decimal places. (2pts)

B) Using this regression equation, find the approximate gross earnings, in millions of dollars, generated by 523 theaters. Round answer to two decimal places. (2pts)

C) Find the minimum number of theaters that would it would take to generate at least 10.25 million dollars in gross earnings in one week. (2pts)
12. A) Graph the following function: (2pts)
   \[ f(x) = -|x + 3| + 5 \]

B) State the domain of the given function. (1pt)

C) State the range of the given function. (1pt)

13. Write the equation for the translation of \( y = |x| \). (2 pts) [A2.A.46]
Multiple Choice (2pts each)
Identify the letter of the choice that best completes the statement or answers the question.

1. Simplify: \( \frac{\sqrt[3]{162x^4}}{\sqrt[3]{2x}} \)
   - a. \( 3x\sqrt{3} \)
   - b. \( \sqrt[3]{162x^3} \)
   - c. \( 3x^3\sqrt{3x} \)
   - d. \( 3x\sqrt{3x} \)

2. Solve for \( x \): \( x = 5 + \sqrt{x + 7} \)
   - a. \( \emptyset \)
   - b. \( \{9\} \)
   - c. \( \{-9, -2\} \)
   - d. \( \{9, 2\} \)

3. Solve for \( x \). \( (x + 2)^{\frac{3}{2}} = 8 \)
   - a. \( 34 \)
   - b. \( 6 \)
   - c. \( 30 \)
   - d. \( 10 \)

4. Write the exponential expression \( x^{\frac{3}{4}} \) in radical form.
   - a. \( \sqrt[4]{x^3} \)
   - b. \( \frac{1}{\sqrt[4]{x^4}} \)
   - c. \( \frac{1}{\sqrt[4]{x^3}} \)
   - d. \( 4\sqrt{x^3} \)
5. What is the domain of the function \( f(x) = \sqrt{x + 5} ? \)  
   a. \([-5, \infty)\)  
   b. \((-5, \infty)\)  
   c. \(x = -5\)  
   d. \((0, \infty)\)

6. Write the radical expression \( \frac{8}{\sqrt[7]{x^{15}}} \) in exponential form.  
   a. \(8x^{\frac{7}{15}}\)  
   b. \(8x^{\frac{15}{7}}\)  
   c. \(8x^{\frac{15}{7}}\)  
   d. \(8x^{\frac{7}{15}}\)

7. Evaluate the following function \( f(x) = x^\frac{3}{2} + \frac{3}{4} \sqrt{x^4 - x^{-2}} \) for \( f(4) \).  
   a. 10  
   b. \(\frac{415}{16}\)  
   c. 17  
   d. \(\frac{93}{16}\)

8. Simplify \( \sqrt{6 - \sqrt{3}} \).  
   \[ \frac{\sqrt{6 - \sqrt{3}}}{\sqrt{6 + \sqrt{3}}} \]  
   [A2.N.05, A2.A.15] (4pts)
9. Write the following expression in simplest form with only positive exponents.
\[(a^{-2})(a^{-4})\]  
\[\text{[A2.A.9] (2pts)}\]

10. Express \[5\sqrt{12x^5} - 2\sqrt{27x^5}\] in simplest radical form.  
\[\text{[A2.A.14, A2.N.02, A2.N.04] (4pts)}\]

11. Graph the function on the axes below.  
\[\text{[A2.A.39, A2.A.46] (4pts)}\]

a) \[f(x) = \sqrt{x + 3} - 5.\]

b) State the domain and range of the function.
12. Write \((27a^{-3})^{\frac{2}{3}}\) in simplest form. [A2.N.1,A2.A.13,A2.A.14] (2 pts.)

13. Solve the equation and check. *Only an algebraic solution will receive full credit.*

\[
\sqrt[3]{7x - 3} + 3 = x
\]

[A2.A.22] (6pts)
Multiple Choice (2 points each)

Identify the choice that best completes the statement or answers the question.

1. Suppose \( f(x) = 4x - 2 \) and \( g(x) = -2x + 1 \).
   Find the value of: \( \frac{f(0)}{g(-2)} \)
   a. -10 b. 2 c. -2 d. \( \frac{2}{5} \)

2. What is the range of the function \( f(x) = x^2 - 4x - 12 \)
   a. \([-\infty, -12]\) c. \([-16, \infty)\)
   b. \((-16, \infty)\) d. \(\mathbb{R}\)

3. What is the domain of the function \( f(x) = \frac{2x}{\sqrt{x + 6}} \)?
   a. \([-6, \infty)\) c. \(x = -6\)
   b. \((-6, \infty)\) d. \((0, \infty)\)

