An Algebra Unit on Quadratics for Teachers with Students with EBD

Christopher Williams
The College at Brockport, cwil0427@brockport.edu

Follow this and additional works at: http://digitalcommons.brockport.edu/ehd_theses

Part of the Algebra Commons, Curriculum and Instruction Commons, Educational Methods Commons, and the Secondary Education and Teaching Commons

To learn more about our programs visit: http://www.brockport.edu/ehd/

Repository Citation
Williams, Christopher, 'An Algebra Unit on Quadratics for Teachers with Students with EBD' (2014). Education and Human Development Master's Theses. 500.
http://digitalcommons.brockport.edu/ehd_theses/500

This Thesis is brought to you for free and open access by the Education and Human Development at Digital Commons @Brockport. It has been accepted for inclusion in Education and Human Development Master’s Theses by an authorized administrator of Digital Commons @Brockport. For more information, please contact kmyers@brockport.edu.
An Algebra Unit on Quadratics for Teachers with Students with EBD

Christopher Williams

A thesis submitted to the
Department of Education and Human Development of the
State University of New York College at Brockport
In partial requirements for the degree of
Master of Science in Education

1
Table of Contents

Chapter 1: Introduction ................................................................................................... 3
   Students with EBD ...................................................................................................... 3
   Purpose of Thesis .................................................................................................. 3

Chapter 2: Literature Review ......................................................................................... 5
   Definition of EBD .................................................................................................... 5
   Performance Statistics of Students with EBD .......................................................... 7
   Comparison between EBD and LD Classified Students ......................................... 8
   Theoretical Perspective: Zone of Proximal Development ...................................... 9
   Reform Curricula: The Paradigm Shift from the NCTM State Standards to the CCSS .12
   Academic Strategies for students with EBD ........................................................... 14

Chapter 3: Unit Plan with Lessons ............................................................................... 22
   Quadratic Equations and Quadratic functions Unit Outline .................................... 24
   Objectives / Pre-assessment .................................................................................. 25
   Identify Quadratic Equations ................................................................................ 34
   Solving Quadratic Equations Using the Method of Extraction of Roots .............. 40
   Solving Quadratic Equations by Factoring ......................................................... 51
   Complete the square ............................................................................................ 59
   Solving Quadratic Equations Using the Quadratic Formula ............................... 72
   Characteristics of Quadratic Functions ............................................................... 79
   Graphing Quadratic Functions & Slope ............................................................... 88
   Applications & modeling ..................................................................................... 96

Chapter 4: Discussion .................................................................................................. 120

References .................................................................................................................. 124

Lesson Answer Keys ................................................................................................. 138
Chapter One: Introduction

Students with Emotional and Behavioral Disorder (EBD)

Secondary students identified as having an emotional and behavioral disorder (EBD) often must strive to overcome more difficulties academically, socially, and behaviorally than any other group of students. The academic deficiencies that students with EBD exhibit are significant, leading to their average performance being almost 3.5 grade levels behind their peers by the time they reach high school. Typically, less than one third of students with EBD function at or above grade level in any academic area (Coutinho, 1986; Epstein, Kinder, & Bursuck, 1989; Ryan, Pierce & Mooney, 2008). These students also need to acquire coping mechanisms, social competencies, (often) psychotherapy and behavioral therapy for emotional regulation. Intervention supports from other areas such as mental health, environmental agencies, nursing services and general wellness are often required. (Cannon et al., 2013; Härkäpää et al., 2014, Maag, 2006; Magyar & Pandolfi, 2012; McInerney et al., 2014; van der Worp et al., 2014).

Academically, secondary students with EBD must achieve a level of proficiency at grade level but may also have severe learning gaps. As a result, there may be basic academic skills missing that are essential for effective functioning within the community (Ryan, Pierce & Mooney, 2008). Compared with students with other disabilities, students with EBD fail more courses, earn lower grade point averages, miss more days of school, and are more often retained a grade (Gunter, Coutinho & Cade, 2002; U.S. Department of Education, 1998; Wagner, Blackorby, & Hebbeler, 1993). These factors make it more difficult for students to express, in measurable ways, higher order thinking skills (HOTS) and problem solving competencies in mathematics without sufficient academic supports.

Purpose of this Thesis

The purpose of this thesis is to provide an algebra unit for teachers with students with EBD that
is grounded in Vygotsky’s (1978) Zone of Proximal Development (ZPD). Working with algebra students with EBD in their ZPD by incorporating empirically based teaching methods to develop the curriculum can transform instruction and knowledge from an interpersonal process into an intrapersonal one (Vygotsky, 1998). Such instruction can shift the actual developmental level to the level of potential development; i.e., what a student can do with assistance today is what they can do independently tomorrow. While students with EBD have a well-documented failure rate that is extremely discouraging and well above the rates for students with any other disability, it may be encouraging for teachers of students with EBD to understand that they can have a profound effect on these students and their outcomes.
Chapter Two: Literature Review

Definition of EBD

The definition and classification of students EBD is often a complex and sometimes convoluted process from the definition to the determination. Emotional disturbance (ED) is one of the categories of disability included under the Individuals with Disabilities Education Improvement Act, also known as IDEA (2004). Emotional disturbance falls under the umbrella term of mental illness. Mental illnesses are medical conditions that disrupt a person's thinking, feeling, mood, ability to relate to others and daily functioning. Just as diabetes is a disorder of the pancreas, mental illnesses are medical conditions that often result in a diminished capacity for coping with the ordinary demands of life (NAMI, 2010).

IDEA defines emotional disturbance as follows:

i. The term means a condition exhibiting one or more of the following characteristics over a long period of time and to a marked degree that adversely affects a child’s educational performance:

A. An inability to learn that cannot be explained by intellectual, sensory, or health factors.

B. An inability to build or maintain satisfactory interpersonal relationships with peers and teachers.

C. Inappropriate types of behavior or feelings under normal circumstances.

D. A general pervasive mood of unhappiness or depression.

E. A tendency to develop physical symptoms or fears associated with personal or school problems.

ii. The term includes schizophrenia. The term does not apply to children who are socially maladjusted, unless it is determined that they have an emotional disturbance (IDEA, 2004).

This legal definition presents difficulties with the translation and application, since
distinguishing “emotional disturbance” from “behavioral disorder” is impossible (Forness & Knitzer, 1992). This may lead to confusion when talking about students with ED or students with EBD because they are referencing the same thing, but more recently it has been commonly accepted to use the terms students with EBD over students with ED since this is the primary disability. There still remains a controversy regarding IDEA’s definition for ED with the exclusion of students who are socially maladjusted but not Ed (Kauffman & Landrum, 2009). This is due to the inconsistency of the exclusion of the term, socially maladjusted, from the original definitions and intentions submitted by Bower (1982) who provided the federal definition of ED, which included socially maladjusted. Further, the behavior of a student with EBD is most likely to be interpreted as social maladjustment (conduct disorder, which includes various forms of antisocial behavior and is closely linked to poor socialization); however, this is one of the most serious disabilities in the EBD category (Kauffman and Landrum, 2009). If a determination can be found that the family or community failed to teach an appropriate behavior, the problem may be classified as a social maladjustment, and more often a classification of ED is resisted thus neglecting students of needed resources and additional supports (Costello, Egger, & Angold, 2005; Kauffman & Landrum, 2009). Independent of the legal definition proposed, the underlying structure of all EBD definitions contain three main elements:

- Extreme behavior (not just slightly different from the usual).
- A chronic problem (constant and on-going, which does not resolve quickly).
- Violation of social or cultural expectations (Hallahan et al., 2009).

While simply stated this often encompasses a large range of disorders including but not limited to anxiety disorders, bipolar disorder (sometimes referred to as manic-depressive disorder), conduct disorders, eating disorders, obsessive-compulsive disorder (OCD), psychotic disorders and post-
traumatic stress disorder (PTSD) (NIMH, 2010).

Often, students with EBD are not identified until their problems are severe and protracted because educators are afraid of labeling or of being accused of making a mistake in identification (Kauffman, 1999). Educators appear to be far more willing to decide that the student should be identified as having a learning disability (LD) than they are to identify a student as having EBD (Hallahan et al., 2009). As a consequence, students with EBD are often ignored or mislabeled, and only a small proportion of children with clear evidence of functionally impairing psychiatric disorder receive treatment (Costello et al., 2005; Kauffman & Landrum, 2009).

There are many complications and issues resulting from the identifications for a student with LD and a student with EBD, since both may display identical behaviors. Both groups experience challenges in terms of curriculum and achievement and lag significantly behind their peers (Kavale & Nye, 1986; Reid et al., 2004). As such, both groups lack fundamental prerequisite knowledge necessary to actively engage in grade-level discussions and activities. Professional special educators recognize this deficit and can provide support in the form of strategies, materials, or instruction to facilitate meaningful participation. Many strategies applied to students with EBD can reflect or repeat those of students with LD. Learning is measured through student performance on academic tasks, and performance may not just happen by applying a specific strategy or teaching style to curriculum materials. Teachers should also recognize that the measurable acquirement of knowledge occurs within the zone of proximal development.

Performance Statistics of Students with EBD

The school performance statistics of students with EBD are bleak. They include a 32.1% graduation rate (U.S. Department of Education, 2006) with 70% of these students incarcerated within 3 years of leaving school (U.S. Department of Health and Human Services, 1999). Additionally, 52%
unemployment rates (D'Amico & Marder, 1991), difficulties with substance abuse, and a high need for mental health services (Bullis & Yovanoff, 2006; Lane et al., 2008; Walker et al., 2004). These statistics may seem discouraging, however, it may be encouraging for teachers of students with EBD to understand that they can have a profound effect on these students and their outcomes.

A large portion of the research for students with EBD outlines the challenges that teachers of students with EBD face while attempting to address these students' social, behavioral and academic deficiencies (Wehhy, Lane & Falk, 2003; Mattison & Blader, 2013). It is important for teachers of students with EBD to successfully navigate them through the curriculum, and in this instance: a unit based in 9th grade mathematics reform curricula. Currently, teachers working with reform curricula may find that not enough attention to instructional design is provided for students with math LD (Sayeski & Paulsen, 2010). Additionally, the same is true for students with EBD. Given that between 2% and 20% of the school-age population is likely to have EBD, this is no small problem (Gunter, Coutinho & Cade, 2002; Kauffman, Lloyd, Baker & Reidel, 1995; Lane et al., 2008, NIMH, 2014; Pierce et al., 2004, U.S. Department of Education, NCES, 2013).

Comparison between EBD and LD Classified Students

Compared to students with a LD, learning may further be challenging for students with EBD since more than half of students with EBD also meet the criteria for a learning disability (LD) (Glassberg et al., 1999; Ryan et al., 2008). Research has shown that Students with an EBD display a significant increase in writing deficits as compared to students with a writing LD, and compared to achievement in mathematics and spelling there may be little difference between students with mathematics LD, spelling LD and students with EBD (Kavale & Nye, 1986; Reid et al., 2004). There are several parallels between teaching methods for students with LD and for students with EBD. Both groups often have difficulty in making the connection between seemingly abstract activities. They also
exhibit challenges with cognitive processing demands such as identifying salient information and concepts for problem solving, and, displaying expressive and receptive language concerns. Students with EBD are also typically very poor self regulators (Montague, 1997, 2008; Sayeski & Paulsen, 2010; Swanson, Hoskyn, & Lee, 1999; Wong, Harris, Graham, & Butler, 2003).

During the school day, students with EBD commonly engage in behaviors that are disruptive, non-compliant, exhibit levels of violence (discourtesy, disrespect, intimidation, retaliation, verbal assaults and physical aggression) that negatively influence both their ability to successfully negotiate peer and adult relationships and their educational experience (Cullinan and Sabornie 2004; Gresham et al., 2004; Landrum et al., 2003; Lane et al., 2008; Ried et al., 2004; Walker et al., 1992; Walker et al., 2004). These behaviors often arouse negative feelings in others, alienating schoolmates and adults and ultimately rob students of the opportunity to learn (Kauffman, 2001). Inevitably, these behaviors significantly impair a students’ ability to succeed in school and society (Ried et al., 2004). Because of the difficulties of teaching students with EBD for learning that can change the historical trajectory, it is beneficial to consider a theoretical perspective of teaching and learning.

**Theoretical Perspective: Zone of Proximal Development**

The zone of proximal development (ZPD) is the difference between what a learner can do without help and what a learner can do with help. Vygotsky (1978) defines ZPD as

The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers.

Teachers may use the ZPD to bridge the gap between what a learner can do without help and what a learner can do with assistance.
As Vygotsky described, the ZPD is where creative activity and development take place (Vygotsky, 1986). For teachers that work with students with EBD it is crucial to develop activities within students’ ZPD to encourage both participation (delivery) and success (modify), and help students to develop critical thinking skills to aid learning (collaboration) (Harris, 2010). Using strategies to foster engagement through delivery, modification, and collaboration will aid in a decrease of off-task behaviors and directly relate to positive reinforcement and authenticity [of praise] (Harris, 2010; Reinke, Lewis-Palmer, & Merrell, 2008). Receiving positive reinforcement and praise are both motivators that can lead to increased self-efficacy, interest and ultimately higher academic performance. It is well documented that these motivational variables of self-efficacy, belief, and interest are important factors in academic achievement (Berndt & Miller, 1990; Greene, Miller, Crowson, Duke, & Akey, 2004; Jinks & Lorsbach, 2003; Linnenbrink & Pintrich, 2002; Pintrich & De Groot, 1990).

During instruction, it is necessary to determine at least two levels of development: the actual developmental level and the level of potential development. The distance between the two levels are described as the ZPD (Vygotsky, 1986). For students with EBD, the ZPD may be much greater in terms of distance between developmental level and the level of potential development relative to academic fluency, background knowledge, self-regulation and behavior management (IDEIA 2004; Mooney et al., 2003). Because most students with EBD are functioning below grade level in at least one academic area, they often have a more difficult struggle to reach graduation and compete or perform efficiently in academics with peers (Coutinho, 1986; Epstein, Kinder, & Bursuck, 1989; Ryan, Pierce & Mooney, 2008).

Since the NCTM 1980 Standards, mathematics k-12 instruction has been guided by either the state standards (most often the NCTM Standards) or by the Common Core State Standards (CCSS)
Mathematics standards detail what students are expected to know and do, yet no curriculum can be complete without specifying how students might best be apprenticed into acquiring that knowledge (Atebe, 2011). It is important to understand how much help and feedback is needed for the complexity of the task. This will be dependent on the mastery level of the learner.

Mathematics itself is a complex task. Complex tasks have many different solutions, are ecologically valid, cannot be mastered in a single session and pose a very high load on the learner’s cognitive system (Sweller, van Merrië”nboer, & Paas, 1998; van Merrie”nboer & Sweller, 2005; van Merrienboer et al., 2006). Teachers of mathematics may misinterpret the ZPD, especially in terms of “what a learner can do with help” as providing too much feedback (even giving answers) as ways of protecting students' positive self-esteem from potential threats [ie. Failure] (Smith, Snyder, & Handelsman, 1982). While frequent and complete feedback may indeed have a positive effect on the acquisition curve and performance on retention tests, it impedes or delays problem solving and transfer of learning (van Merrienboer et al., 2006).

The process by which an adult helps a child to carry out a task that is above the child’s level to perform on its own is called scaffolding (Newman & Holzman, 1993). As scaffolding occurs, a great learning support is provided at the time new concepts are introduced and the support is slowly taken from the student as he or she masters the content (Lamport et al., 2012). Often the content increases in complexity and difficulty and the amount of scaffolding is adjusted individually, based on the learner's needs, misconceptions, or lack of skill development (Lawson, 1980). Some forms of scaffolding will assist with problem decomposition and other higher order skills to allow the learner to focus on domain specific schema formation, while other forms of scaffolding will assist the learner in domain specific skills or answers so that they can practice the overall approach to the problem (Collins et al., 1989; Murray & Arroyo, 2002).
Again the process of scaffolding in offering assistance to meet students' specific needs, or the amount of feedback needs to be constantly evaluated or it may hinder the problem solving process and the transfer of learning. For example, a teacher's intervention may be misdirected and cause more confusion than clarification, or may deny students the opportunity to resolve their own difficulties (Granott, 1993; Goos et al., 2002). Decisions also need to be made about the timing of such interventions, and, indeed, whether to intervene at all (Goos et al., 2002; Teasley & Roschelle, 1993). There are finely tuned appraisals to be made about the timing, amount, and type of assistance to provide, if a delicate balance between encouraging persistence and avoiding frustration is to be achieved (Goos et al., 2002).

Teacher's who effectively apply scaffolding to the curriculum understand that it is a process and the process is a mutual adjustment and appropriation of ideas rather than a simple transfer of information and skills from teacher to learner (Brown et al., 1993; Packer, 1993; Wertsch, 1984). While specific academic strategies and even curriculum may prove consistent across a broad range of learners scaffolding often is not (Wertsch, 1984). This is critical for student's with EBD to begin to acquire skills and align their levels of potential development closer to that of their peers. Through the zone of proximal development occurs a shift from “teacher tells” to an investigative approach. This is not only important for learning to happen, but provides a foundation for the implementation of mathematics reform curricula.

Reform Curricula: The Paradigm Shift from the NCTM State Standards to the CCSS

With the implementation of the Common Core State Standards (CCSS), from the NYS NCTM standards, New York has new instructional shifts to consider. These instructional shifts are a part of the CCSS curriculum and include Focus, Coherence, Fluency, Deep Understanding, Application and Dual Intensity (Nctm.org, 2011; Corestandards.org, 2013 ; Engageny.org, 2012). As Corestandards.org states
“focus will help students gain strong foundations, including a solid understanding of concepts, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the classroom.” (2013). The question posed (or should be posed) for teachers working with student's with EBD is what if they didn't gain? The common core standards are designed around coherent progressions from grade to grade. Each standard is not a new event, but an extension of previous learning (Nctm.org, 2011; Corestandards.org, 2013 ; Engageny.org, 2012). What will this mean for Deep Understanding, Application and Dual Intensity for students significantly lagging behind their peers, or who have missed entire topics and even grades? It is not uncommon for students to be socially promoted (the process of graduating a student to the next grade level based on age and not ability) into high school through one or more grade levels (Koretz, 2008). Questions like these are needed for teachers working with students with EBD and in developing curriculum embedded in current standards.

The implementation of the new standards also means less resources may be available for a particular subject matter. Few [of the 40 states surveyed] have the necessary staffing levels, staff experience and resources to provide Common Core training for teachers and principals (Frizzell, 2014). This may challenge teachers with students with EBD to have to develop new materials to help students reach grade level abilities. Teachers of students with EBD must help students overcome previous gaps but also develop critical thinking and mathematical modeling abilities in relation to their peers. Due to the significant learning gaps and behavioral challenges students with EBD may have a much wider zone of proximal development to meet the criteria for a successful graduation, and require differing amounts of scaffolding to occur throughout the lessons. Teachers of students with EBD will often rely on several learning strategies to develop, teach or supplement new curriculum while teaching past materials that are grounded in the new theories.
Academic Strategies for students with EBD

Academic strategies are methods and processes to incorporate empirically based teaching methods into classrooms to maximize teaching effectiveness by increasing student engagement, and addressing the academic deficits of students with EBD. The following academic strategies have had positive impacts on improving the academic skills and performance of students with EBD and increasing engagement in school (Ryan, Pierce & Mooney, 2008; Nelson, Benner, & Mooney, 2008), with the hope of improving graduation rates (Mooney, Epstein, Reid, & Nelson, 2003).

Strategies are broken into two categories, Content support (C) and Pedagogical support (P). Content support are strategies that directly change the subjects or topics being taught in the classroom. Content supports help to develop particular content knowledge that can assist in promoting greater participation for students with EBD. Pedagogical Supports changes how instruction is delivered and can be made to increase a student’s ability to be actively involved and learning. Some strategies may be labeled as both (B). These categories are used to show that when applied may have an impact on the amount of time needed in a lesson, or may potentially increase the time needed to complete a unit.