4. Let \( f(x) = x^2 + 4 \) and \( g(x) = \sqrt{1 - x} \), what is the value of \( f(g(-3)) \)?
   a. \(2i\sqrt{3}\) b. 8 c. 2 d. 13
5. What is the inverse of the function $f(x) = 7x + 2$?  

   a. $f^{-1}(x) = 7x - 2$  
   b. $f^{-1}(x) = \frac{2x}{7}$  
   c. $f^{-1}(x) = \frac{2-x}{7}$  
   d. $f^{-1}(x) = \frac{x-2}{7}$

6. Which of the following functions has a limited domain?  

   a. $y = \frac{3}{4}x + 5$  
   b. $y = [2x + 5]$  
   c. $y = x^2 + 2x - 5$  
   d. $y = \frac{x+4}{x^2 - 5}$

7. Determine which of the following represents the inverse of the given function.  

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>4</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>-1</td>
</tr>
</tbody>
</table>

   a.  
   | $x$ | 3  | 2  | 7  | -1 |
   | $y$ | 0  | -4 | -9 | -10 |

   b.  
   | $x$ | 0  | -4 | -9 | -10 |
   | $y$ | 3  | 2  | 7  | -1 |

   c.  
   | $x$ | 3  | 2  | 7  | -1 |
   | $y$ | 0  | 4  | 9  | 10 |

   d.  
   | $x$ | 0  | 4  | 9  | 10 |
   | $y$ | -3 | -2 | -7 | 1 |
8. The accompanying graph illustrates the presence of a certain strain of bacteria at various pH levels. [A2.A.51]

![Graph showing number of bacteria colonies vs. pH]

What is the range of this set of data?

a. $5 \leq y \leq 70$

b. $5 \leq x \leq 70$

c. $5 \leq x \leq 9$

d. $0 \leq y \leq 70$

Short Answer

9. Given $h(x) = |3x - 1|$ Find $h(-2)$ [A2.A.41] (2pts)

10. Given $f(x) = x^\frac{3}{2}$ and $g(x) = 8x^{-\frac{1}{2}}$, find $(g \circ f)(27)$. [A2.A.42] (2pts)
11. Given: $f(x) = 2x - 5$ and $g(x) = x^2 + 2x.$

A.) Find $(g \circ f)(2).$

B.) Find the rule for $(g \circ f)(x).$

C.) Evaluate your answer in part B of this question at $x = 2$. Compare this answer to the one you found in part A. Explain why they are the same or different.
12. A.) Determine if the function in the diagram below is onto. Explain why or why not. [A2.A.43] (2pts)

B.) Determine if the function $f(x) = 2x + 6$ is one to one, onto or both. Explain your decision. (2pts)

13. If $g(x) = 2x + 1$, find the value of $(g \circ g^{-1})(5)$. [A2.A.44, A2.A.45] (2pts)
14. A.) Graph the function \( f(x) = (x + 3)^2 + 2 \), for \( x \geq -3 \)

B.) State the range of \( f \).

C.) Find \( f^{-1} \). Write the equation and graph \( f^{-1} \).

D.) State the domain and range of \( f^{-1} \).
15. The graphs below are of the functions $y = f(x)$ and $y = g(x)$. Evaluate the following based on the graphs. (3pts)

A.) $g(f(-4))$

B.) For what $x$ values does $g(x) = 3$?
Multiple Choice

Identify the answer that best completes the question. (2 pts each)

1. Find a quadratic model for the set of values: (-2, -12), (0, -4), (4, -60)
   1) \(y = 4x^2 - 2x + 3\) 
   2) \(y = 3x^2 + 2x + 4\) 
   3) \(y = -2x^2 - 3x - 4\) 
   4) \(y = -3x^2 - 2x - 4\)

2. Which of the following equations has imaginary roots?
   1) \(3x^2 + 6x + 3 = 0\) 
   2) \(2x^2 - 4x - 7 = 0\) 
   3) \(4x^2 - 3x + 5 = 0\) 
   4) \(-x^2 + 2x + 5 = 0\)

3. Factor the following expression:
   \(5x^2 - 22x - 15\)
   1) \((5x + 3)(x + 5)\) 
   2) \((5x + 3)(x - 5)\) 
   3) \((x + 3)(5x - 5)\) 
   4) \((5x - 5)(x - 3)\)

4. Find the roots of the function \(f(x) = 5x^2 + 6x - 9\), to the nearest hundredth.
   1) 2.07, -0.87 
   2) 1.74, -4.14 
   3) 1.47, -1.47 
   4) 0.87, -2.07

5. For which value of \(k\) will the following equation have real and equal roots?
   \(4x^2 - 4x + k = 0\)
   1) -1 
   2) 1 
   3) 3 
   4) 2
6. Use vertex form to write the equation of the parabola.