The strategies are arranged in 3 tables outlined by Ryan, Pierce & Mooney. (2008) developed by Epstein, Nelson, Trout, & Mooney (2005) who summarized the intervention literature targeted at improving the academic skills and performance of students with EBD served in public schools. In the research they assessed the efficacy of three types of academic interventions (I.e., teacher-mediated, peer-mediated and self-mediated) for students with EBD (Mooney, Ryan, Uhing, Reid, & Epstein, 2005; Pierce, Reid, & Epstein, 2004; Ryan et al., 2004). While some interventions have been removed (not applicable for high school), or have not demonstrated efficacy in educating students with EBD; other interventions have been added or changed based on empirical evidence. The changes have been noted below.
Some of the changes (in the interventions listed) may have overlapping qualities in them (modeling and direct instruction both show how something is done) but each are unique in further developed qualities or strategies. Further, education often adopts 'buzz words' to delineate changes and methodologies occurring. Current buzz words and trends in working with students with EBD often associated with academic interventions is explicit instruction and direct instruction. Explicit instruction is a combination of previewing, structured academic tasks, direct instruction, ongoing progress monitoring (see below), and for scaffolding to occur within the lesson. Explicit instruction often refers to the systematic sequencing of instructional procedures in a lesson (Amer, 2013). More importantly, it involves the degree of clarity in the learners' constructions and their deliberate use of a particular concept, strategy or procedure, calling to consciousness what is being taught and strives to clarify for learners the expectations the teachers have for their learning (Bomer, 1998; Serafini, 2004). Therefore, explicit instruction has been omitted as a strategy, but should be viewed as a collection of strategies.

Direct instruction refers to guided procedures used in instructional design often for teaching difficult subject matter. These procedures include studying worked examples, partial solutions, goal setting problems, and more (van Merriënboer & Sweller, 2005). Direct instruction is precisely showing, guiding or explaining how something is done. Often, direct instruction is referred in contradiction to discovery-learning, but one must be able to work within their ZPD to reach their potential development. Since direct instruction also includes prompts for self-explanation for developing procedural learning, procedural transfer and conceptual knowledge it is often overlooked as a technique to foster and hasten discovery-learning within the context of a time frame for lesson to occur. It is well documented that prompts to self-explain, when compared with no-prompts, lead to immediate improvement in procedural learning (Bielaczyc, Pirolli, & Brown, 1995; Pine & Messer, 2000) as well as procedural transfer (Aleven & Koedinger, 2002; Atkinson, Renkl, & Merrill, 2003; Renkl, Stark,
Direct instruction can also provide organizing schemas for novices in a domain that help coordinate information in working memory (Sweller, Van Merrienboer & Paas, 1998). For example, one might provide direct instruction in recommended strategies (e.g., rather than directing students to “draw a diagram,” teach students the process of how to draw a diagram) (Sayeski & Paulsen, 2010). The table below (beginning on next page) outlines the teacher-mediated interventions. These interventions are those in which the teacher takes responsibility for treatment, through manipulation of antecedents and/or consequences. Changes to the original table presented by Ryan, Pierce & Mooney are also noted.

**Changes:**

The definition for previewing was altered to include mathematics review and improving performance in special education (Sayeski & Paulsen, 2010). The definition for Structured academic tasks was widened to include cumulative review with the mixing of different problem types (van Merriënboer & Sweller, 2005). Direct instruction, ongoing progress monitoring, authentic tasks and motivational activities were added. Teacher-Mediated antecedent focused interventions were simply added to Teacher-mediated interventions category.

### Table 1: Teacher Mediated Interventions

<table>
<thead>
<tr>
<th>C/P</th>
<th>Intervention</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Modeling, rehearsal, and feedback</td>
<td>A process wherein teachers model a skill, have the student rehearse the skill, and provide direct feedback about the student’s performance.</td>
</tr>
<tr>
<td>C</td>
<td>Previewing</td>
<td>A comprehension strategy that involves activating prior knowledge, predicting, and setting a purpose in reviewing skills and concepts central to the lesson to improve performance.</td>
</tr>
<tr>
<td>C</td>
<td>Structured academic tasks</td>
<td>A process wherein teachers require students to complete specific tasks in a sequential order. Also includes cumulative review with the mixing of different problem types.</td>
</tr>
</tbody>
</table>
C Sequential prompting A strategy to use multiple levels of prompts (administered in order from most independent to most dependent) to increase academic performance.

C Direct instruction A process for instruction to precisely show, guide or explain how something is done.

C Incorporating student interest A strategy to incorporate student interest in the development and content of lesson.

B Ongoing progress monitoring A process for collecting data to determine if students are making appropriate progress on essential skills or if reteaching and more practice needs to occur (evaluative data).

C Authentic tasks A teaching method that encourages students to explore, discuss, construct concepts, develop projects and connect them meaningfully to real life problems.

P Token reinforcement Teachers provided points or tokens to students.

P Use of free time Teacher provides increasing amounts of free time to students based on increasing number of sight words learned.

P Academic contracting Teachers contracted with student to earn specified reinforcer for predetermined levels of academic improvement.

B Motivational activities Activities that increase involvement in the learning process of mathematics.

(Ryan, Pierce & Mooney, 2008)

**Reasoning of Changes**

Ongoing progress monitoring was supplemented for teacher planning strategies. Teacher planning strategies was defined as a process wherein teachers are trained to use daily planning procedures based on trend analysis and error analysis (Ryan, Pierce & Mooney, 2008). Ongoing progress monitoring relies on the collection of data and assessments to guide instructional decisions. Developing planning procedures based on trend and error analysis may lead to more accurate assessments. The goal of planning and monitoring progress is to provide valid assessments. Ongoing progress monitoring is the practice of collecting ongoing data on student progress in the curriculum (L. S. Fuchs & Fuchs, 1998). While Ongoing progress monitoring can be parallel to formative assessments, Ongoing progress monitoring can be evaluative and weight bearing to the student's grades. Using
systematic and ongoing progress monitoring may help teachers effect positive outcomes in math for students with EBD (Hodge, Riccomini & Buford, 2006) and can be essential for teachers of students with special needs and math learning disabilities (Sayeski & Paulsen, 2010).

Ongoing progress monitoring will also help the teacher to gain measures on the actual developmental level and better understand the level of potential development in order to adjust instruction to meet the needs of the students. Without understanding what student's know, it is impossible to pinpoint an accurate developmental level. Further, if direct assessment and Ongoing progress monitoring of mathematical outcomes is not conducted, the effectiveness of the modifications or interventions is unclear (Hodge, Riccomini & Buford, 2006).

Authentic tasks were added both as a intervention for students with EBD and as a part of reform mathematics increased ability to cultivate understanding and motivation. Part of reform mathematics curricula is to seek patterns from applied examples and real life situations and create models of systems abstracted from real world objects (ie: applied mathematics) (Schoenfeld, 1992). While much research has interchanged, 'real world', ‘authentic’, ‘real life’ and ‘situated’ to mean the same thing they generally have wide and varying definitions (Beswick, 2011).

Authentic learning is a teaching method that encourages students to explore, discuss, construct concepts, develop projects and connect them meaningfully to real life problems (Donovan, Bransford, & Pellegrino, 1999). Authentic learning has real life value, enables students to actively construct knowledge, and is used in order to solve problems and complete open ended tasks (KOÇYİĞİT & ZEMBAT, 2013). Therefore authentic tasks are not simply real-world examples presented as word problems since to provide the authenticity of the task there is no ready-made algorithm (Kramarski et al., 2002).
Authentic tasks was an added strategy since participation in authentic performance tasks may lead to improved academic performance (Gallagher, Stepie, & Rosentbal, 1992; Scbneider, Krajcik, Marx, & Soloway, 2001; Tbohas, 2000). Just as Incorporating student interest can be a motivational strategy, students' may connect deeper to lessons they will experience in their daily lives. Further, authentic tasks can both support academic goals and social learning goals (Barnes & Urbankowski, 2014). Authentic tasks offer enriching opportunities for other interventions for students with EBD like group work, cooperative learning, peer assessment, Goal setting / Self-evaluation and self-monitoring (Patrick & Middleton, 2002).

While motivational activities may have the most overlap with other interventions there were strategies within that may not have fit precisely in one or the other categories. Motivational activities are tasks that focus on factors such as self-efficacy (Bandura, 1997), expectancy and task value (Wigfield and Eccles, 2000), goal orientation (Ames, 1992), self concept (Guay, Marsh, and Boivin, 2003) and attributions (Skinner, Wellborn, and Connell, 1990). Motivational activities generally have self-selected goals or assigned goals, however self-selected goals have a significantly higher improvement in mathematics computation than assigned goals for activities (Fuchs, Bahr & Rieth, 1989). Further, participation in motivational activity goal selection improves performance outcomes, and may enhance the sense of potential accomplishment with which students with LD and students with EBD approach learning tasks (Sayeski & Paulsen, 2010; Fuchs, Bahr & Rieth, 1989). Specific motivational activities may include mathematical softwares, beat your own score, or graphing of student progress (Fuchs et al., 2008).

The next table outlines the peer-mediated interventions. These interventions are those that require students to implement teacher selected instruction for their peers as opposed to the more traditional method of teacher-led instruction (Hoff & Robinson, 2002). There are no changes to the
Table 2: Peer-Mediated Interventions

<table>
<thead>
<tr>
<th>C/P</th>
<th>Intervention</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Classwide peer tutoring (CWPT)</td>
<td>Entire class simultaneously participates in tutoring dyads. During each tutoring session, students can participate as both tutor and tutee, or they can participate as either the tutor or tutee.</td>
</tr>
<tr>
<td>P</td>
<td>Cooperative learning</td>
<td>Small teams composed of students with different levels of ability use a variety of learning activities to improve the team’s understanding of a subject. Each member of a team is responsible not only for learning what is taught but also for helping teammates learn.</td>
</tr>
<tr>
<td>P</td>
<td>Peer tutoring</td>
<td>Students who need remedial support are paired with select tutors (perhaps highly skilled peers, peers also in need of remedial work, or cross-age tutors). Each member of the dyad may receive and provide tutoring in the same content area, or tutors can provide instruction in a content area in which they are highly skilled.</td>
</tr>
<tr>
<td>C</td>
<td>Peer assessment</td>
<td>Peers are used to assess the products or outcomes of learning of other students of similar status.</td>
</tr>
<tr>
<td>P</td>
<td>Peer reinforcement</td>
<td>Peers provide reinforcement for appropriate responses within the natural environment. The purpose is to reinforce appropriate behaviors of students with disabilities by their peers.</td>
</tr>
</tbody>
</table>

(Ryan, Pierce & Mooney, 2008)

The last category of strategies are self-mediated strategies. Self-mediated interventions are those in which the students themselves are responsible for providing academic instruction. These strategies can be critical for students with EBD since they are typically such poor self-regulators. However poor self-regulation can benefit from strategy instruction and self-regulation training (Montague, 1997, 2008; Sayeski & Paulsen, 2010; Swanson, Hoskyn, & Lee, 1999; Wong, Harris, Graham, & Butler, 2003). While developing these strategies for students to use it is important to note that most are time consuming in teaching the strategy itself, however they typically lead to a higher independence of learning and if implemented accurately have demonstrated their ability to produce large academic gains for students with EBD across subject areas (Carr & Punzo, 1993; Skinner, Ford, & Yunker, 1991; Skinner et al., 1992). Generally, as the student becomes more proficient these
strategies typically fade.

**Changes:**

Self-monitoring was changed to Self-monitoring / self-monitoring checklists with the addition of academic self-monitoring. Goal setting / Self-evaluation was incorporated into one intervention with the basis that the criterion can often be academically motivated besides behaviorally.

**Table 3: Self-Mediated Interventions**

<table>
<thead>
<tr>
<th>C/P</th>
<th>Intervention</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Self-monitoring / self-monitoring checklists</td>
<td>A two-stage process of observing and recording one’s behavior wherein the student: (a) discriminates occurrence/nonoccurrence of a target behavior; and (b) self-records some aspect of the target behavior. Checklists can prompt students regarding the steps needed to solve a particular type of problem.</td>
</tr>
<tr>
<td>B</td>
<td>Goal setting / Self-evaluation</td>
<td>A process wherein a student self-selects a behavioral target (e.g., term paper completion), which serves to structure student effort, provide information on progress, and motivate performance.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A process wherein a student compares her/his performance to a previously established criterion set by student or teacher (e.g., improvement of performance over time) and is awarded reinforcement based on achieving the criterion.</td>
</tr>
<tr>
<td>P</td>
<td>Self-instruction</td>
<td>A procedure wherein a student uses self-statements to direct behavior.</td>
</tr>
<tr>
<td>B</td>
<td>Strategy instruction</td>
<td>A process wherein a student is taught a series of steps to independently follow in solving a problem or achieving an outcome.</td>
</tr>
</tbody>
</table>

(Ryan, Pierce & Mooney, 2008)
Chapter 3: Unit Plan with Lessons

In developing a unit in algebra for students with EBD it is important to consider the previous skills learned as well as the direction for creating the next unit. Given the amount of available curriculum in the current subject matter it is important to determine what knowledge the student has available and speed or pacing of the new information. Sometimes additional work can be presented as academic intervention strategy (AIS) work. Other times the curriculum may have to be adopted to fit additional time for development in problem decomposition and other higher order skills or domain specific skills.

This unit follows a unit in factoring polynomials and previous to that linear equations and functions. This allows for student recollection of factoring coefficients from quadratics, ordering terms, computing terms, solving for variables and presenting functions in various forms; ie. equations, graphs, table representation or a collection of data to be analyzed and modeled. Students who have struggled with these units may need additional focus on these skills in terms of fluency, definitions, application or reasoning. If they have struggled or are struggling it should be evident from their previous unit assessments, and even from the pre-assessment of this unit. This follows from the strategy, ongoing progress monitoring which is needed to direct teaching for what the student needs in order to be successful with their learning. The unit directs learning from the NYS CCSS and the NYS regents outlines, as well as generating ideas, problem solving abilities and patterns that will be needed as a structure as the student progresses through high school mathematics and into college.

The unit outline (below) follows from the strategy, structured academic tasks, as each task is sequenced to increase in difficulty and variability. The lessons follow with explanations for how the strategies and scaffolding are applied. Different books order these lessons in different ways and even exclude some or many of the topics from the units from their pages, or combine some topics as one
lesson without accentuating major points of learning or skill development. In many of the lessons there may be focus on skill development for particular problem types, however it is just as important to develop more challenging independent problems for students with higher expertise in problem solving, and for students with less expertise to begin to develop their own higher order thinking skills. For each lesson it is implied that any needed additional fluency in solving problem types can be added through the use of solving on-line problem types or through additional examples.

The lessons not created for this thesis are numbers 11 and 19, the mid and final review and assessments. Due to the nature of the objectives sheet (number 1) the review should be targeted towards the objectives students may have struggled with or not obtained with minimal strategy review for more easily acquired skills. Similarly the assessments should follow according to the objectives outline to determine the proficiency of student's knowledge with more variability included, rather than modeling the assessments with a standardized approach.

While the lessons are put together into a workbook format they do not need to presented to students in this fashion. It was the clearest way to assemble the needed outlines, problem types, links, videos and tasks. However, it also makes it beneficial to have it in this format for students with low attendance rates. The lessons also do not follow APA formatting in order to meet the needs of teachers and their printing needs.

It is also important to note that the strategies for token reinforcement, use of free time and academic contracting for teaching the lessons of this thesis were built into the structure of the school through PBIS (Positive Behavior Intervention and Supports). Every student has a tracked academic goal (ie: complete more homework, increase time in class, increase participation, etc..) and receives PBIS tickets from staff in the room for meeting classroom objectives. Meeting the ticket objectives for the students leads to school wide and individual rewards.
Quadratic Equations and Quadratic functions Unit Outline

1. Objectives / Pre-assessment

2. Identify Quadratic Equations (Standard / Intercept / Vertex) and the Zero Factor Property / Zero Product Property

3. Solving Quadratic Equations Using the Method of Extraction of Roots (Solve a Quadratic Equation Using Square Roots)

4. Assessment

5. Solving Quadratic Equations by Factoring

6. Assessment

7. Complete the Square With Coefficient 1

8. Assessment

9. Complete the Square Any Case.

10. Solving Quadratic Equations Using the Quadratic Formula

11. Mid Unit review and Assessment

12. Characteristics of Quadratic Functions (Axis of Symmetry, y-Intercept, Vertex, Discriminant & Roots)

13. Assessment

14. Graphing Quadratic Functions (Table / By Characteristics)

15. Assessment

16. Slope

17. Applications & modeling (Estimation, Distance, Volume, Velocity and Height, Consecutive
Integers, Finance)

18. Modeling Project

19. Summary of Key Concepts / Exercise Review and Proficiency Assessment

1. Objectives / Pre-assessment

The objectives are outlined for students as a self monitoring checklist. In the first column students can add a check if they feel they have demonstrated proficiency and this can assessed by the teacher in the second column. This allows for a constant review and demonstration of understanding, especially if inconsistencies occur. The objectives outline the the entire unit, and allows for targeting specific skills to meet the overarching standards. The objectives are all written with “I can” statements in order for students to internalize the objectives rather than having a statement that they are more likely to disregard. For each lesson students are asked to evaluate their objectives.

The objective “I can use my calculator to graph a quadratic.” was added to 8.3 without explicit instruction in the lesson. This is due to students spending a fair amount of instructional time in the linear unit to develop calculator skills. However, if this objective can not be obtained from previous lessons then it may need to be developed into an instructional lesson.

One issue most teachers have with many books, worksheets and other resources for instructing students with EBD is not enough problem types in one booklet. This thesis tries to compensate for that issue by extracting several problem types from Weiler and Matula's book Common Core Algebra I (Weiler & Matula, 2014), Ellis and Burzynski's book Elementary Algebra (Ellis & Burzynski, 2012), and self constructed problems, with word problems in the project from engage NY’s website (Engageny.org, 2012). All three resources are on-line and free to teachers.
### 1. Objectives

<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher</th>
<th>Lesson</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can give an accurate definition for a quadratic function.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can identify quadratic functions by equations or by graphs.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can place a quadratic in standard form.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can identify the leading coefficient of a quadratic function.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can state the y-intercept of a quadratic from the graph or from an equation.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can use the zero factor property to solve equation in this form.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can identify the vertex (turning point) from the graph of a parabola.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can give an accurate definition of a parabola.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can evaluate f(x) for a given x value.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can describe f(x) for a given value is a point on the graph of the parabola.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can state the domain of a parabola from the graph.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can state the range of a parabola from the graph.</td>
</tr>
<tr>
<td>8.2</td>
<td></td>
<td></td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can explain what a root is for a quadratic function.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can show the rule for square roots.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can solve quadratic equations using the Square Root Method given the form.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can sketch the roots of a quadratic using the Square Root Method.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td><strong>I can use my calculator to graph a quadratic.</strong></td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can explain my steps and error check solutions using the Square Root Method</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can develop a checklist to assist me in problem solving.</td>
</tr>
<tr>
<td>8.3</td>
<td></td>
<td></td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
<tr>
<td>8.4</td>
<td></td>
<td></td>
<td>I can try several methods to develop an answer when faced with a challenge.</td>
</tr>
<tr>
<td>8.4</td>
<td></td>
<td></td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
<tr>
<td>8.5</td>
<td></td>
<td></td>
<td>I can multiply two binomials (FOIL)</td>
</tr>
<tr>
<td>8.5</td>
<td></td>
<td></td>
<td>I can solve quadratic equations by factoring (Find the roots for quadratic equations by using the zero product law)</td>
</tr>
<tr>
<td>8.5</td>
<td></td>
<td></td>
<td>I can research and explain uses for quadratic functions.</td>
</tr>
<tr>
<td>8.5</td>
<td></td>
<td></td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
<tr>
<td>Section</td>
<td>Objective</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.6</td>
<td>I can try several methods to develop an answer when faced with a challenge.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.6</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can solve quadratic equations using the method of completing the square with coefficient 1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can convert the standard form of a quadratic to the vertex form of a quadratic.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can identify the vertex from the vertex form of a quadratic.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can demonstrate a visual representation of completing the square.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can use the extraction of roots to find the x values when completing the square.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can develop a checklist to assist me in problem solving.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.8</td>
<td>I can try several methods to develop an answer when faced with a challenge.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.8</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9</td>
<td>I can solve all quadratic equations using the method of completing the square.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9</td>
<td>I can identify the maximum and minimum from the vertex form.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9</td>
<td>I can work through difficult problems to obtain an answer.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td>I can identify the quadratic formula.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td>I can use the quadratic formula to solve for the roots of a quadratic.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td>I can complete my project objectives.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can define a parabola.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can state the y-intercept given a graph.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can state the y-intercept given a quadratic equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can define the axis of symmetry.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can label the axis of symmetry given a graph of a parabola.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can determine the axis of symmetry given a quadratic equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>I can label the vertex on a graph.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Integrated Algebra

<table>
<thead>
<tr>
<th>8.11</th>
<th>I can state whether the vertex is a minimum or maximum from a graph.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.11</td>
<td>I can determine the vertex given a quadratic equation.</td>
</tr>
<tr>
<td>8.11</td>
<td>I can describe the way the parabola opens by the leading coefficient.</td>
</tr>
<tr>
<td>8.11</td>
<td>I can describe the discriminant to others.</td>
</tr>
<tr>
<td>8.11</td>
<td>I can calculate the discriminant given a quadratic equation.</td>
</tr>
<tr>
<td>8.11</td>
<td>I can label the roots of a quadratic on a graph</td>
</tr>
<tr>
<td>8.11</td>
<td>I can determine the roots of a quadratic using various methods</td>
</tr>
<tr>
<td>8.11</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.12</th>
<th>I can work appropriately with others to complete my objectives.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.12</td>
<td>I can work through difficult problems to obtain an answer.</td>
</tr>
<tr>
<td>8.12</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.13</th>
<th>I can complete a table of coordinate values given a quadratic equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.13</td>
<td>I can interpret a table as a representation of a graph.</td>
</tr>
<tr>
<td>8.13</td>
<td>I can interpret a table as a representation of a quadratic equation.</td>
</tr>
<tr>
<td>8.13</td>
<td>I can graph a quadratic using both the characteristics and creating coordinate points.</td>
</tr>
<tr>
<td>8.13</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.14</th>
<th>I can work appropriately with others to complete my objectives.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.14</td>
<td>I can work through difficult problems to obtain an answer.</td>
</tr>
<tr>
<td>8.14</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.15</th>
<th>I can research information from the Internet</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.15</td>
<td>I can evaluate the average rate of change from two points.</td>
</tr>
<tr>
<td>8.15</td>
<td>I can identify patterns that may/may not occur from the average rate of change of a parabola.</td>
</tr>
<tr>
<td>8.15</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.16</th>
<th>I can work appropriately with others to complete my objectives.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.16</td>
<td>I can model word problems using variables</td>
</tr>
<tr>
<td>8.16</td>
<td>I can model word problems through picture representations.</td>
</tr>
<tr>
<td>8.16</td>
<td>I can develop a checklist to assist me in problem solving. (5-step method)</td>
</tr>
<tr>
<td>8.16</td>
<td>I can solve quadratic word problems through algebraic manipulation.</td>
</tr>
<tr>
<td>8.16</td>
<td>I can check to see if the answer makes sense.</td>
</tr>
<tr>
<td>------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>8.16</td>
<td>I can develop strategies to begin a problem and error check as I continue.</td>
</tr>
<tr>
<td>8.16</td>
<td>I can problem solve my solutions that do not make sense.</td>
</tr>
<tr>
<td>8.16</td>
<td>I can evaluate my objectives to see if I am meeting them.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can develop a product from research.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can make estimations on given data.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can estimate demand curves to produce a viable unit sale price structure.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can utilize mathematical graphing software to model mathematics.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can utilize several technology programs to model mathematical data.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can develop technology for increasing study habits.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can solve quadratic word problems to emulate cost estimation.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can work appropriately with others to complete my objectives.</td>
</tr>
<tr>
<td>8.17</td>
<td>I can present my topic following the rubric guidelines.</td>
</tr>
<tr>
<td>8.18</td>
<td>I can review my objectives.</td>
</tr>
</tbody>
</table>
1. Pre-Assessment: Apply your knowledge.

1) Factoring is the opposite of: ________________________________

2) Circle each linear equation.
   a. 2x-9y+3=0
   b. x=27+3
   c. y=x^3
   d. 17=y
   e. 4+y=2x-3x^2
   f. y=|x|

3) What is a quadratic? ____________________________________________
   ____________________________________________

4) Which of the following products is equivalent to the trinomial \( x^2 - 5x - 24 \)?
   a. \((x-12)(x-2)\)
   b. \((x-8)(x+3)\)
   c. \((x+12)(x-2)\)
   d. \((x+8)(x-3)\)

5) Factor completely: \( 2x^2 - 18 \)

6) What is the GCF of 24 and 36?

7) What is the LCM of 24 and 36?
8) Answer the following questions, Given: \( m = 12x^3 + 18x^5 + 27 - 3x^4 - 6x^2 \)

a. How many terms does \( m \) have? __________________________________________

b. Is \( m \) ordered correctly? _________________________________________________

c. If not (part b), how could you order it? _______________________________________

d. What could you factor from \( m \)? _________________________________________

e. What are the coefficients of \( m \)? _________________________________________

9) What is a parabola? _________________________________________________________

_____________________________________________________________________________

10) What is the difference between the domain of \( y = x - 2 \) and \([x = \{-1,0,1,2,3,4,5,6\}; y = \{1,2,3,4,5,6,7,8\}]\)

_____________________________________________________________________________

_____________________________________________________________________________

11) List as many words that you know for the following symbols:

<table>
<thead>
<tr>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>*</th>
<th>/</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12) What is the slope of a line? ________________________________

13) How do you determine what the slope of a line is? ________________________________

14) Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

From NYS Regents Common core exam 08/14
15) Let \( f(x) = -2x^2 \) and \( g(x) = 2x - 4 \). On the set of axes below, draw the graphs of \( y = f(x) \) and \( y = g(x) \). From NYS Regents Common core exam 08/14

Using this graph, determine and state all values of \( x \) for which \( f(x) = g(x) \).
2. **Identify Quadratic Equations**

The Pre-assessment acts as the tool for previewing skills in order to move into the next area. It may be necessary to add lessons based on the outcomes of the assessment, however it is encouraged to spend minimal time within these lessons as the construction of the unit will help to develop these skills even for students with low scores.

The lesson for identifying quadratic equations focuses heavily on sequential prompting, direct instruction, modeling, rehearsal, and feedback, class wide peer tutoring (CWPT), and incorporating student interest (on-line). It is assumed that instructors have a knowledgeable grasp of these strategies and implementing them therefore it is outlined when each begins and ends. For instance when sequential prompting begins and given the problem $3x^2 + 2x - 1 = 0$ it may be tedious to add that the instructor should pose the questions to students; 1. How many terms in this equation? 2. What is the degree of the terms? If they can not answer then an explanation followed by, How many x's does each term have? 3. Which term has the highest degree? 4. What determines a quadratic as opposed to other polynomials? And finally, 5. Is this a quadratic? This is one part of the original question posed by the directions and modeled by the strategy intended.

Similarly, modeling, rehearsal, and feedback is often thought of the I do, we do and you do strategy. This often allows the instructor to model the first problem with self talk as they complete the problem. The second problem they may pose more questions with less results on the white/smart board, and finally circulate around the room as students complete the last problem (or sets of problems).

Students often gravitate toward on-line problem sets as opposed to a constructed worksheet. However, many students tend to just click answers rather than solving the problems. One benefit for the on-line problems introduced in the homework is the computer rewards students for receiving the correct answers without hints, encouraging them to answer correctly.
2. Introduction to quadratic Functions and equations.

Begin think-pair-share (different groups for each section)

A quadratic function is ____________________________________________________________

The domain is: __________________________________________________________________
The codomain is: ________________________________________________________________
The range is: ___________________________________________________________________

Exercise #1: Read the definition above and answer the following questions.

(a) Why is it important for the leading coefficient to be nonzero?

(b) Circle the choices below that are quadratic equations.

\[ y = x^2 - 3 \]
\[ y = -2x^2 + 10x^3 - 4 \]
\[ y = x^2 + \sqrt{x} + 7 \]
\[ y = 10 - x^2 \]

(c) Given the quadratic equation \( y = 14 - 3x^2 + 7x - 4 \) write it in standard form and state the value for the leading coefficient.

(d) If \( f(x) = 2x^2 - 3x + 1 \), then find the value of \( f(-2) \). What point must lie on this quadratic equation?

A parabola is: __________________________________________________________________
The y-intercept is: _______________________________________________________________
The vertex is: __________________________________________________________________

End think-pair-share. Begin modeling, rehearsal, and feedback with guiding questions.
Exercise #2: Look at the graphs below and state the y-intercept and vertex of each graph.

<table>
<thead>
<tr>
<th>Graph 1</th>
<th>Graph 2</th>
<th>Graph 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept</td>
<td>y-intercept</td>
<td>y-intercept</td>
</tr>
<tr>
<td>vertex</td>
<td>vertex</td>
<td>vertex</td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td>( , )</td>
</tr>
<tr>
<td>Domain</td>
<td>Domain</td>
<td>Domain</td>
</tr>
<tr>
<td>Range</td>
<td>Range</td>
<td>Range</td>
</tr>
</tbody>
</table>

End modeling, rehearsal, and feedback with guiding questions. Begin sequential prompting.

Exercise #3: Determine if the equation is a quadratic, and if so, determine the coefficients and the constant of each quadratic. Consider $0=ax^2+bx+c$

(a) $3x^2+2x-1=0$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

(b) $5x^2+8x = 0$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

(c) $x^2+7 = 0$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

(d) $-6x^2=2x+3$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

(e) $3x+2=0$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

(f) $8x^2 + \frac{3}{x} - 5=0$
   $a = \underline{\hspace{2cm}}$  , $b = \underline{\hspace{2cm}}$  , $c = \underline{\hspace{2cm}}$

End sequential prompting. Begin partners.
Exercise #4: What can you conjecture about the value of the constant $c$ and the y-intercept from the examples above?

Exercise #5: What is the y-intercept given the quadratic function:

(a) $3x^2 + 4x - 7 = y$
(b) $-2 + 7x^2 + 4x + y = 0$
(c) $2y + 16x^2 = 12x - 20$
(d) $4y = 36x^2$

End partners. Begin modeling, rehearsal, and feedback and direct instruction with guiding questions.

Given: $y = ax^2 + bx + c$ where $a \neq 0$; when $x=0$ then $y=c$

C is the constant and the y-intercept.

Zero Factor Property

If two numbers $a$ and $b$ are multiplied together and the resulting product is zero, then at least one of the numbers must be zero.
If $ab = 0$, then $a=0$ or $b=0$ or both $a=0$ and $b=0$

Exercise #6: Use the zero factor property to solve each equation.

(a) $2x = 0$
(b) $-9x^2 = 0$
(c) $x - 1 = 0$
(d) $x(x + 8) = 0$
(e) $(x + 4)(x + 3) = 0$
(f) $(x + 11)(4x - 5) = 0$
(g) $6(a-4) = 0$
(h) $(1 - 9x)(-11x - 3) = 0$
(i) $(7a - 2)^2 = 0$
Integrated Algebra

Name: ___________________________________  Date: _________________

Independent Problems


Part #2: Determine which of the following is a quadratic equation, and if so, determine the coefficients and the constant of each quadratic.

(a) 3x = 2
   a = ________ , b = ________ , c = ________

(b) 3x^2 + 8x +2 = -8x - 2
   a = ________ , b = ________ , c = ________

(c) 6(2)^3 = 7
   a = ________ , b = ________ , c = ________

(d) \( \frac{2}{x} - 5x^2 + 6 = 4x^2 + 8 \)
   a = ________ , b = ________ , c = ________

(e) 12x^2 + 8x – 16 = 0 - 8
   a = ________ , b = ________ , c = ________

(f) 9x + 2x – 1x + 3x^2 = 0
   a = ________ , b = ________ , c = ________

Multiple choice: Which of the following is a quadratic function?

(a) y + 3x = 2  
(b) y = x^2 – 3

(c) y = x^3 – 5x + 3  
(d) \( y - \frac{7}{x} = -x^2 + 6 \)

Part #3: Write the following in standard form.

(a) 4x^2 + 3x + 1 = -8x – 2 + y
   Standard form: ________________________________

(b) 3y = 18x^2 -3x + 21
   Standard form: ________________________________

(c) -8 - 3x^2 + 6x + y = 0
   Standard form: ________________________________
Part #4: Use the zero factor property to solve each equation.

(a) 4m = 0  
(b) 3(k + 7) = 0

(c) −5(x + 4) = 0  
(d) y(y − 1) = 0

(e) (y − 4) (y − 8) = 0  
(f) (y + 6) (2y + 1) = 0

(g) (6m + 5) (11m − 6) = 0  
(h) (7a + 6) (7a − 6) = 0

(i) (5m − 6)^2 = 0  
(j) (m − 3)^2 = 0

Part #5: Evaluate f(x) to determine the point that lies on the quadratic's graph.

(a) f(x) = 3x^2 + 5x − 1; for f(-3) Point: (      ,       )

(b) f(x) = -x^2 + 8x − 16; for f(2) Point: (      ,       )

(c) f(x) = x^2 + x − 1; for f(-1) Point: (      ,       )

Multiple choice: Which of the following points lies on the graph of y = x^2 − 5?

(a) (3 , -2)  
(b) (5 , 0)

(c) (-2 , -1)  
(d) (-1 , -6)
3. Solving Quadratic Equations Using the Method of Extraction of Roots

This lesson begins with the strategy previewing. Students have already solved radical equations and they can activate their prior knowledge of the steps needed to perform the operations for solving quadratic equations using the method of extraction of roots. The next part leads with direct instruction making connections to the equation and the graph as two representations. The final exercise allows students to develop their own checklist in solving these types of problems. This strategy in conjunction with the objective checklist allows students to self-monitor their work and how they can begin to develop more independence with problem solving. As students become more proficient with solving these types of problems they can gradually abandon the checklist. If students create their own checklists then they generally feel more involved and utilize it more efficiently.

Students at this point should be proficient with solving literal equations and utilizing opposite operations to do so and the distributive property. If they are completely lost in these subjects (as students with EBD may have several school placements which enhances learning gaps) it may be necessary to scaffold these skills in depending on their mastery level, or provide remediation in AIS or reteaching. Additional links are provided for more problems as well as a secondary worksheet for these problem types. Since this skill continues to be needed for the next lessons it is important that students have a higher proficiency with developing this.
3. Solving quadratic equations using the extraction of roots.

<table>
<thead>
<tr>
<th>Previewing: solving a radical equation</th>
<th>Now you try: $\sqrt{2x+9} - 1 = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>solve: $\sqrt{x-3} + 2 = 10$</td>
<td></td>
</tr>
<tr>
<td>Recall: step 1: isolate the square root by itself on one side of the equal sign.</td>
<td></td>
</tr>
<tr>
<td>So: $\sqrt{x-3} = 8$</td>
<td></td>
</tr>
<tr>
<td>Recall: step 2: square both sides</td>
<td></td>
</tr>
<tr>
<td>So: $x - 3 = 64$</td>
<td></td>
</tr>
<tr>
<td>Recall: step 3: solve for $x$</td>
<td></td>
</tr>
<tr>
<td>So: $x = 67$</td>
<td></td>
</tr>
</tbody>
</table>

End Previewing. Begin Direct Instruction.

A square root is: ______________________________________________________________

A quadratic root is: _____________________________________________________________

The rule of square roots: $x^2 = a > 0 \iff x = \pm \sqrt{a}$
If $a \geq 0$ then the solutions are $\pm \sqrt{a}$
If $a < 0$ then no real number solution exists

Exercise #1: Solve the following quadratic equations using the Square Root Method.

(a) $x^2 = 9$

The roots are where the parabola intersects the x-axis.

root 1: $x =$ _________________

root 2: $x =$ _________________

If $x^2 = 9$ then $x^2 - 9 = 0$ and $(x - 3)(x + 3) = 0$. Therefore $x = 3$ and $x = -3$

(b) $x^2 - 1 = 24$

(c) $55 = 3x^2 + 7$
**Steps:** solving a quadratic equations using the Square Root Method

solve: \[-7(x^2 - 10)^2 - 6 = -258\]

**Step 1:** isolate the squared term by itself on one side of the equal sign. (add 6 and divide by -7)

So: ________________________

**Step 2:** take the square root of both sides and DONT FORGET: it has two solutions \((x = \pm \sqrt{a})\)

So: ________________________

**Step 3:** evaluate the radical

So: ________________________

**Step 4:** consider two cases and solve for \(x^2\)

<table>
<thead>
<tr>
<th>Case 1: ________________________</th>
<th>Case 2: ________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>We get: ________________________</td>
<td>We get: ________________________</td>
</tr>
<tr>
<td>Now use the zero factor property</td>
<td>Now use the zero factor property</td>
</tr>
<tr>
<td>so: (x = _<em><strong><strong>) and (x = _</strong></strong></em>)</td>
<td>so: (x = _<em><strong><strong>) and (x = _</strong></strong></em>)</td>
</tr>
</tbody>
</table>

**Step 5:** If the answer is not coming out right, try something different.

The solutions to this quadratic equations are \(x = \______\), \(x = \______\), \(x = \______\) and \(x = \______\).

**Step 6:** check. Make sure your answer makes sense.

*Here we can see we have two parabolas:*

\[y = \______\]

and

\[y = \______\]

from

\[y = -7(x^2 - 10)^2 - 6 + 258\]

By graphing all 3 we can see the roots match up!

Sketch The graphs

**Exercise #2:** Create a checklist that will help you complete the steps.
Independent Problems:

Part #1: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/e/solving_quadratics_by_taking_the_square_root

Part #2: solving a quadratic equations using the Square Root Method
(a) \( k^2 = 75 \) 
(b) \( x^2 = 21 \)
(c) \( x^2 + 8 = 28 \) 
(d) \(-6m^2 = -414\)
(e) \(-5x^2 - 10 = 490 \) 
(f) \( 3 - 4x^2 = -85 \)

Part #3: solving a quadratic equations using the Square Root Method and provide a graph.
(a) \(-7(-x^2 + 1) + 6(x^2 - 1) = -17\)
(b) \(-2 (x^2 \ - 7)^2 + 3 = -47\)

(c) \(7(x^2 + 3) - 3(2x^2 - 2) = 127\)
3.b Additional: More solving quadratic equations using the extraction of roots.