1) \( y = (x - 2)^2 + 3 \)
2) \( y = (x + 2)^2 + 3 \)
3) \( y = (x - 2)^2 - 3 \)
4) \( y = (x + 2)^2 - 3 \)

7. Which graph represents the solution set for the inequality \( x^2 - x - 20 < 0 \)?

1) \[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]
2) \[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]
3) \[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]
4) \[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]
8. Solve for $x$ by any appropriate method: 

$$48x^3 - 75x = 0$$

9. Solve $2x^2 - 5x \geq 12$ and represent the solution set in interval notation OR set builder notation. 

$$[A2.A.4]$$

(3pts)
10. Solve the quadratic equation by completing the square. Answer must be expressed in simplest radical form. (4pts) [A2.A.24]

\[ 2x^2 + 10x + 4 = 0 \]

11. Given the function: \( f(x) = 5x^2 + 10x + 60. \) [A.A.4] (3pts)

a. Find the vertex.

b. Write the equation of the axis of symmetry.

c. Write the function in vertex form.
12. Given the following equation: \( x^2 + 4x - 8 = 0 \) [A2.A.2, A2.A.24, A2.A.25] (4pts)

a. Find the value of the discriminant:

b. Describe the roots:

c. Find the roots in simplest radical form using the quadratic formula: [only a solution obtained by using the quad formula will be accepted]
13. Solve the following system of equations algebraically and check. [A2.A.3] (6pts)

\[ y = x^2 - 6x \]
\[ y + x = -4 \]
14. A manufacturer determines that the profit for a certain number of drills sold, \( P \), it can make is given by the formula \( P = -3d^2 + 100d - 150 \), where \( d \) is the selling price of the drills in dollars.

\[ \text{[A.A.41, A2.A.4]} (6\text{pts}) \]

a. What is the maximum profit? (round answer to nearest hundredth)

b. At what selling price will the manufacturer attain the greatest profit? (round to the nearest hundredth)

c. For what selling prices does the manufacturer make a profit? (round values to the nearest hundredth)

d. For what selling price(s) will the manufacturer make a profit of \$400? (round to the nearest hundredth).
A2T_05_CmplNmbrs_05_P1112

Multiple Choice (2pts. each)
Identify the choice that best completes the statement or answers the question.

___ 1. The expression \( \frac{\sqrt{-50}}{\sqrt{-2}} \) is equivalent to

1) 5 3) -5
2) 5i 4) -5i

___ 2. When simplified, \((1+2i)^2 - 4i\) equals

1) 1 + 4i 3) 1 - 4i
2) 5 4) -3

___ 3. What is the sum of the roots of the equation \(3x^2 - 2x + 5 = 0\)

1) \(-\frac{2}{3}\) 3) \(-\frac{5}{3}\)
2) \(\frac{2}{3}\) 4) \(\frac{5}{3}\)

___ 4. Express in terms of \(i\) in simplest form: \(4\sqrt{-18} + \frac{3}{2} \sqrt{-32}\)

1) 18i\sqrt{2} 3) 6i\sqrt{50}
2) 36i 4) 36i\sqrt{2}
5. The product of $\frac{12}{3} \cdot 3^2$ is

1) 36
2) 36i
3) -36
4) -36i

6. Find the magnitude of $10 + 24i$

1) 26i
2) 26
3) 676
4) 676i

7. In which quadrant does the sum of $(4 - 3i)$ and $(5 - 6i)$

1) I
2) II
3) III
4) IV

Part II - Short Answer

8. Perform the indicated operation and express your answer in simplest $a + bi$ form.

$$\frac{4 + \sqrt{-18}}{2 + \sqrt{-50}}$$

1) $A2.N.7, A2.N.9$
2) $A2.A.13, A2.N.8$
3) $A2.N.8$
4) $A2.A.13, A2.N.6, A2.A.9$ (4pts)
9. Express the sum of $4 + \frac{2}{3} \sqrt{-45}$ and $6 - 3\sqrt{-405}$ in simplest $a + bi$ form.

[A2.A.13, A2.N.9, A2.N.6] (3pts)

10. Find the multiplicative inverse of $6 - 3i$ and express in simplest $a + bi$ form.

[A2.A.14, A2.N.8, A2.N.9] (3pts)
11. A) What complex number does vector $A$ represent? 

B) Graph $-3 + 5i$ in the complex plane. Label it $B$.

12. Perform the operation below. Express answer in simplest form.

$$\sqrt{-1} \div \sqrt{8}$$
13. A) If one root of a quadratic equation is $-4 + 6i$, what is the other root?


B) Write a quadratic equation that has these roots.

14. Solve for $x$ and express the roots in simplest $a + bi$ form:


$$6x^2 = -4x - 1$$