The rule of square roots: \( x^2 = a > 0 \iff x = \pm \sqrt{a} \)
If \( a \geq 0 \) then the solutions are \( \pm \sqrt{a} \)
If \( a < 0 \) then no real number solution exists

**Exercise #1:** Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \( 25x^2 = 36 \)  
(b) \( 4n^2 = 36 \)  
(c) \( 5x^2 - 15z^2z^2 = 0 \); for \( x \)  
(d) \( 14a^2 = 235 \)  
(e) \( 6x^2 + 36 = 0 \)  
(f) \( 6n^2 = 864 \)  
(g) \( 9z^2 - 121 = 0 \)  
(h) \( 4n^2 = 24m^2p^8 \); for \( n \)  
(i) \( 5p^2q^2 = 45p^2 \); for \( q \)  
(j) \( 16m^2 = -2206 \)
Exercise #2: Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \((x + 2)^2 = 81\) 
(b) \((a + 3)^2 = 5\)

(c) \((a + 6)^2 = 64\) 
(d) \((m - 4)^2 = 15\)

(e) \((y - 7)^2 = 49\) 
(f) \((k - 1)^2 = 12\)

(g) \((x - 11)^2 = 0\) 
(h) \((a - 6)^2 = 6\)
Independent Problems:

**Part #1:** Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) $x^2 = 81$ 
(b) $b^2 - 4 = 0$

(c) $z^2 = 3$ 
(d) $m^2 - 11 = 14$

(e) $3x^2 - 27 = 0$ 
(f) $2n^2 - 25 = 25$

(g) $x^2 = 9b^2$; for $x$ 
(h) $x^2 = 9b^2$; for $b$

(i) $k^2 = p^2q^2r^2$; for $k$ 
(j) $2y^2 = 2a^2n^2$; for $y$

(k) $x^2 - z^2 = 0$; for $x$ 
(l) $5a^2 - 10b^2 = 0$; for $a$
Part #2: Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \((x - 1)^2 = 4\)  
(b) \((x - 2)^2 = 9\)

(c) \((x + 5)^2 - 12 = 5\)  
(d) \((x + 3)^2 = 5\)

(e) \((x - 3)^2 = -10\)  
(f) \((x - 7)^2 = 45\)

(g) \((x + 1)^2 = a ; \text{ for } x\)  
(h) \((t - 5)^2 = b ; \text{ for } t\)

(i) \((s^2 + 1)^2 = a^2 ; \text{ for } s\)  
(j) \((x + c)^2 = z^6 ; \text{ for } x\)

(k) \((m + c)^2 = j ; \text{ for } m\)  
(l) \((x^2 + n^4)^2 = a^6 ; \text{ for } x\)
4. Assessment

The assessments (number 4, 6, 8, 13 and 15) all follow the same strategies for students to use self-instruction and to help incorporate student interest, and increase the likelihood of motivational activities. The later assessments also include partner work with half the assessment to assist struggling students with a type of peer tutoring and cooperative learning. Utilizing the computer students tend to display less anxiety about the assessments, and get a quick feedback about their progress. Because of the similarities between assessments the outline is addressed here but each is included below.
4. Apply your knowledge.

Students must get 5 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 5 questions is recorded at the top of the screen are the points they receive.

Green check = 5 points
light bulb (hints) = 2 points
x mark = 2 points
total = 25 points

Part 2: Factor the GCF from each polynomial.

(1) \(22g^6 + 44g^8\)  
(2) \(48x^{31} - 54x^{41}\)

(3) \(2y^3 - 10y^2 - 18y\)  
(4) \(12a^3b^4 + 8a^2b\)

Part 3: Factor the Difference of Perfect squares.

(1) \(144 - p^2\)  
(2) \(36 - 25x^2\)  
(3) \(4z^2 - 8\)

(4) \(25c^2 - 81\)  
(5) \(w^2 + 100\)  
(6) \(9n^2 - 196\)
5. Solving Quadratic Equations by Factoring

This lesson uses several different strategies to be able to differentiate student learning and allow for sufficient time to scaffold skills to students. The lesson begins with direct instruction and allows students to see the pacing and the nature of the problems. This allows for completed problem types as well as partial completion problem types. Students will move toward greater independence through modeling, rehearsal and feedback. This strategy will give them the ability to attempt problems by themselves. Students who demonstrate higher proficiency can assist those who are struggling with the material through cooperative learning. Then students are asked to develop this skill in relation to graphing, which can be directed by sequential prompting.

In this lesson students are asked to Complete a checklist that will help you solve these equations. For students who have a harder time gaining this skill the checklist will help them to develop the patterns of problem solving. If sufficient time is spent developing this checklist it also may allow the next lessons to move at a faster pace.

The homework is also scaffolded to develop the next lessons through structured academic tasks. As the unit is building to developing the skills for modeling quadratics, there is a small presentation task to assist students in developing their research and presentation skills.
5. Solving quadratic equations by factoring.

Often books refer to solving a quadratic equation by factoring to mean the same thing as finding the roots. While finding the roots of a quadratic is an important skill, it does not encompass the entirety of what a quadratic is modeling.

(Remember: the roots are where the quadratic intersects the x-axis) Quadratic equations can be solved by factoring, by completing the square, by using the quadratic formula, trig solutions, Vieta's solutions, Bramagupta's methods, or by graphing. In this lesson we will focus on specific quadratics that can be factored (not all quadratic equations can be solved in this method)

Begin Direct Instruction.

<table>
<thead>
<tr>
<th>FOIL</th>
<th>FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 7)(x – 2) = 0</td>
<td>1) Make parenthesis</td>
</tr>
<tr>
<td>x² + 5x - 14 = 0</td>
<td>2) Find square root of the first term (when 1)</td>
</tr>
<tr>
<td>(       ) (       ) = 0</td>
<td>3) Choose the signs</td>
</tr>
<tr>
<td>14 = 1, 2, 7, 14</td>
<td>4) Find factors of last term</td>
</tr>
<tr>
<td>Sign    Sign *  +</td>
<td>5) Choose factors that add to middle term</td>
</tr>
<tr>
<td>+       +</td>
<td>6) solve for x (twice)</td>
</tr>
<tr>
<td>-       -</td>
<td>7) MAKE CONNECTIONS</td>
</tr>
<tr>
<td>+       -</td>
<td>?</td>
</tr>
</tbody>
</table>

Exercise #1: Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law). Sometimes you will be instructed to solve by factoring, find the roots, or find the zeros.

The zero product law’s importance to mathematics cannot be overstated. It finally allows us, in certain situations, to solve equations that are higher-order polynomials than just linear. Of course, for it to work, we must have two conditions met: (1) we must have the equation set equal to zero and (2) we must be able to factor the expression equal to zero.

End Direct Instruction. Begin modeling, rehearsal and feedback.

<table>
<thead>
<tr>
<th>I do</th>
<th>We do</th>
<th>You do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor: x² + x – 6 = 0</td>
<td>Find the roots: x² + x – 12 = 0</td>
<td>Find the zeros: x² + 6x + 8 = 0</td>
</tr>
<tr>
<td>(a) x² + 5x + 6 = 0</td>
<td>(b) x² – x – 12 = 0</td>
<td></td>
</tr>
</tbody>
</table>
End modeling, rehearsal and feedback. Begin cooperative learning.

**Exercise #2:** Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law).

(a) \(x^2 - 11x = -24\)

(b) \(10x^2 + x - 21 = 0\) (not 1)

(c) \(x^2 + 6x + 9 = 0\)

(d) \(3x^2 + 90 - 39x = 0\)

(e) \(x^3 + 6x^2 - 16x = 0\)

(f) \(-20x + 2x^2 = -32\)

(g) \(3x^2 - 9x + 6 = 0\)

(h) \(4x^3 + 16x^2 + 64 = 4x^3 + x^2 + 10x^2 + 56\)

End cooperative learning. Begin sequential prompting.

**Exercise #3:** Complete a checklist that will help you solve these equations.

**Exercise #4:** Find the zeroes of the quadratic function \(y = 3x^2 - 6x - 24\) algebraically. Sketch a graph of the zeroes and y-intercept on the graph.
Independent Problems:

Part #2: Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law).

(a) \(2x^2 + 12x + 18 = 0\) 

(b) \(x^3 - 8x^2 + 16x = 0\)

(c) \(x^2 + 23x = 50\)

(d) \(-8x = 20 + x^2\)

(e) \(x^2 - 18x + 32 = 0\)

(f) \(2x = x^2 + 3x - 10\)

(g) \(x^2 - x - 56 = 0\)

(h) \(8 = x^2 - x + 2\)

(i) \(x^4 = 3x^3 + 10x^2\)

(j) \(2x^2 - 22x - 24 = 0\)
Integrated Algebra

Name: ___________________________ Date: ________________

Part #4: History: The ancient civilizations of Babylonia, Egypt, Greece, China, and India used the method for finding the roots in order to calculate land taxes, finances, and solve problems relating the areas and sides of rectangles, this holds true today in finance, path and trajectory, changes in temperature, or structure, and area formations.

5 minute Presentation: Research and find an example of quadratics in use throughout history and explain the significance of the example you have found.

Multiple Choice: The roots of $x^2 - 6x - 16 = 0$ can be found by factoring as

(1) {-16, 6}  (3) {-2, 8}
(2) {-8, 2}    (4) {6, 16}

Multiple Choice: The equation $(2x - 3)(x + 7) = 0$ has a solution set of

(1) {-7, 3}  (3) {-7, 3}
(2) {3, 7}    (4) {$\frac{1}{2}$, -3}

Part #5: Find the roots of each of the following equations by factoring:

(a) $x^2 - 36 = 0$
(b) $x^2 + 12x + 27 = 0$

(c) $3x^2 + 5x - 2 = 0$
(d) $20x^2 - 10x = 0$

(e) $10x^2 + x - 21 = 0$
(f) $4x^2 - 16x - 84 = 0$
**Part #6:** A baking soda rocket is fired upwards with an initial speed of 80 feet per second. Its height, \( h \), above the ground in feet can be modeled using the equation:

\[ h(t) = -16t^2 + 80t \]

where \( t \) is the time since launch in seconds.

At what time, \( t > 0 \), does the rocket hit the ground? Find algebraically using factoring.

---

**Part #7:**


Factoring with other leading coefficients than 1

**Part #6:** Find the zeroes of the quadratic function

\[ y = x^2 - 4x - 5 \]

algebraically. Sketch a graph of the zeroes and \( y\)-intercept on the graph.

---

**Part #7:** The two quadratic equations below have the same solutions. Can you determine why? Completely factor both to see what they have in common.

\[ x^2 - 7x + 12 = 0 \]

\[ 3x^2 - 21x + 36 = 0 \]

---

**Part #8:**

**ONLINE:** [http://www.ixl.com/math/algebra-1/factor-quadratics-special-cases](http://www.ixl.com/math/algebra-1/factor-quadratics-special-cases)

Factoring with special cases
Part #4: History: The ancient civilizations of Babylonia, Egypt, Greece, China, and India used the method for finding the roots in order to calculate land taxes, finances, and solve problems relating the areas and sides of rectangles, this holds true today in finance, path and trajectory, changes in temperature, or structure, and area formations.

5 minute Presentation: Research and find an example of quadratics in use throughout history and explain the significance of the example you have found.

Rubric: Every class member is expected to complete a rubric for each presenter. Grades are determined both on your presentation and level at which you participate in scoring other class members, making positive critiques, and participating if asked.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Points/Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Organization</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Audience cannot understand presentation because there is no sequence of information.</td>
</tr>
<tr>
<td>2</td>
<td>Audience has difficulty following presentation because student jumps around.</td>
</tr>
<tr>
<td>3</td>
<td>Student presents information in logical sequence which audience can follow.</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

| **Content Knowledge** | | |
| 1 | Students show no understanding of mathematical concepts within the presentation | Students demonstrate a complete and comprehensive understanding of the mathematical concepts in the presentation |
| 2 | Students are visibly uncomfortable with the mathematical concepts of the presentation | |
| 3 | Students are at ease with the mathematical concepts of the presentation but lack a deep conceptual understanding | |
| 4 | | |

| **Visuals** | | |
| 1 | Students use no visuals | The visuals used supported audience understanding |
| 2 | Students occasionally use visuals that rarely support the presentation and audience understanding | |
| 3 | Students use visuals that are related to the presentation but did not completely support audience understanding | |
| 4 | | |

| **Mechanics** | | |
| 1 | Students presentation contained four or more spelling, grammatical or mathematical errors | Presentation had no spelling, grammatical or mathematical errors |
| 2 | Presentation had three spelling, grammatical or mathematical errors | |
| 3 | Presentation had no more than two spelling, grammatical or mathematical errors | |
| 4 | | |

| **Delivery** | | |
| 1 | Student mumbles, incorrectly pronounces terms, and speaks too quietly for students in the back of class to hear. | Student used a clear voice and correct, precise pronunciation of terms. |
| 2 | Student incorrectly pronounces terms. Audience members have difficulty hearing presentation. | |
| 3 | Student's voice is clear. Student pronounces most words correctly. | |
| 4 | | |

**Total**

Presenters:

Comments: Please add any additional comments that may aide the presenter in their next project.
6. Apply your knowledge.

**Quiz #1:** ONLINE: https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/e/solving_quadratics_by_factoring_2

Students must get 5 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 5 questions is recorded at the top of the screen are the points they receive.

- Green check = 5 points
- Light bulb (hints) = 2 points
- X mark = 2 points
- Total = 25 points

**Quiz #2:** ONLINE: http://www.ixl.com/math/algebra-1/solve-a-quadratic-equation-by-factoring

Score is recorded as the smart score.
7. & 9. Complete the square

With these lessons students develop the idea of completing the square through a completed problem type and study the mechanics and process of proof behind solving it through direct instruction. After the first problem, students are asked to watch a video for the visual representation for completing the square. This links the idea for students on 'how to do it' with 'why it is being done'. From the process of introducing a new skill students are asked to preview a couple of skills they have learned from previous units. This area may also be thought of as a motivational strategy because students can compete with themselves and use cooperative learning in this area. If students are not proficient on these skills it may be necessary to re-teach or perform some remedial skills tasks in AIS for students.

Students generally move at a faster pace through these lessons than the previous one if they have developed some proficiency with the material and the pedagogical nature of the problems. During cooperative learning there are usually some motivational activities built in the process, such as lab tables or games where students compete with themselves to gain the right answer. Often if students are allowed to move freely about the room and complete a station in partners or groups they are more animate about completing the tasks.
7. Completing the square with coefficient 1.

Begin Direct Instruction.

Suppose we wish to solve the quadratic equation \(x^2 - 3x - 1 = 0\): Since the equation is not of the form \(x^2 = a\); we cannot use extraction of roots. Next, we try factoring, but after a few trials we see that \(x^2 - 3x - 1 = 0\) is not factorable. We need another method for solving quadratic equations.

The method we shall study is based on perfect square trinomials and extraction of roots. The method is called solving quadratic equations by completing the square. Consider the equation \(x^2 + 6x + 5 = 0\): This quadratic equation could be solved by factoring, but we'll use the method of completing the square. We will explain the method in detail after we look at this example. First we'll rewrite the equation as

\[
x^2 + 6x + 5 = 0
\]

\[
x^2 + 6x = -5
\]

Then, we'll add 9 to each side. We get

\[
x^2 + 6x + 9 = -5 + 9
\]

The left side factors as a perfect square trinomial.

\[
(x + 3)^2 = 4
\]

In this form the quadratic function is written in its shifted or vertex form as

\[
y = (x + 3)^2 - 4
\]

We can solve this by extraction of roots.

\[
x + 3 = \pm\sqrt{4}
\]

\[
x + 3 = 2 \quad x + 3 = -2
\]

\[
x = 2 - 3 \quad x = -2 - 3
\]

\[
x = -1 \quad x = -5
\]

Notice that when the roots are rational numbers, the equation is factorable. The big question is, How did we know to add 9 to each side of the equation? We can convert any quadratic trinomial appearing in an equation into a perfect square trinomial if we know what number to add to both sides. We can determine that particular number by observing the following situation: Consider the square of the binomial and the resulting perfect square trinomial

\[
(x + p)^2 = x^2 + 2px + p^2
\]

Notice that the constant term (the number we are looking for) can be obtained from the linear term 2px: If we take one half the coefficient of \(x; = \frac{2p}{2} = p\); and square it, we get the constant term \(p^2\): This is true for every perfect square trinomial with leading coefficient 1.
Exercise #1: Given the function \( y = (x - 3)^2 + 2 \) do the following:
(a) Give the coordinates of the turning point. 
(b) Determine the range by drawing a rough sketch.

Taking the standard form \( y = ax^2 + bx + c \) where \( a, b \) and \( c \) are real numbers and \( a \neq 0 \) and putting it into its vertex (shifted) form relies on several skills.

End Direct Instruction. Begin previewing.

Review #1: Write each of the following as an equivalent trinomial.
(a) \((x + 5)^2\) 
(b) \((x - 1)^2\) 
(c) \((x + 4)^2\)

End previewing. Begin sequential prompting.

Exercise #2: Given each trinomial in Review #1 of the form \( ax^2 + bx + c \), what is true about the relationship between the value of \( b \) and the value of \( c \)? Illustrate.

End sequential prompting. Begin previewing.

Review #2: Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.
(a) \( x^2 + 20x + 100 \) 
(b) \( x^2 - 6x + 9 \) 
(c) \( x^2 + 2x + 1 \)

End previewing. Begin sequential prompting.

Exercise #3: If we graph the parabola \( y = x^2 - 4x - 1 \) we see the vertex is at \((2, -5)\), and \( h = \)_____, \( k = \)______ which means the vertex form is \( y = (x \text{_____})^2 \text{_____} \) so if we have \( x^2 - 4x - 1 = 0 \) or \( x^2 - 4x = 1 \) then we add \( x^2 - 4x + \text{_____} = 1 + \text{_____} \) to get \( (x + \text{_____})^2 = \text{_____} \) or \( (x + \text{_____})^2 \text{_____} = 0 \)

Completing the square algorithm
Given: \( y = ax^2 + bx + c \) where \( a = 1 \) then
1. find \( \frac{1}{2} \) the value of \( b \); \( \frac{b}{2} \). 
2. square this term; \( \left(\frac{b}{2}\right)^2 \) 
3. Add or subtract it
Exercise #4: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s turning point and solve this by extraction of roots. The last two problems will involve fractions. Stick with it!

(a) \( y = x^2 + 6x - 2 \)  
(b) \( y = x^2 - 2x + 11 \)  
(c) \( y = x^2 - 2x + 27 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

\( x = \),  
\( x = \),  
\( x = \),

(d) \( y = x^2 + 8x \)  
(e) \( y = x^2 + 5x + 4 \)  
(f) \( y = x^2 - 9x - 2 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

\( x = \),  
\( x = \),  
\( x = \),

(g) More complex solutions:  
(h) A factorable coefficient of a:  
\( y = x^2 - 3x - 1 \)  
\( y = 3x^2 - 36x - 39 \)

vertex = ( , )  
vertex = ( , )

\( x = \),  
\( x = \),

End Modeling, rehearsal and feedback. Begin sequential prompting.

Exercise #5: Complete a checklist that will help you complete the square.
Integrated Algebra

Name: ___________________________________  Date: __________________

Independent Problems:

Part #1: Find each of the following products in standard form.

(a) \((x + 4)^2\)  
(b) \((x - 5)^2\)  
(c) \((x + 8)^2\)  

(d) \((x - 7)^2\)  
(e) \((x + 2)^2\)  
(f) \((x - 10)^2\)  

Part #2: Each of the following trinomials is a perfect square. Write it in factored form, i.e., \((x + a)^2\) or \((x - a)^2\)

(a) \(x^2 + 6x + 9\)  
(b) \(x^2 - 22x + 121\)  
(c) \(x^2 + 10x + 25\)  

(d) \(x^2 + 30x + 225\)  
(e) \(x^2 - 2x + 1\)  
(f) \(x^2 - 18x + 81\)  


Part #4: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/e/completing_the_square_in_quadratic_expressions
Part #5: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s turning point and solve this by extraction of roots.

(a) \( y = x^2 - 12x + 40 \)  
(b) \( y = x^2 + 4x + 14 \)  
(c) \( y = x^2 - 24x + 146 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

x =  ,  
\( x =  \),  
\( x =  \),

(d) \( y = x^2 - 2x - 48 \)  
(e) \( y = x^2 + 3x - 5 \)  
(f) \( y = x^2 + 4x + 7 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

x =  ,  
\( x =  \),  
\( x =  \),

(a) \( y = x^2 - 10x \)  
(b) \( y = x^2 + 14x + 13 \)  
(c) \( y = x^2 + 7x + 12 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

x =  ,  
\( x =  \),  
\( x =  \),

(d) \( y = x^2 - 6x \)  
(e) \( y = x^2 - 2x - 24 \)  
(f) \( y = x^2 - 5x - 6 \)

vertex = ( , )  
vertex = ( , )  
vertex = ( , )

x =  ,  
\( x =  \),  
\( x =  \),
Part #6: Answer the following questions based on the information provided.

A cable is attached at the same height from two poles and hangs between them such that its height above the ground, \( y \), in inches, can be modeled using the equation: 
\[
y = x^2 - 16x + 67
\]
where \( x \) represents the horizontal distance from the left pole, in feet.

(a) What height is point A above the ground? Show your work and use proper units.

(b) Write the equation in vertex form.

(c) What is the difference in the heights of points A and B? Show your analysis and include units.

(d) What is the horizontal distance that separates points A and B? Explain your reasoning.

Part #7: Use the vertex form of each of the following quadratic functions to determine which has the lowest \( y \)-value.

\[
y = x^2 - 8x + 6 \quad y = x^2 + 6x + 1
\]
Integrated Algebra

Name: ___________________________  Date: ______________

8. Apply your knowledge.

Quiz #1: ONLINE: http://www.ixl.com/math/algebra-1/complete-the-square

Score is recorded as the smart score.

Quiz #2: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/e/completing_the_square_1

Students must get 3 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 3 questions is recorded at the top of the screen are the points they receive.

Green check = 5 points
light bulb (hints) = 2 points
x mark = 2 points
total = 15 points
9. **Completing the square with coefficient other than 1.**

**Begin Direct instruction.**

**Method 1:**
Consider the quadratic equation: \(7x^2 - 5x - 1 = 0\).

By the previous method we would move the constant to the right side of the equation to get:
\[7x^2 - 5x = 1\]

Then we would add \(\left(\frac{b}{2}\right)^2\) to both sides which would be + \(\frac{25}{4}\)

However \(7x^2 - 5x + \frac{25}{4}\) can not be written as \((ax - \frac{5}{2})^2\) since we need a * a to be 7. Impossible!

so we modify.

1. Write the equation so that the constant term appears on the right side of equation.
\[7x^2 - 5x = 1\]

2. If the leading coefficient is different from 1, divide each term of the equation by that coefficient.
\[x^2 - \frac{5}{7}x = \frac{1}{7}\]

3. Take one half of the coefficient of the linear term, square it, then add it to both sides of the equation.
\[\left(\frac{\frac{5}{7}}{2}\right)^2 = \frac{5}{14}\]

4. Square it.
\[\left(\frac{5}{14}\right)^2 = \frac{25}{196}\]

5. Add it to both sides of the equation.
\[x^2 - \frac{5}{7}x + \frac{25}{196} = \frac{1}{7} + \frac{25}{196}\]

4. The trinomial on the left is now a perfect square trinomial and can be factored as \((\cdot)^2\): The rest term in the parentheses is the square root of the quadratic term. The last term in the parentheses is one-half the coefficient of the linear term.
\[\left(x - \frac{5}{14}\right)^2 = \frac{53}{196}\]

5. Solve this equation by extraction of roots.
\[x = \frac{5}{14} \pm \frac{\sqrt{53}}{\sqrt{196}}\]

6. Since there is a square root in the denominator, you must rationalize the denominator.
\[x = \frac{5}{14} \pm \frac{\sqrt{53}}{14}\]

7. Check to determine if you can simplify the square root. In this case we can not.

8. Write your two solutions.
\[X = \frac{5 + \sqrt{53}}{14} \approx 0.877\] and \[x = \frac{5 - \sqrt{53}}{14} \approx -0.163\]

**SPECIAL CASE:** consider \(2x^2 + x + 4 = 0\)

When we perform steps 1-4 from above we get:
\[(x - \frac{1}{4})^2 = -2 + \left(\frac{1}{4}\right)^2\text{ OR } -2 + \left(\frac{1}{16}\right) \approx -1.938\]

so RHS = (-)

Since we know that the square of any number is positive, this equation has **no real number solution.**
End direct instruction. Begin cooperative learning.

Exercise #1: Solve each of the following quadratic equations using the method of completing the square
(a) \( y = 2x^2 - 12x + 11 \)  
(b) \( y = 5x^2 + 20x + 23 \)

\[ x = \quad , \quad x = \quad , \]

(c) \( y = -2x^2 + 4x + 7 \)  
(d) \( y = 6x^2 - 24x + 14 \)

\[ x = \quad , \quad x = \quad , \]

(e) \( y = -x^2 - 12x - 33 \)  
(f) \( y = 5x^2 - 2x - 24 \)

\[ x = \quad , \quad x = \quad , \]
End cooperative learning. Begin direct instruction.

Method 2:
The difficulty with the method above is understanding the form of $y = a(x-h)^2 + k$.

From above we get $(x - \frac{5}{14})^2 = \frac{53}{196}$ or alternately $y = (x - \frac{5}{14})^2 - \frac{53}{196}$

so $a$ will always be 1. However we can simplify this by multiplying both sides by the factor of 7 (the number we factor) to get: $7(x - \frac{5}{14})^2 = \frac{53}{28}$ or alternately $y = 7(x - \frac{5}{14})^2 - \frac{53}{28}$

which is the form we want. Since $a=7$ is positive, the parabola opens up from the vertex, and the vertex is a minimum.

Exercise #2: Edit your last checklist to complete the square when the leading coefficient is not 1.

Given $y = a(x-h)^2 + k$

If $a \geq 0$ then the vertex is a minimum. If $a < 0$ then the vertex is a maximum.

End direct instruction. Begin modeling, rehearsal and feedback.

Exercise #3: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s vertex (turning point), state whether the vertex is a maximum or minimum and solve this by extraction of roots.

(a) $y = 5x^2 - 2x - 4$
(b) $y = 4x^2 + 5x + 1$

(c) $y = 2x^2 + 2x - 1$
(d) $y = 4x^2 - 8x - 16$

- vertex = ( , )
- x = ,
- Max or Min

- vertex = ( , )
- x = ,
- Max or Min

- vertex = ( , )
- x = ,
- Max or Min

- vertex = ( , )
- x = ,
- Max or Min
Independent Problems:

Completing the square with leading coefficient other than 1.

**Part #2:** Write each quadratic in vertex form by Completing the Square, identify the quadratic’s vertex (turning point), state whether the vertex is a maximum or minimum and solve this by extraction of roots.

(a) \( y = 9x^2 + 12x - 5 \)  
(b) \( y = 16x^2 - 8x - 3 \)

\[
\begin{align*}
\text{vertex } &= ( \ , \ ) \\
\text{x } &= \ , \\
\text{Max or Min } &= \\
\text{Max or Min } &= \\
\text{(c) } y &= 2x^2 + 5x - 4 \\
\text{vertex } &= ( \ , \ ) \\
\text{x } &= \ , \\
\text{Max or Min } &= \\
\text{Max or Min } &= \\
\text{(d) } y &= 3x^2 + 2x - 18 \\
\text{vertex } &= ( \ , \ ) \\
\text{x } &= \ , \\
\text{Max or Min } &= \\
\text{Max or Min } &= \\
\end{align*}
\]

**Part #3:** Use the method completing the square to write each of the following quadratic functions in the form \( y = a(x - h)^2 + k \) Then, identify the turning point and whether it is a maximum or minimum.

(a) \( y = 3x^2 - 12x + 17 \)  
(b) \( y = -5x^2 + 40x - 70 \)
Multiple choice: Which of the following equations models the graph shown at the right? Explain how you made your choice?

(1) \( y = (x - 1)^2 - 5 \)

(2) \( y = -3(x + 1)^2 - 5 \)

(3) \( y = (x + 1)^2 - 5 \)

(4) \( y = 2(x - 1)^2 - 5 \)

Explanation:

Part 4: Answer the following questions based on the information given.

The vertical height of projectiles above level ground can be modeled by equations in the form:

\[ h(t) = -16(t - t_{\text{max}})^2 + h_{\text{max}} \]

where \( h_{\text{max}} \) is the maximum height in feet and \( t_{\text{max}} \) is the time, in seconds, when it occurs.

(a) A given projectile has a height function given by
\[ h(t) = -16(t - 8)^2 + 156 \]
What is its maximum height and at what time, \( t \), does it occur?

(b) A projectile has a height function given by
\[ h(t) = -16t^2 + 160t + 120 \]
Write this in vertex form (from a)

(c) What is the maximum height and at what time does it occur for the projectile from (b)?

(d) At what height does the projectile in (b) start above the ground? Show the work that leads to your answer.

(e) Using your calculator, sketch a graph of the height on the axes below for the projectile from (b). Mark your answers from (c) and (d) on the graph.
10. Solving Quadratic Equations Using the Quadratic Formula

This lesson develops the skill to be able to use the quadratic formula and develop an understanding of the uses and history of it. Students begin by previewing the skills they have learned and move into developing the quadratic formula through modeling and direct instruction. One strategy not listed in the lesson is additional motivational activities. During this lesson students were asked by their physical education teacher a series of questions (given by the mathematics teacher) to demonstrate proficiency with the understanding of the quadratic formula in a pickle-ball game. Those who answered first and correctly were allowed to serve first.

The project in this lesson is scaffolded from previous lessons and as a structured academic task leads to a more developed final project result. The ability to complete a project is also a part of the structured academic tasks as is the ability to efficiently work with others in various settings.
10. Solving Quadratic Equations Using the Quadratic Formula

Begin previewing.

Recall that a quadratic is in the standard form of \( 0 = ax^2 + bx + c \) where \( a \neq 0 \) and \( a, b \) and \( c \) are real numbers.

**Exercise 1:** Identify the real numbers for coefficients \( a \) and \( b \), and the constant term \( c \).

(a) \( 0 = 2x^2 - 12x + 11 \)  
(b) \( 0 = 3x^2 + 5x + 2 \)  
(b) \( 0 = 1x^2 + 3x - 1 \)

\[ a = \underline{_______} \quad a = \underline{_______} \quad a = \underline{_______} \]
\[ b = \underline{_______} \quad b = \underline{_______} \quad b = \underline{_______} \]
\[ c = \underline{_______} \quad c = \underline{_______} \quad c = \underline{_______} \]

(d) \( 0 = 6x^2 - 2 \)  
(e) \( 0 = 3x^2 + 7x \)  
(f) \( 12 = x^2 + 10x \)

\[ a = \underline{_______} \quad a = \underline{_______} \quad a = \underline{_______} \]
\[ b = \underline{_______} \quad b = \underline{_______} \quad b = \underline{_______} \]
\[ c = \underline{_______} \quad c = \underline{_______} \quad c = \underline{_______} \]

When a quadratic equation is written in standard form so that the values \( a, b \) and \( c \) are readily determined, the equation can be solved using the quadratic formula. The values that satisfy the equation are found by substituting the values \( a; b; \) and \( c \) into the formula.

**Quadratic Formula:** Given \( y = ax^2 + bx + c \) where \( a \neq 0 \),

the zeroes can be found by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Keep in mind that the plus or minus symbol, is just a shorthand way of denoting two possibilities.

**Exercise 2:** Solve the quadratic equations using the quadratic formula and express your answers in simplest radical form

<table>
<thead>
<tr>
<th>I do</th>
<th>We do</th>
<th>You do</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( 0 = 12x^2 - 2x - 1 )</td>
<td>(b) ( 0 = 2x^2 - 9x + 4 )</td>
<td>(c) ( 0 = x^2 + 8x + 3 )</td>
</tr>
</tbody>
</table>
Exercise 3: Solve $ax^2 + bx + c = 0$ for $x$ by completing the square.

Exercise 4: Consider the quadratic equation $2x^2 - 7x - 15 = 0$

(a) Find the solutions to this equation either by factoring or completing the square.

(a) Find the solutions to this equation using the quadratic formula.

Exercise 5: A projectile is fired vertically from the top of a 60 foot tall building. Its height in feet above the ground after $t$ seconds is given by the formula $h = -16t^2 + 20t + 60$

At what time, $t$, does the ball hit the ground? Solve by using the quadratic formula to the nearest tenth of a second.
Integrated Algebra

Name:___________________________________ Date:_________________

Independent Problems:


Part 2: Identify the real numbers for coefficients a and b, and the constant term c.

(a) $0 = 4x^2 - 3x + 5$  
(b) $-9 + 3x^2 = -9x$  
(b) $1 = -2x^2 + 4x$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
</tbody>
</table>

(d) $0 = x^2 - 2x$  
(e) $5x - 3 = -3x^2$  
(f) $0 = -3x^2 - 11x - 2$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
</tbody>
</table>

Part 3: Solve the quadratic equations using the quadratic formula and express your answers in simplest radical form

(a) $0 = 2x^2 + 3x - 7$  
(b) $0 = 5x^2 - 2x - 1$  
(c) $0 = -3x^2 + x$

(d) $0 = x^2 - 2x - 3$  
(e) $0 = x^2 + 5x + 6$  
(f) $0 = -6x^2 - 1x + 2$
Multiple choice: If the quadratic formula is used to solve the equation $x^2 - 4x - 41 = 0$, the correct roots are:

(a) $4 \pm 3\sqrt{10}$  
(b) $-4 \pm 3\sqrt{10}$  
(c) $2 \pm 3\sqrt{5}$  
(d) $-2 \pm 3\sqrt{5}$

Part 5: Answer the following questions based on the information given.
The flow of oil in a pipe, in gallons per hour, can be modeled using the function $F(t) = -2t^2 + 20t + 11$

(a) Using the quadratic formula, find, to the nearest tenth of an hour, the time when the flow stops (is zero). Show your work.

(b) Use the process of completing the square to write $F(t)$ in its vertex form. Then, identify the peak flow and the time at which it happens.

Part 6 History:  Watch: http://www.youtube.com/watch?v=vy6vjQ-wlgE
Create a 5-10 minute presentation that showcases some part of the quadratic formula. Explain the significance of this development, how it became developed, or the uses (how to). Construct using: http://www.kizoa.com or other presentation builder. See Rubric for scoring.
### Part 6 History

Watch: [http://www.youtube.com/watch?v=vy6vjQ-wlgE](http://www.youtube.com/watch?v=vy6vjQ-wlgE)

Create a 5-10 minute presentation that showcases some part of the quadratic formula. Explain the significance of this development, how it became developed, or the uses (how to). Construct using: [http://www.kizoa.com](http://www.kizoa.com) or other presentation builder. See Rubric for scoring.

**Rubric:** Every class member is expected to complete a rubric for each presenter. Grades are determined both on your presentation and level at which you participate in scoring other class members, making positive critiques, and participating if asked.

#### Presenters:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Points/Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>Content opening</td>
<td>Does not introduce self</td>
</tr>
<tr>
<td>Overview</td>
<td>No overview</td>
</tr>
<tr>
<td><strong>Content of Presentation</strong></td>
<td></td>
</tr>
<tr>
<td>Mathematical Concepts</td>
<td>Students shows no understanding of mathematical concepts within the presentation</td>
</tr>
<tr>
<td>Mathematical Procedures</td>
<td>Student has difficulty explaining mathematical procedures.</td>
</tr>
<tr>
<td>Questions</td>
<td>Student cannot answer questions about topic</td>
</tr>
<tr>
<td>Organization</td>
<td>Audience cannot understand presentation because there is no sequence of information.</td>
</tr>
<tr>
<td>Completeness of Content</td>
<td>One or more points left out</td>
</tr>
<tr>
<td><strong>Vocal Skills</strong></td>
<td></td>
</tr>
<tr>
<td>Enthusiasm</td>
<td>Shows absolutely no interest in topic presented</td>
</tr>
<tr>
<td>Elocution</td>
<td>Mumbles or speaks too quietly for a majority of audience to hear</td>
</tr>
</tbody>
</table>
## Integrated Algebra

Name: ________________________  Date: ________________

<table>
<thead>
<tr>
<th>Nonverbal Skills</th>
<th>Eye Contact</th>
<th>Displays minimal eye contact with audience, while reading mostly from notes</th>
<th>Occasionally looks at someone or some groups during presentation, but returns to notes</th>
<th>Constantly looks at someone or some groups at all/most times during presentation, while seldom looking at notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eye Contact</td>
<td>Does not attempt to look at audience at all, reads notes the entire time</td>
<td><strong>Body Language</strong></td>
<td>No movement or descriptive gestures; Has an deadpan expression; Slumps or sits</td>
<td>Very little movement or descriptive gestures; Occasionally has deadpan expression; Occasionally slumps</td>
</tr>
<tr>
<td>Body Language</td>
<td>No movement or descriptive gestures; Has an deadpan expression; Slumps or sits</td>
<td><strong>Poise</strong></td>
<td>Tension or nervousness is obvious; has trouble recovering from mistakes</td>
<td>Displays mild tension; has trouble recovering from mistakes</td>
</tr>
<tr>
<td>Poise</td>
<td>Makes minor mistakes, but quickly recovers from them, with little or no tension</td>
<td><strong>Creativity</strong></td>
<td>Use of visuals</td>
<td>Students use no visuals</td>
</tr>
<tr>
<td>Use of visuals</td>
<td>Students use no visuals</td>
<td><strong>Use of audio</strong></td>
<td>Students use no audio</td>
<td>Students use of audio is completely distracting from presentation.</td>
</tr>
<tr>
<td>Use of audio</td>
<td>Students use no audio</td>
<td><strong>Comments</strong>: Please add any additional comments that may aide the presenter in their next project.</td>
<td>Students use of audio is completely distracting from presentation.</td>
<td>Students use of audio did not completely support audience understanding, somewhat distracting</td>
</tr>
</tbody>
</table>

**Your Name:** ________________________
12. Characteristics of Quadratic Functions

This lesson is cumulative for all the skills that students have developed in quadratics. Some areas students may have a good grasp on, and through a variety of cooperative learning techniques students can enhance their knowledge, speed and fluency with the subject matter. The problem types increase in difficulty and students generally move from successful to being challenged. Because of the nature of the lesson teachers may be able to more accurately assess students learning needs with a continued progress monitoring in each section.
12. Characteristics of Quadratic Functions

Begin previewing.

A parabola is: ____________________________________________________________________________________

Exercise #1: Answer the following from the graph of the quadratic function.

<table>
<thead>
<tr>
<th>y-intercept</th>
<th>vertex</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

End previewing. Begin cooperative learning.

The y intercept:

Exercise #2: What is the y-intercept given the quadratic function:

(a) $7x^2 + 2x - 3 = y$
(b) $-7 + 16x^2 + 2x + y = 0$
(c) $2y - 20x^2 = 8x - 4$
(d) $4y = 40x^2$
(e) $y = (x + 4)^2$
(f) $y = (x - 5)^2$
(g) $y = (x + 1)^2 - 5$
(h) $y = 2(x - 1)^2 - 5$
End cooperative learning. Begin direct instruction.

The axis of symmetry:
The axis of symmetry is:

Exercise #3: State the axis of symmetry given the graph:

Axis of symmetry: $x = \ ___________ $

Exercise #4: State the axis of symmetry given the quadratic function:

(a) $7x^2 + 2x - 3 = y$

(b) $-7 + 16x^2 + 2x + y = 0$

(c) $2y - 20x^2 = 8x - 4$

(d) $4y = 40x^2$

(e) $y = (x + 4)^2$

(f) $y = (x - 5)^2$

(g) $y = (x + 1)^2 - 5$

(h) $y = 2(x - 1)^2 - 5$

Axis of symmetry: Given $y = ax^2 + bx + c$ where $a \neq 0$, the axis of symmetry can be found by $x = \frac{-b}{2a}$
End direct instruction. Begin cooperative learning.

The vertex:

Exercise #5: State the vertex of the parabola given the graph:

<table>
<thead>
<tr>
<th>Graph 1</th>
<th>Graph 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
</tr>
</tbody>
</table>

Vertex: $( $, $ )

Vertex: $( $, $ )

Vertex: $( $, $ )

Vertex: $( $, $ )

Exercise #6: State the vertex given the quadratic function. Determine if function has a maximum or minimum:

(a) $7x^2 + 2x - 3 = y$
(b) $-7 + 16x^2 + 2x + y = 0$
(c) $2y - 20x^2 = 8x - 4$
(d) $4y = 40x^2$
(e) $y = (x + 4)^2$
(f) $y = (x - 5)^2$
(g) $y = (x + 1)^2 - 5$
(h) $y = 2(x - 1)^2 - 5$
End cooperative learning. Begin modeling, rehearsal and feedback.

**The discriminant:**

The discriminant tells you: ____________________________

---

The discriminant: Given \( y = ax^2 + bx + c \) where \( a \neq 0 \),
the discriminant can be found by \( \sqrt{b^2 - 4ac} \)

**Exercise #7:** Calculate the discriminant from the quadratic function. State the number of roots:

(a) \( 7x^2 + 2x - 3 = y \)   
(b) \( -7 + 16x^2 + 2x + y = 0 \)

(c) \( 2y - 20x^2 = 8x - 4 \)   
(d) \( 4y = 40x^2 \)

(e) \( y = (x + 4)^2 \)   
(f) \( y = (x + 1)^2 - 5 \)

(g) \( y = 2x^2 + 3x - 7 \)   
(h) \( y = 5x^2 - 2x - 1 \)

(i) \( y = -3x^2 + x \)   
(j) \( 0 = x^2 - 2x - 3 \)

(k) \( y = x^2 + 5x + 6 \)   
(l) \( y = -6x^2 - 1x + 2 \)
End modeling, rehearsal and feedback. Begin cooperative learning.

**Quadratic roots:**

**Exercise #8:** Determine the roots of the quadratic from the graph

<table>
<thead>
<tr>
<th>x=</th>
<th>x=</th>
<th>x=</th>
</tr>
</thead>
</table>

**Exercise #8:** Determine the roots of the quadratic. Hint: you have already calculated the discriminant.

(a) \(7x^2 + 2x - 3 = y\)  
(b) \(-7 + 16x^2 + 2x + y = 0\)

(c) \(2y - 20x^2 = 8x - 4\)  
(d) \(4y = 40x^2\)

(e) \(y = (x + 4)^2\)  
(f) \(y = (x + 1)^2 - 5\)

(g) \(y = 2x^2 + 3x - 7\)  
(h) \(y = 5x^2 - 2x - 1\)

(i) \(y = -3x^2 + x\)  
(j) \(0 = x^2 - 2x - 3\)

(k) \(y = x^2 + 5x + 6\)  
(l) \(y = -6x^2 - 1x + 2\)
**Independent Problems:**

**Part #1:** (Review) ONLINE: http://www.ixl.com/math/algebra-1/characteristics-of-quadratic-functions

**Part #2:** From the graph determine the \( y \)-intercept, the axis of symmetry, the vertex, if it is a maximum or minimum, tell if the discriminant is positive, negative or zero, and determine the roots:

<table>
<thead>
<tr>
<th>Graph 1</th>
<th>Graph 2</th>
<th>Graph 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept:</td>
<td>y-intercept:</td>
<td>y-intercept:</td>
</tr>
<tr>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
</tr>
<tr>
<td>vertex:</td>
<td>vertex:</td>
<td>vertex:</td>
</tr>
<tr>
<td>max / min:</td>
<td>max / min:</td>
<td>max / min:</td>
</tr>
<tr>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
</tr>
<tr>
<td>roots:</td>
<td>roots:</td>
<td>roots:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph 4</th>
<th>Graph 5</th>
<th>Graph 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept:</td>
<td>y-intercept:</td>
<td>y-intercept:</td>
</tr>
<tr>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
</tr>
<tr>
<td>vertex:</td>
<td>vertex:</td>
<td>vertex:</td>
</tr>
<tr>
<td>max / min:</td>
<td>max / min:</td>
<td>max / min:</td>
</tr>
<tr>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
</tr>
<tr>
<td>roots:</td>
<td>roots:</td>
<td>roots:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph 7</th>
<th>Graph 8</th>
<th>Graph 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept:</td>
<td>y-intercept:</td>
<td>y-intercept:</td>
</tr>
<tr>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
<td>axis of symmetry:</td>
</tr>
<tr>
<td>vertex:</td>
<td>vertex:</td>
<td>vertex:</td>
</tr>
<tr>
<td>max / min:</td>
<td>max / min:</td>
<td>max / min:</td>
</tr>
<tr>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
<td>(+),(-),(0) discriminant:</td>
</tr>
<tr>
<td>roots:</td>
<td>roots:</td>
<td>roots:</td>
</tr>
</tbody>
</table>
Part #4: From the quadratic function determine the y-intercept, the axis of symmetry, the vertex, if it is a maximum or minimum, tell if the discriminant is positive, negative or zero, and determine the roots:

(a) $3x^2 + 5x - 2 = y$

(b) $y = x^2 - 4x - 5$

(c) $x^2 - 7x + 12 = y$

(d) $2x = x^2 - y + 3x - 10$

(e) $y = (x + 5)^2$

(f) $y = (x + 2)^2 - 3$
13. Apply your knowledge.

Work with a partner for part 1.

Score is recorded as completed without hints.

Students must get 3 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 3 questions is recorded at the top of the screen are the points they receive.

Green check = 5 points
light bulb (hints) = 2 points
x mark = 2 points
total = 15 points
14. & 16. Graphing Quadratic Functions & Slope

These lessons focus more on visual representations. This allows for students to utilize their knowledge of the characteristics of the quadratic function and develop the model for the entire function. The strategies follow from previous lessons so students should have familiarity with them and there should be less time developing the strategies (such as cooperative learning) so students can more accurately and independently utilize them.

The lesson for slope is often overlooked in many books and on-line outlines. It is included here because it uses the idea of linear and quadratic functions in a way that may help students grasp the idea of finding patterns and structure. The lesson is also outlined to assist students with research methods that they will be asked to complete with their final project. This allows the teacher to use ongoing progress monitoring and sequential prompting to help students to complete the assignment.
14. Graphing Quadratic Functions

Begin direct instruction and sequential prompting.

**Exercise #1:** Label the x and y axis and give the graph a scale then complete the following questions.

(a) Given $y = x^2$, determine, graph and label the
y-intercept: ___________________________
axis of symmetry: _______________________
vertex: _________________________________
max / min: _____________________________
$(+), (-), (0)$ discriminant: ____________
roots: _________________________________

(b) Complete the table below

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(c) Why is the middle value of the table the input $(0,0)$?

(d) What $y$ values are determined when you choose two values $|x|$ distance away from 0?

End direct instruction. Begin modeling, rehearsal and feedback.

**Exercise #2:** Determine the middle value of the table based on the quadratic given and complete the table.

(a) $y = x^2 - 2x - 8$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) $y = x^2 + 6x + 9$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
End modeling, rehearsal and feedback. Begin cooperative learning.

**Exercise #3:** Consider the quadratic \( y = x^2 + 5x - 14 \)

(a) Determine the middle value of the table based on the quadratic given and complete the table.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Why is there no x and y axis pre-made on the graph?

(d) Plot the points you determined from a.

(c) State the range of this function

(d) Over what domain interval is the function increasing?

(e) Label the axis of symmetry, the vertex, the roots and label the equation on the graph.

**Exercise #4:** Suppose you were given \( y = 7x^2 - 5x - 1 \). From completing the square or using the quadratic formula we can see that the roots are \( x = \frac{5 \pm \sqrt{53}}{14} \). Are there any integer values we could use for x?

Complete the table for these values.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Integrated Algebra

Name: ___________________________  Date: __________________

Independent Problems:


Part #2: The quadratic function f(x) has selected values shown in the table below

(a) What are the coordinates of the turning point?

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-2</td>
</tr>
</tbody>
</table>

(b) What is the range of the quadratic function?

(c) Challenge: Construct the equation of the quadratic equation in standard form. Hint: start with the vertex form.

Part #3: The quadratic function f(x) has selected values shown in the table below

(a) Given \( y = x^2 - x - 5 \), determine, graph and label the
y-intercept:
axis of symmetry:
vertex:
max / min:
(+) , (-) , (0) discriminant:
roots:

(b) Complete the table below

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multiple Choice: A quadratic function is partially given in the table below. Which of the following are the coordinates of its turning point?

(1) (0, 6)  
(2) (10, 2)  
(3) (3, 15)  
(4) (7, 1)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Part 5: A quadratic function $g(x)$ is shown partially in the table below. The turning point of the function has the coordinates (3, 8) Think about how outputs repeat in a quadratic function and answer the following.

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>g(x)</td>
<td>24</td>
<td>0</td>
<td>-6</td>
<td>-8</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Fill in the missing output values from the table.  
(b) What are the zeroes of the function?  
(c) What is this function’s $y$-intercept?  
(d) For the domain interval $1 \leq x \leq 7$, what is the range of the function?

Part 7: The height of an object that is traveling through the air can be well modeled by a quadratic function that opens downward. An object is fired upward and its height in feet above the ground is given by: $f(x) = -16x^2 + 64x + 80$ where the input, $x$, is the time, in seconds, the object has been in the air

(a) Using your calculator, sketch a graph of the object’s height for all times where it is at or above the ground.  
(b) What is its maximum height in feet?  
(c) At what time does it hit the ground?  
(d) Over what time interval is its height increasing.
15. Apply your knowledge.

Work with a partner for part 1.

Quiz #1: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/e/parabola_intuition_1

Score is recorded as completed without hints.

Quiz #2: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/e/graphing_parabolas_2

Students must get 3 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 3 questions is recorded at the top of the screen are the points they receive.

Green check = 5 points
light bulb (hints) = 2 points
x mark = 2 points
total = 15 points
16. Slope of a Quadratic Functions
Begin cooperative learning and peer tutoring.

Exercise #1: ONLINE: http://mathbitsnotebook.com/Algebra1/Quadratics/QDSlope.html

End cooperative learning and peer tutoring. Begin direct instruction.

Exercise #2: Consider the quadratic $y = x^2 + 4x + 4$

(a) Calculate the average rate of change

<table>
<thead>
<tr>
<th>points being used</th>
<th>average rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Graph the points on the graph provided.

(b) Is there a pattern developing regarding the average rate of change?

Exercise #2: Consider the quadratic whose graph is shown.

(a) Find the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

The average rate of change finds the slope of the line through the points (1,0) and (3,-4) as shown.
Since the average rate of change is negative, we know that the function values for the quadratic function, $y = f(x)$ are mostly decreasing on the interval from $x = 1$ to $x=3$. 
Independent Problems:

**Part #1:** Consider the function $f(x) = 3x^2 + 6x + 4$.

(a) Find the average rate of change of the function from $x = 1$ to $x = 4$.

(b) Sketch a graph of $f(x)$ and a graph of the line connecting the corresponding points at $x = 1$ and $x = 4$.

(c) What does the average rate of change of $f(x)$ from $x = 1$ to $x = 4$ calculated in part (a) represent in your picture in part (b)?

**Part #2:** Given the function $f(x) = x^2 + 4x - 1$, find the average rate of change from $x=1$ to $x=5$ and interpret what the average rate of change tells you about the function.

**Part #3:** Partner Work:

With your elbow partner, find the average rate of change for the following functions over the given interval. Then, answer the conclusion question.

(a) $f(x) = 3x + 5$ from $x=2$ to $x=3$

(b) $f(x) = x^2 + 4$ from $x=2$ to $x=3$

(c) $f(x) = 3^x$ from $x=2$ to $x=3$

(d) Write a conclusion (with no less than 4 complete sentences) regarding how you can use the average rate of change to compare the three functions on the given interval.
17. & 18. Applications & modeling

The applications lesson begins with cooperative learning and creating stations that the students need to revolve around in partners or small groups. During the process groups and partners can be interchanged to meet the needs of student learning. For the first part of this lesson the instructor may have to assist more with modeling and rehearsal to get students to complete the problems. The tasks are all contextual, and vary in applications.

In the second part of the lesson a strategy for creating a checklist is introduced. Students should be familiar with the strategy at this point and have little difficulty in assembling their own checklist. This will allow for greater independence through the rest of the problem set.

The modeling project may be distributed over the entire unit. It is broken into pieces to accommodate just that. For instance, in the first part students are asked to invent a product, or research one that can be made, distributed and sold in the school. Since this act of brainstorming may take some time it is best to introduce the idea at the beginning of the unit, with daily questions to see which students have arrived at a product. Within the project several types of technology are used. Utilizing peer tutoring throughout the lesson will help students with less proficiency develop their project with more capable peers. The ability to use the technology and create a successful presentation is also scaffolded in the previous lessons through the use of structured academic tasks.

The project is an authentic task, so answers need to be carefully studied and models can be changed to fit the needs of particular products. Because of the authenticity of the task students can develop their product and gain a great deal of exposure to sales, marketing and manufacturing through the task. If the product moves to a development phase students can look back on their predictions to see their accuracy of the model. This also allows them to adjust and redevelop the model to fit the actuality of sales.
17. Applications of Quadratic Functions

Stations: In small groups or partners go around to the different stations and apply your knowledge.

**Exercise #1:** Consider a rectangle whose area is 45 square feet. If we know that the length is one less than twice the width, then we would like to find the dimensions of the rectangle.

(a) If we represent the width of the rectangle using the variable \( w \), then write an expression for the length of the rectangle in terms of \( w \).

(b) Set up an equation that could be used to solve for the width, \( w \), based on the area.

(c) Solve the equation to find both dimensions. Why is one of the solutions for \( w \) not viable?

**Exercise #2:** A square has one side increased in length by two inches and an adjacent side decreased in length by two inches. If the resulting rectangle has an area of 60 square inches, what was the area of the original square? First, draw some possible squares and rectangles to see if you can solve by guess-and-check. Then, solve it algebraically.
Exercise #3: There are two rational numbers that have the property that when the product of seven less than three times the number and one more than the number is found it is equal to two less than ten times the number. Find the two rational numbers that fit this description.

Exercise #4: Find all sets of consecutive integers such that their product is eight less than ten times the smaller integer.

Exercise #5: Brendon claims that the number five has the property that the product of three less than it with one more than it is the same as the three times one less than it. Show that Brendon’s claim is true and algebraically find the other number for which this is true.
The Five Step method for solving word problems

Step 1: Let \( x \) (or some other letter) represent the unknown quantity.
Step 2: Translate the verbal expression to mathematical symbols and form an equation.
Step 3: Solve this equation.
Step 4: Check the solution by substituting the result into the equation found in step 2.
Step 5: Write a conclusion.

**Exercise #6:** A producer of personal computer mouse covers determines that the number \( N \) of covers sold is related to the price \( x \) of a cover by \( N = 35x - x^2 \): At what price should the producer price a mouse cover in order to sell 216 of them?

**Exercise #7:** It is estimated that \( t \) years from now the population of a particular city will be \( P = t^2 - 24t + 96,000 \): How many years from now will the population be 95,865?

**Exercise #8:** The length of a rectangle is 4 inches more than twice its width. The area is 30 square inches. Find the dimensions (length and width).
Exercise #9: The product of two consecutive integers is 156. Find them.

Exercise #10: A box with no top and a square base is to be made by cutting out 2-inch squares from each corner and folding up the sides of a piece of a square cardboard. The volume of the box is to be 8 cubic inches. What size should the piece of cardboard be?
Exercise #11: A study of the air quality in a particular city by an environmental group suggests that \( t \) years from now the level of carbon monoxide, in parts per million, in the air will be

\[ A = 0.3t^2 + 0.1t + 4.2 \]

(a) What is the level, in parts per million, of carbon monoxide in the air now?

(b) How many years from now will the level of carbon monoxide be at 8 parts per million?

Exercise #12: A contractor is to pour a concrete walkway around a swimming pool that is 20 feet wide and 40 feet long. The area of the walkway is to be 544 square feet. If the walkway is to be of uniform width, how wide should the contractor make it?
Independent Problems:

**Part #1:** The product of two consecutive positive even integers is 14 more than their sum. Set up an equation that can used to find the two numbers and solve it.

**Part #2:** The length of a rectangle is 4 less than twice the width. The area of the rectangle is 70. Find the width, w, of the rectangle algebraically. Explain why one of the solutions for w is not viable.

**Part #3:** Two sets of three consecutive integers have a property that the product of the larger two is one less than seven times the smallest. Set up and solve an equation that can be used to find both sets of integers.
Part #4: A curious pattern occurs in a group of people who all shake hands with one another. It turns out that you can predict the number of handshakes that will occur if you know the number of people.

If we are in a room of 5 people, we can determine the number of handshakes by this line of reasoning:

The first person will shake 4 hands (she won’t shake her own).
The second person will shake 3 hands (he won’t shake his own or the hand of the first person, they already shook).
The third person will shake 2 hands (same reasoning).
The fourth person will shake 1 hand (that of the fifth person).
The fifth person will shake 0 hands. So there will be a total of \(1+2+3+4=10\) handshakes

(a) Determine the number of handshakes, \(h\), that will occur for each number of people, \(n\), in a particular room.

<table>
<thead>
<tr>
<th>(n) (people)</th>
<th>Calculation</th>
<th>(h) (handshakes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(1 + 2 + 3 + 4 = 10)</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Prestel poses the formula \(\frac{n(n-1)}{2}\). Determine if the formula is correct in this instance.

(c) Assuming Prestel’s formula is correct, algebraically determine the number of people in a room if there are 66 handshakes that occur.
Part #5: A manufacturer of cloth personal computer dust covers notices that the number $N$ of covers sold is related to the price of covers by $N = 30x - x^2$: At what price should the manufacturer price the covers in order to sell 216 of them?

Part #6: The length of a rectangle is 3 feet more than twice its width. The area is 14 square feet. Find the dimensions.

Part #7: The area of a triangle is 24 square meters. The base is 2 meters longer than the height. Find the base and height. The formula for the area of a triangle is $A = \frac{1}{2} bh$. 
Part #8: The product of two consecutive integers is 210. Find them

Part #9: Four is added to an integer and that sum is tripled. When this result is multiplied by the original integer, the product is –12. Find the integer.

Part #10: A box with no top and a square base is to be made by cutting 3-inch squares from each corner and folding up the sides of a piece of cardboard. The volume of the box is to be 48 cubic inches. What size should the piece of cardboard be?
Part #11: A study of the air quality in a particular city by an environmental group suggests that t years from now the level of carbon monoxide, in parts per million, in the air will be
\[ A = 0.2t^2 + 0.1t + 5.1 \]

(a) What is the level, in parts per million, now?

(b) How many years from now will the level of carbon monoxide be at 8 parts per million?
Round to the nearest tenth.

Part #12: A contractor is to pour a concrete walkway around a swimming pool that is 15 feet wide and 25 feet long. The area of the walkway is to be 276 square feet. If the walkway is to be of uniform width, how wide should the contractor make it?
18. Project: Modeling with Quadratic Functions

Part #1: Goal: Students will research, model, create and sell a product for their school fund raising activities. They will develop the maximum profit through the use of quadratics and present their product to the sales committee explaining why this product will be viable and successful.

Watch how professionals market their products for financing:
http://www.youtube.com/watch?v=7C0LP1ehmVk

Directions: Research the product you want to sell for fund raising activities. Give your product a name. Make a guess as to what your customer would pay for this product. Find all the components you would need to make your product and complete the information below.

My Product Name: ________________________________________________________________

Estimated Product cost to consumer: $ ______________________________________________

Directions: Create a spreadsheet labeled “components” to reflect the table below.

Components:
A: Enter the amount needed for one product under QTY
B: Enter the unit cost as exactly one component times the QTY
C: Enter the amount sold by manufacturer/retail in one package for PKG Qty
D: Enter the cost of the package from the manufacturer/retail under total $

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>QTY</th>
<th>Unit cost</th>
<th>PKG Qty</th>
<th>Total $</th>
</tr>
</thead>
</table>
Part #2: Goal: make estimations on your initial costs based on the product you want to sell.

Estimations: From the spreadsheet in part 1 calculate:

A: The minimum number of products to be constructed from PKG Qty: __________________________
   How many components would not be used? ____________  Excess $ ___________

B: The Maximum number of products to be constructed from PKG Qty: __________________________
   How many components would not be used? ____________  Excess $ ___________

C: What is the total cost for one product? ________________  This is the cost per item.

Initial Cost:

Research a definition for initial cost and complete below:

Definition: ______________________________________________________________________________
_______________________________________________________________________________________

Spread sheet application:
Add a new tab on your spreadsheet to reflect initial costs:

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Construct your product to use the lowest excess cost available. Add this amount to the Initial cost.

Your Initial cost $ _____________________
Part #3: Goal: Develop an understanding for demand curves and modeling these using graphing software.

Demand Curve:

An example:

$700,000 for manufacturing set-up costs, advertising, etc
$110 to make each bike

Based on similar bikes, you can expect sales to follow this "Demand Curve":

- Unit Sales = 70,000 - 200P

Where "P" is the price.

For example, if you set the price:
- at $0, you would just give away 70,000 bikes
- at $350, you would not sell any bikes at all.
- at $300 you might sell \(70,000 - 200 \times 300 = 10,000\) bikes

Developing a demand curve usually entails understanding previous sales or estimating projection sales by market demands. The coordinate points would be an estimated range from (free item, max units sold) and (max price – no one would pay, min units sold – usually zero)

\[ y = mx + b, \ b \text{ is the maximum units sold at zero, calculate m (slope) from the max,min point. (rise/run)} \]

The linear function of the demand curve is NOT an absolute. It is a starting point to develop and model the ideal product sales. Unit sales will often be tweaked as other developments arise.

Graphing software application: Geogebra
Model your Demand curve

From this model present the linear equation of the unit sales.

What is your Unit Sales: ________________________________________________________________

Use “p” as the price.
Part #4: Goal: Understand vocabulary, and set up of various types of business applications.

Developing modeling skills.

Read the following information about Business Applications:

Many business contexts can be modeled with quadratic functions. This is because the expressions representing price (price per item), the cost (cost per item), and the quantity (number of items sold) are typically linear. The product of any two of those linear expressions will produce a quadratic expression that can be used as a model for the business context.

The variables used in business applications are not as traditionally accepted as variables are in physics applications, but there are some obvious reasons to use c for cost, p for price, and q for quantity (all lowercase letters). For total production cost we often use C for the variable, R for total revenue, and P for total profit (all uppercase letters).

Business Application Vocabulary:

**Unit Price (Price per Unit):** The price per item a business sets to sell its product, sometimes represented as a linear expression.

**Quantity:** The number of items sold, sometimes represented as a linear expression.

**Revenue:** The total income based on sales (but without considering the cost of doing business).

**Unit Cost (Cost per Unit) or Production Cost:** The cost of producing one item, sometimes represented as a linear expression.

**Profit:** The amount of money a business makes on the sale of its product. Profit is determined by taking the total revenue (the quantity sold multiplied by the price per unit) and subtracting the total cost to produce the items (the quantity sold multiplied by the production cost per unit): Profit = Total Revenue − Total Production Cost.

The following business formulas will be used in this next examples:

Total Production Costs = (cost per unit)(quantity of items sold)

Total Revenue = (price per unit)(quantity of items sold)

Profit = Total Revenue − Total Production Costs

Quizlet application:
Create note cards for the vocabulary words on quizlet.com, then test your knowledge.
Part #5: Goal: Apply knowledge of business application to word problems.

Exercise #1: A theater decided to sell special event tickets to benefit a local charity at $10 per ticket. The theater can seat up to 1,000 people and they expect to be able to sell all 1,000 seats for the event. To maximize the revenue for this event, a research company volunteered to do a survey to find out if they could increase the ticket price without losing revenue. The results showed that for each $1 increase in ticket price, 20 fewer tickets will be sold.

a. Let x represent the number of $1.00 price-per-ticket increases. Write an expression to represent the expected price for each ticket:

b. Use the survey results to write an expression representing the possible number of tickets sold.

c. Using x as the number of $1-ticket price increases and the expression representing price per ticket, write the function, R(x), to represent the total revenue in terms of the number of $1-ticket price increases.

d. How many $1-ticket price increases will produce the maximum revenue? (i.e., what value for x produces the maximum R?)
e. What is the price of the ticket that will provide the maximum revenue?

f. What is the maximum revenue?

g. How many tickets will the theater sell to reach the maximum revenue?

h. How much more will the theater make for the charity by using the results of the survey to price the tickets than they would had they sold the tickets for their original $10 price?
Exercise 2: Amazing Photography Studio takes school pictures and charges $20 for each class picture. The company sells an average of 12 class pictures in each classroom. They would like to have a special sale that will help them sell more pictures and actually increase their revenue. They hired a business analyst to determine how to do that. The analyst determined that for every reduction of $2 in the cost of the class picture, there would be an additional 5 pictures sold per classroom.

a. Write a function to represent the revenue for each classroom for the special sale.

b. What should the special sale price be?

c. How much more will the studio make than they would have without the sale?
Integrated Algebra

Group work Questions:

a. What is the relevance of the vertex in business applications?

b. Katrina developed an app (application) that she sells for $5 per download. She has free space on a website that will let her sell 500 downloads. According to some research she did, for each $1 increase in download price, 10 fewer apps are sold. Determine the price that will maximize her profit.
Part #6: Goal: Complete your product analysis for your presentation.

Directions: Complete the following information from the data you have gathered and by the definitions.

________________________________________________ initial cost.

________________________________________________ cost per item.

________________________________________________ unit sales.

Calculate:

________________________________________________ dollar sales = (unit sales)(price=p)

________________________________________________ cost = ( initial cost + (cost per item)(unit sales)

________________________________________________ profit = dollar sales – costs

Graphing software application: Geogebra
Model your profit

Profit is your quadratic. Graph the quadratic showing the roots, vertex, and line of symmetry from steps below:

Maximum profit (1):
(a) Find the roots of the quadratic using the quadratic equation:

(b) Find the average of the roots: (This is the sales price.)
(c) Find the axis of symmetry using $x = -\frac{b}{2a}$ (this should be the same number as above)

(d) Evaluate the sales price in the profit quadratic. (This is maximum profit)

(e) Evaluate the sales price in the cost. (This is minimum cost at maximum profit)

(f) Evaluate the sales price in the unit sales. (This is maximum sales at maximum profit)

(g) How many units need to be sold? 

(h) How much will this cost? 

(i) How much profit will be made? 

Part #7: Goal: Create a presentation using the standards of the rubric.

Video: Product analysis 7:40

Create a presentation Using the product analysis, graphs and data from the software above. Describe your product, why this product is profitable, why it should be the next product made, and how you can validate this.

Presentation software application: kizoa.com
Use kizoa or other presenting software to showcase your product.
Create a presentation Using the product analysis, graphs and data from the software above. Describe your product, why this product is profitable, why it should be the next product made, and how you can validate this.

**Rubric:** Every class member is expected to complete a rubric for each presenter. Grades are determined both on your presentation and level at which you participate in scoring other class members, making positive critiques, and participating if asked.

**Presenters:**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Points/Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Presentation Timing:</strong></td>
<td></td>
</tr>
<tr>
<td>Time Frame</td>
<td></td>
</tr>
<tr>
<td>Presentation is less than</td>
<td></td>
</tr>
<tr>
<td>minimum time allotted</td>
<td></td>
</tr>
<tr>
<td>Presentation is more than</td>
<td></td>
</tr>
<tr>
<td>maximum time allotted</td>
<td></td>
</tr>
<tr>
<td>Presentation falls outside</td>
<td></td>
</tr>
<tr>
<td>required time frame with</td>
<td></td>
</tr>
<tr>
<td>purpose.</td>
<td></td>
</tr>
<tr>
<td>Presentation falls within</td>
<td></td>
</tr>
<tr>
<td>required time frame</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>Content opening</td>
<td></td>
</tr>
<tr>
<td>Does not introduce self</td>
<td></td>
</tr>
<tr>
<td>Introduces self briefly</td>
<td></td>
</tr>
<tr>
<td>Introduces self somewhat</td>
<td></td>
</tr>
<tr>
<td>Introduces self thoroughly</td>
<td></td>
</tr>
<tr>
<td>Overview</td>
<td></td>
</tr>
<tr>
<td>No overview</td>
<td></td>
</tr>
<tr>
<td>Provides adequate overview of topic</td>
<td></td>
</tr>
<tr>
<td>Overview of topic with some points missing</td>
<td></td>
</tr>
<tr>
<td>Provides thorough overview of topic</td>
<td></td>
</tr>
<tr>
<td><strong>Content of Presentation</strong></td>
<td></td>
</tr>
<tr>
<td>Mathematical Concepts</td>
<td></td>
</tr>
<tr>
<td>Students shows no understanding of mathematical concepts within the presentation</td>
<td></td>
</tr>
<tr>
<td>Students are visibly uncomfortable with the mathematical concepts of the presentation</td>
<td></td>
</tr>
<tr>
<td>Students are at ease with the mathematical concepts of the presentation but lack a deep conceptual understanding</td>
<td></td>
</tr>
<tr>
<td>Students demonstrate a complete and comprehensive understanding of the mathematical concepts in the presentation</td>
<td></td>
</tr>
<tr>
<td>Mathematical Procedures</td>
<td></td>
</tr>
<tr>
<td>Student has difficulty explaining mathematical procedures.</td>
<td></td>
</tr>
<tr>
<td>Student explains mathematical procedures without difficulty</td>
<td></td>
</tr>
<tr>
<td>Student explains mathematical procedures without difficulty and provides partial explanations for why mathematical procedures are valid or appropriate.</td>
<td></td>
</tr>
<tr>
<td>Student explains mathematical procedures without difficulty and provides full explanations for why mathematical procedures are valid or appropriate.</td>
<td></td>
</tr>
<tr>
<td>Mathematical Modeling</td>
<td></td>
</tr>
<tr>
<td>Student uses little or no mathematical models from project.</td>
<td></td>
</tr>
<tr>
<td>Student uses some mathematical models from project</td>
<td></td>
</tr>
<tr>
<td>Students uses most mathematical models from project</td>
<td></td>
</tr>
<tr>
<td>Student uses all mathematical models from project</td>
<td></td>
</tr>
<tr>
<td>Technology Integration</td>
<td></td>
</tr>
<tr>
<td>Student incorporates no other types of technology/software into presentation</td>
<td></td>
</tr>
<tr>
<td>Student incorporates one or two types of technology/software into presentation</td>
<td></td>
</tr>
<tr>
<td>Student incorporates a couple of types of technology/software into presentation</td>
<td></td>
</tr>
<tr>
<td>Student incorporates several types of technology/software into presentation</td>
<td></td>
</tr>
<tr>
<td>Questions</td>
<td></td>
</tr>
<tr>
<td>Student cannot answer questions about topic</td>
<td></td>
</tr>
<tr>
<td>Student answers only rudimentary class questions with explanations and elaborations</td>
<td></td>
</tr>
<tr>
<td>Student answers most class questions with sufficient explanations and elaborations</td>
<td></td>
</tr>
<tr>
<td>Student answers all class questions with explanations and elaborations</td>
<td></td>
</tr>
<tr>
<td>Organization</td>
<td></td>
</tr>
<tr>
<td>Audience cannot understand presentation because there is no sequence of information.</td>
<td></td>
</tr>
<tr>
<td>Audience has difficulty following presentation because student jumps around.</td>
<td></td>
</tr>
<tr>
<td>Student presents information in logical sequence which audience can follow.</td>
<td></td>
</tr>
<tr>
<td>Student presents information in logical, interesting sequence which audience can follow.</td>
<td></td>
</tr>
<tr>
<td>Completeness of Content</td>
<td></td>
</tr>
<tr>
<td>One or more points left out</td>
<td></td>
</tr>
<tr>
<td>Majority of points glossed over</td>
<td></td>
</tr>
<tr>
<td>Majority of points covered in depth; some points glossed over</td>
<td></td>
</tr>
<tr>
<td>Thoroughly explains all points</td>
<td></td>
</tr>
</tbody>
</table>
**Vocal Skills**

<table>
<thead>
<tr>
<th>Enthusiasm</th>
<th>Shows absolutely no interest in topic presented</th>
<th>Shows some negativity toward topic presented</th>
<th>Occasionally shows positive feelings about topic during presentation</th>
<th>Demonstrates a strong positive feeling about topic during all/most of presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elocution</td>
<td>Mumbles or speaks too quietly for a majority of audience to hear</td>
<td>Speaks in a low voice and enunciates so that audience members have difficulty hearing presentation</td>
<td>Uses clear voice and enunciates properly so that audience members can hear presentation most of the time</td>
<td>Uses clear voice and enunciates properly so that all audience members can hear presentation</td>
</tr>
</tbody>
</table>

**Nonverbal Skills**

<table>
<thead>
<tr>
<th>Eye Contact</th>
<th>Does not attempt to look at audience at all, reads notes the entire time</th>
<th>Displays minimal eye contact with audience, while reading mostly from notes</th>
<th>Occasionally looks at someone or some groups during presentation, but returns to notes</th>
<th>Constantly looks at someone or some groups at all/most times during presentation, while seldom looking at notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Language</td>
<td>No movement or descriptive gestures; Has an deadpan expression; Slumps or sits</td>
<td>Very little movement or descriptive gestures; Occasionally has deadpan expression; Occasionally slumps</td>
<td>Made movement or descriptive gestures that enhanced presentation; Rarely has deadpan expression; Rarely slumps</td>
<td>Movements seem fluid; Displays appropriate expression (never deadpan); Stands up straight with both feet on the floor</td>
</tr>
</tbody>
</table>

| Poise | Tension or nervousness is obvious; has trouble recovering from mistakes | Displays mild tension; has trouble recovering from mistakes | Makes minor mistakes, but quickly recovers from them, with little or no tension | Displays relaxed, self-confident nature about mistakes |

**Creativity**

<table>
<thead>
<tr>
<th>Use of visuals</th>
<th>Students use no visuals</th>
<th>Students occasionally use visuals that rarely support the presentation and audience understanding</th>
<th>Students use visuals that are related to the presentation but did not completely support audience understanding</th>
<th>The visuals used supported audience understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use of audio</td>
<td>Students use no audio</td>
<td>Students use of audio is completely distracting from presentation.</td>
<td>Students use of audio did not completely support audience understanding, somewhat distracting</td>
<td>The audio used supported audience understanding, engaged audience.</td>
</tr>
</tbody>
</table>

**Comments:** Please add any additional comments that may aide the presenter in their next project.

**Your Name:**
Chapter 4: Discussion

The research for the strategies in this thesis are widely explained and well documented about their success on improving academic performance. The purpose of this thesis is not to explain the strategies, or to provide the strategies levels of successes with students with EBD. From above, the purpose of this thesis is to provide an algebra unit for teachers with students with EBD that is grounded in Vygotsky’s (1978) Zone of Proximal Development (ZPD). Working with algebra students with EBD in their ZPD by incorporating empirically based teaching methods to develop the curriculum and transform instruction and knowledge from a interpersonal process into an intrapersonal one (Vygotsky, 1998). Such instruction can shift the actual developmental level to the level of potential development; ie. what a student can do with assistance today is what they can do independently tomorrow.

One piece of lesson planning the reader may have noted that was omitted from general lesson plans is the timing of the lesson. Each lesson was not designed within a particular time frame, rather each lesson was designed to develop proficiency for a particular domain specific skill. Also, within the lessons are broken into segments for developing either a particular domain specific skill or working on problem decomposition. Further, while not comprehensive, it was noted what skills may be lacking, or what skills may need to be re-taught or spent time on in remedial education time such as AIS.

Each lesson has incorporated some type of explicit direct instruction either through direct instruction or modeling, rehearsal and feedback. For students with EBD the benefits of explicit direct instruction hold true for establishing simple routines and content-based lessons. (Gunter et al., 2002; Salend & Sylvestre, 2005) This strategy not only assists with academic gain but behavior management as well. These strategies allow the teacher with opportunities to provide numerous opportunities for scaffolded questions and sequential prompting. This allows teachers to pose questions that would ensure responses with at least 80% accuracy before having students move on to independent practice.
(which is recommended). (Niesyn, 2009) Having students give correct responses not only leads to increased desirable behaviors (Barton-Arwood, Wehby, & Falk, 2005; Conroy & Davis, 2000; Gunter et al., 2002; Gunter et al., 2000; Gunter & Reed, 1997) but increased participation and an increase in praise which also increases the likelihood of desirable behaviors and further participatory responses. (Landrum et al., 2003; Niesyn, 2009)

While each lesson has a large problem set for independent work it does not need to be followed that students will complete all the problems, or that the problems need to be presented in their current form (on a worksheet). In teaching this unit, students would often complete more homework and choose to complete more when given the choice for choosing their own homework. In particular, for lesson 7. completing the square, only one column on the second from last page, and the last page was going to be assigned because students had demonstrated a high accuracy rate in class, however, when given the choice of which problems to complete, most had finished all the problems. While it has been shown that giving shorter assignments are positively associated with relieving students’ stress (Conroy & Davis, 2000; Gunter et al., 2000; Weaster, 2004) it may be more opportune to have a large sample set of problems and allow students to choose their own homework sets.

While in this setting each student has a tracked academic goal with consistent feedback to help students self management their progress, the checklists that students create in the lessons is another means of developing self mediated interventions. Self mediated interventions have demonstrated their ability to produce large academic gains for students with EBD across subject areas. (Ryan, Pierce & Mooney, 2008) Self-management is a viable instructional strategy to teach academic skills (Callahan et al., 1998; Glomb & West, 1990). Consequently, self-management skills may be used to enhance metacognitive and strategy-based instruction. Another reason why the timing of the lesson was omitted was because these skills take time to develop. However once completed, students become increasingly
faster with creating and using them.

Often with meeting the student's needs for education each lesson for each class may be slightly different. This is why it is important to understand that there is not always one specific strategy that will be the defining factor, but for each lesson a multitude of strategies will be imposed with some working concurrently to develop students to their potential levels, and often these strategies will swap from one problem type to the next, or may not be introduced as part of the lesson at all, but rather a classroom philosophy. For instance, in this class, peer tutoring is a common practice for students both at lunch time and during AIS (which is school wide for the last period of the day). Peers who participate are assigned to assist other peers outside of class in return for extra credit, additional activities, or prizes. Peer tutoring is one of the most frequently cited instructional strategies for decreasing negative behavior and increasing positive behavior for students with EBD. (Niesyn, 2009) Peer tutoring has been reported to improve both academic and behavioral deficits as well as student engagement and response rates for all students, including those with special needs (Barton-Arwood et al., 2005; Gunter et al., 2000; Landrum et al., 2003). Therefore while cooperative learning and class wide peer tutoring (both types of peer tutoring) are listed in the lessons themselves, peer tutoring occurs outside the lesson development.

One of the most critical elements of this unit is the objectives. This is the main element that allows the teacher to understand the actual developmental level and make key instructional choices to moving the students to the student's potential developmental level (ie: grade level). Further it acts as the student's guide for developing and increasing self-management skills throughout the unit as well as a more fluid guide for developing lessons to meet the students academic and behavioral needs. Utilizing the strategy for structured academic tasks allows for the objectives to be evaluated consistently and for other objectives to be incorporated throughout the unit. For instance, if a class struggles with group
work it may be advantageous to develop the objective “I can work appropriately with a partner” and outline exactly what that looks like either through modeling, rehearsal, and feedback, peer tutoring or reinforcement, or even self-monitoring / self-monitoring checklists, or a combination of strategies.

The rubrics and projects often need to be scaffolded throughout the year as well as through the unit. Often, it is advantageous to present a general rubric (as in lesson 5) and pinpoint where students need more or less help in areas to develop a presentation and refine these categories (as in lesson 18) to showcase both strengths and weaknesses. For example, in the rubric for lesson 5 the general category is about delivery with broad references to vocal skills and non verbal skills, but in lesson 10 each of these skills are broken into eye contact, body language, poise, enthusiasm and elocution; with more specific definitions for what is required. In some classes it may be necessary to have a rubric for audience members with categories for raising hands, speaks when called upon, makes appropriate comments, body gestures, and other categories as needed. Students who have more exposure to different types of tasks and have the ability to successively demonstrate growth in each category will generally put forth more effort for each category as they are presented.

In conclusion, meeting the goals for teaching students with EBD is often the most challenging for educators because of the monumental challenges they face both in academics and with behaviors. Further, while most curriculum is written for students at grade level it is often up to teachers of students with EBD to piece together both the curriculum and the strategies that work effectively from an overwhelming variety of sources usually with limited time and without an overview for constructing an entire lesson or unit, much less a entire curriculum year for students. Hopefully through this unit teachers of students with EBD will be able to have a more developed resource for instructing mathematics and with giving their students a more successful outcome both in school and as young adults in the community.


Coutinho, M. J. (1986). Reading achievement of students identified as behaviorally disordered at the secondary level. Behavioral Disorders, 11, 200-207.


Engageny.org Modules. (2012). New York State P-12 Common Core Learning Standards for Mathematics | EngageNY. [online] Available at: https://www.engageny.org/resource/high-


Fuchs, L. S., Fuchs, D., & Karns, K. (2001). Enhancing kindergartners’ mathematical development:


Harris, L. (2010). Delivering, modifying or collaborating? Examining three teacher conceptions of how to facilitate student engagement. Teachers & Teaching, 16(1), 131-151.


Psychologists.


Practice, 13(2), 1045-1054.


Disorders. Psychology In The Schools, 49(10), 975-987. doi:10.1002/pits.21645


Nctm.org, (2011). Math Common Core Coalition. [online] Available at:


Lesson Answer Keys
1. Pre-Assessment

1) Factoring is the opposite of: The distributive property

2) Circle each linear equation. (In bold)
   a. $2x-9y+3=0$
   b. $x=27+3$
   c. $y=x^3$
   d. $17=y$
   e. $4+y=2x-3x^2$
   f. $y=|x|$

3) What is a quadratic? Ans will vary: An equation of degree two. Students may write a quadratic in standard form, ie: $f(x)=ax^2+bx+c$. A polynomial with degree 2.

4) Which of the following products is equivalent to the trinomial $x^2 - 5x - 24$?
   a. $(x-12)(x-2)$
   b. $(x-8)(x+3)$
   c. $(x+12)(x-2)$
   d. $(x+8)(x-3)$

5) Factor completely: $2x^2-18$
   $=2(x^2-9)$
   $=2(x-3)(x+3)$

6) What is the GCF of 24 and 36? 12

7) What is the LCM of 24 and 36? 72
8) Answer the following questions, given: \( m = 12x^3 + 18x^5 + 27 - 3x^4 - 6x^2 \)

   a. How many terms does \( m \) have? 5 showing, but 6 since \( x = 0 \)
   b. Is \( m \) ordered correctly? no
   c. If not (part b), how could you order it? \( 18x^5 - 3x^4 + 12x^3 - 6x^2 + 27 \)
   d. What could you factor from \( m \)? 3
   e. What are the coefficients of \( m \)? 18, -3, 12, -6

9) What is a parabola? Ans will vary: The curve or shape of the quadratic function.

10) What is the difference between the domain of \( y = x - 2 \) and \( [x = \{-1, 0, 1, 2, 3, 4, 5, 6\}; y = \{1, 2, 3, 4, 5, 6, 7, 8\}] \)
    One is infinite, one is finite, they would be represented in different forms, ie. Interval or set notation.

11) List as many words that you know for the following symbols:

<table>
<thead>
<tr>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition, altogether, both, in all, sum, total, increase by, more than, raise, combine, together</td>
<td>subtraction, minus, less than, decreased by, difference, reduce, dropped, diminished</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>*</th>
<th>/</th>
</tr>
</thead>
<tbody>
<tr>
<td>times, multiplication, every, at this rate, of, per, area, by, volume, product, twice, doubled, tripled</td>
<td>division, ratio, quotient, split, cut, average, per, part</td>
</tr>
</tbody>
</table>
12) What is the slope of a line? It's the rate of change.

13) How do you determine what the slope of a line is? Use slope formula from two points, change of y over change of x; Graph it and determine change of y over change of x.

14) Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

From NYS Regents Common core exam 08/14

$2x^3 + 17x^2 + 25x - 50$ and correct work is shown.
15) Let \( f(x) = -2x^2 \) and \( g(x) = 2x - 4 \). On the set of axes below, draw the graphs of \( y = f(x) \) and \( y = g(x) \). From NYS Regents Common core exam 08/14

Using this graph, determine and state all values of \( x \) for which \( f(x) = g(x) \).

Both functions are graphed correctly, and 2 and 1 are stated.
2. Introduction to quadratic Functions and equations.

Begin think-pair-share (different groups for each section)

A quadratic function is a type of a polynomial with degree 2
The domain is: the x values (independent variables)
The codomain is: possible y values
The range is: actual y values (dependent variables)

Exercise #1: Read the definition above and answer the following questions.

(a) Why is it important for the leading coefficient to be nonzero?

It would not be a quadratic

(b) Circle the choices below that are quadratic equations.

\[ y = x^2 - 3 \]
\[ y = -2x^2 + 10x^3 - 4 \]
\[ y = x^2 + \sqrt{x} + 7 \]
\[ y = 10 - x^2 \]

(c) Given the quadratic equation \( y = 14 - 3x^2 + 7x - 4 \) write it in standard form and state the value for the leading coefficient.

\[ y = 3x^2 + 7x + 10 \]
leading coefficient = 3

(d) If \( f(x) = 2x^2 - 3x + 1 \), then find the value of \( f(-2) \). What point must lie on this quadratic equation?

\[ f(x) = 8 + 6 + 1 = 15 \]
\( (-2, 15) \)

A parabola is: the curve (or shape) of the quadratic
The y-intercept is: where the parabola crosses the y line
The vertex is: the turning point (maximum)(minimum)

End think-pair-share. Begin modeling, rehearsal, and feedback with guiding questions.

<table>
<thead>
<tr>
<th></th>
<th>y-intercept</th>
<th>vertex</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>(3, -9)</td>
<td>(-(\infty), (\infty))</td>
<td>[-9, (\infty)]</td>
</tr>
</tbody>
</table>
Exercise #2: Look at the graphs below and state the y-intercept and vertex of each graph.

<table>
<thead>
<tr>
<th>y-intercept</th>
<th>y-intercept</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>vertex</th>
<th>vertex</th>
<th>vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0, 0 )</td>
<td>( -3, 9 )</td>
<td>( -1, -6 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain</th>
<th>Domain</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-∞, ∞)</td>
<td>(-∞, ∞)</td>
<td>(-∞, ∞)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range</th>
<th>Range</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 0, ∞ )</td>
<td>( -∞, 9 ]</td>
<td>[ -6, ∞ )</td>
</tr>
</tbody>
</table>

Exercise #3: Determine if the equation is a quadratic, and if so, determine the coefficients and the constant of each quadratic. Consider 0=ax^2+bx+c

(a) 3x^2+2x-1= 0
    a = 3 , b = 2 , c = -1

(b) 5x^2+8x = 0
    a = 5 , b = 8 , c = 0

(c) x^2+7 = 0
    a = 1 , b = 0 , c = 7

(d) -6x^2=2x+3
    a = -6 , b = 2 , c = 3

(e) 3x+2=0
    a = _________ , b = _________ , c = _________

(f) 8 x^2 + 3 \frac{3}{x} - 5 = 0
    a = _________ , b = _________ , c = _________

Exercise #4: What can you conjecture about the value of the constant c and the y-intercept from the examples above? The y intercept is the constant value when x=0
Exercise #5: What is the y-intercept given the quadratic function:

(a) \(3x^2 + 4x - 7 = y\)  
\[-7\]

(b) \(-2 + 7x^2 + 4x + y = 0\)  
\[2\]

(c) \(2y + 16x^2 = 12x - 20\)  
\[-10\]

(d) \(4y = 36x^2\)  
\[0\]

End partners. Begin modeling, rehearsal, and feedback and direct instruction with guiding questions.

<table>
<thead>
<tr>
<th>Zero Factor Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two numbers (a) and (b) are multiplied together and the resulting product is zero, then at least one of the numbers must be zero. If (ab = 0), then (a=0) or (b=0) or both (a=0) and (b=0).</td>
</tr>
</tbody>
</table>

Exercise #6: Use the zero factor property to solve each equation.

(a) \(2x = 0\)  
x = 0

(b) \(-9x^2 = 0\)  
x = 0  \(\text{then} \ x = 0\)

(c) \(x - 1 = 0\)  
x = 1

(d) \(x(x + 8) = 0\)  
x = 0  \(\text{and} \ x = -8\)

(e) \((x + 4)(x + 3) = 0\)  
x = -4  \(\text{and} \ x = -3\)

(f) \((x + 11)(4x - 5) = 0\)  
x = -11  \(\text{and} \ x = 5/4\)

(g) \(6a - 4 = 0\)  
a = 4

(h) \((1 - 9x)(-11x - 3) = 0\)  
x = 1/9  \(\text{and} \ x = -3/11\)

(i) \((7a - 2)^2 = 0\)  
a = 2/7
Independent Problems

Part #2: Determine which of the following is a quadratic equation, and if so, determine the coefficients and the constant of each quadratic.

(a) \(3x = 2\)
\[a = \underline{\hspace{1cm}}\ , \ b = \underline{\hspace{1cm}}\ , \ c = \underline{\hspace{1cm}}\]

(b) \(3x^2 + 8x - 2 = -8x - 2\)
\[a = 3\ , \ b = 16\ , \ c = 4\]

(c) \(6(2)^3 = 7\)
\[a = \underline{\hspace{1cm}}\ , \ b = \underline{\hspace{1cm}}\ , \ c = \underline{\hspace{1cm}}\]

(d) \(\frac{2}{x} - 5x^2 + 6 = 4x^2 + 8\)
\[a = \underline{\hspace{1cm}}\ , \ b = \underline{\hspace{1cm}}\ , \ c = \underline{\hspace{1cm}}\]

(e) \(12x^2 + 8x - 16 = 0 - 8\)
\[a = 12\ , \ b = 8\ , \ c = -8\]

(f) \(9x + 2x - 1x + 3x^2 = 0\)
\[a = 3\ , \ b = 10\ , \ c = 0\]

Multiple choice: Which of the following is a quadratic function?

(a) \(y + 3x = 2\)  
(b) \(y = x^2 - 3\)

(c) \(y = x^3 - 5x + 3\)  
(d) \(y - \frac{7}{x} = -x^2 + 6\)

Part #3: Write the following in standard form.

(a) \(4x^2 + 3x + 1 = -8x - 2 + y\)  
Standard form: \(y = 4x^2 + 11x + 3\)

(b) \(3y = 18x^2 - 3x + 21\)  
Standard form: \(y = 6x^2 - 1x + 7\)

(c) \(-8 - 3x^2 + 6x + y = 0\)  
Standard form: \(y = 3x^2 - 6x + 8\)
Part #4: Use the zero factor property to solve each equation.

(a) $4m = 0$
    $m = 0$

(b) $3(k + 7) = 0$
    $k = -7$

(c) $-5(x + 4) = 0$
    $x = -4$

(d) $y(y - 1) = 0$
    $y = 1$ and $y = 0$

(e) $(y - 4)(y - 8) = 0$
    $y = 4$ and $y = 8$

(f) $(y + 6)(2y + 1) = 0$
    $y = -6$ and $y = -\frac{1}{2}$

(g) $(6m + 5)(11m - 6) = 0$
    $m = -\frac{5}{6}$ and $m = \frac{6}{11}$

(h) $(7a + 6)(7a - 6) = 0$
    $a = -\frac{6}{7}$ and $a = \frac{6}{7}$

(i) $(5m - 6)^2 = 0$
    $m = \frac{6}{5}$

(j) $(m - 3)^2 = 0$
    $m = 3$

Part #5: Evaluate $f(x)$ to determine the point that lies on the quadratic's graph.

(a) $f(x) = 3x^2 + 5x - 1$; for $f(-3)$ Point: $(-3, 11)$

(b) $f(x) = -x^2 + 8x - 16$; for $f(2)$ Point: $(2, -4)$

(c) $f(x) = x^2 + x - 1$; for $f(-1)$ Point: $(-1, -1)$

Multiple choice: Which of the following points lies on the graph of $y = x^2 - 5$?

(a) $(3, -2)$
(b) $(5, 0)$
(c) $(-2, -1)$
(d) $(-1, -6)$
3. Solving quadratic equations using the extraction of roots.

**Previewing:** solving a radical equation

solve: \( \sqrt{x-3} + 2 = 10 \)

Recall: step 1: isolate the square root by itself on one side of the equal sign.

So: \( \sqrt{x-3} = 8 \)

Recall: step 2: square both sides

So: \( x - 3 = 64 \)

Recall: step 3: solve for \( x \)

So: \( x = 67 \)

Now you try: \( \sqrt{2x+9} - 1 = 2 \)

1: \( \sqrt{2x+9} = 3 \)

2: \( 2x + 9 = 9 \)

3: \( x = 0 \)

A square root is: a number \( x \) is a number \( y \) such that \( y^2 = x \); therefore \( y*y = x \).

A quadratic root is: The solutions for \( x \) of a quadratic equation (generally thought of as the \( x \)-intercepts)

| The rule of square roots: \( x^2 = a > 0 \) ↔ \( x = \pm \sqrt{a} \) |
|------------------------------|-------------------------|
| If \( a \geq 0 \) then the solutions are \( \pm \sqrt{a} \) | If \( a < 0 \) then no real number solution exists |

**Exercise #1:** Solve the following quadratic equations using the Square Root Method.

(a) \( x^2 = 9 \)

\( x = \pm \sqrt{9} \)

\( x = \pm 3 \)

The roots are where the parabola intersects the \( x \)-axis.

root 1: \( x = -3 \)

root 2: \( x = 3 \)

If \( x^2 = 9 \) then \( x^2 - 9 = 0 \) and \( (x - 3)(x + 3) = 0 \). Therefore \( x = 3 \) and \( x = -3 \)

(b) \( x^2 - 1 = 24 \)

\( x = \pm 5 \)

(c) \( 55 = 3x^2 + 7 \)

\( x = \pm 4 \)

steps: solving a quadratic equations using the Square Root Method
solve: \(-7(x^2 - 10)^2 - 6 = -258\)

**Step 1:** isolate the squared term by itself on one side of the equal sign. (add 6 and divide by -7)
So: \((x^2 - 10)^2 = 36\)

**Step 2:** take the square root of both sides and **DON'T FORGET:** it has two solutions (\(x = \pm \sqrt{a}\))
So: \(x^2 - 10 = \pm 6\)

**Step 3:** evaluate the radical
So: \(x^2 = 10 \pm 6\)

**Step 4:** consider two cases and solve for \(x^2\)

<table>
<thead>
<tr>
<th>Case 1: (x^2 = 10 + 6)</th>
<th>Case 2: (x^2 = 10 - 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>We get: (x = \pm 4)</td>
<td>We get: (x = \pm 2)</td>
</tr>
<tr>
<td>Now use the zero factor property</td>
<td>Now use the zero factor property</td>
</tr>
<tr>
<td>so: (x = -4) and (x = 4)</td>
<td>so: (x = -2) and (x = 2)</td>
</tr>
</tbody>
</table>

**Step 5:** If the answer is not coming out right, try something different.
The solutions to this quadratic equations are \(x = -4\), \(x = -2\), \(x = 2\) and \(x = 4\).

**Step 6:** check. Make sure your answer makes sense.

Here we can see we have two parabolas:

\[ y = x^2 - 16 = 0 \]

and

\[ y = x^2 - 4 = 0 \]

from

\[ y = -7(x^2 - 10)^2 - 6 + 258 \]

By graphing all 3 we can see the roots match up!

**Exercise #2:** Create a checklist that will help you complete the steps.
Independent Problems:

Part #2: solving a quadratic equations using the Square Root Method

(a) \( k^2 = 75 \)
\[
x = \pm 5 \sqrt{3}
\]

(b) \( x^2 = 21 \)
\[
x = \pm \sqrt{21}
\]

(c) \( x^2 + 8 = 28 \)
\[
x = \pm 2 \sqrt{5}
\]

(d) \( -6m^2 = -414 \)
\[
x = \pm \sqrt{69}
\]

(e) \( -5x^2 - 10 = 490 \)
\[
x = \pm 10
\]

(f) \( 3 - 4x^2 = -85 \)
\[
x = \frac{\pm \sqrt{82}}{2}
\]

Part #3: solving a quadratic equations using the Square Root Method and provide a graph.

(a) \( -7(-x^2 + 1) + 6(x^2 - 1) = -17 \)
\[
7x^2 - 7 + 6x^2 - 6 = -17
\]
\[
13x^2 - 13 = -17
\]
\[
13x^2 = -4
\]
no solution (-)
(b) \(-2 (x^2 - 7)^2 + 3 = -47\)

\((x^2 - 7)^2 = 25\)

\(x^2 - 7 = \pm 5\)

\(x^2 = 12\) or \(x^2 = 2\)

\(x = \pm 2\sqrt{3}\)

OR

\(x = \pm \sqrt{2}\)

(c) \(7(x^2 + 3) - 3(2x^2 - 2) = 127\)

\(7x^2 + 21 - 6x^2 + 6 = 127\)

\(x^2 + 27 = 127\)

\(x^2 = 100\)

\(x = -10\) OR \(x = 10\)
3.b Additional: More solving quadratic equations using the extraction of roots.

The rule of square roots:  \( x^2 = a > 0 \)  \( \leftrightarrow \)  \( x = \pm \sqrt{a} \)

If \( a \geq 0 \) then the solutions are  \( \pm \sqrt{a} \)

If \( a < 0 \) then no real number solution exists

**Exercise #1:** Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \( 25x^2 = 36 \)  
\( x = \frac{\pm 6}{5} \)

(b) \( 4n^2 = 36 \)  
\( n = \frac{\pm 6}{2} \)

(c) \( 5x^2 - 15z^2 = 0 \); for \( x \)  
\( x = \pm \sqrt{3} z \)

(d) \( 14a^2 = 235 \)  
\( a = \pm \sqrt{\frac{235}{14}} \)

(e) \( 6x^2 + 36 = 0 \)  
\( x = \pm \sqrt{6} \)

(f) \( 6n^2 = 864 \)  
\( n = \pm 12 \)

(g) \( 9z^2 - 121 = 0 \)  
\( z = \pm \frac{11}{3} \)

(h) \( 4n^2 = 24m^2p^8 \); for \( n \)  
\( n = \pm \sqrt{6} mp^4 \)

(i) \( 5p^2q^2 = 45p^2 \); for \( q \)  
\( q = \pm 3 \)

(j) \( 16m^2 = -2206 \)  
\( m = \frac{\pm \sqrt{235}}{4} \)
Exercise #2: Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \((x + 2)^2 = 81\)
\[x = 7 \text{ and } x = 11\]

(b) \((a + 3)^2 = 5\)
\[a = -3 + \sqrt{5} \text{ and } a = -3 - \sqrt{5}\]

(c) \((a + 6)^2 = 64\)
\[a = -2 \text{ and } a = 14\]

(d) \((m - 4)^2 = 15\)
\[m = 4 + \sqrt{15} \text{ and } m = 4 - \sqrt{15}\]

(e) \((y - 7)^2 = 49\)
\[y = 0 \text{ and } y = 14\]

(f) \((k - 1)^2 = 12\)
\[k = 1 + \sqrt{12} \text{ and } k = 1 - \sqrt{12}\]

(g) \((x - 11)^2 = 0\)
\[x = 11\]

(h) \((a - 6)^2 = 6\)
\[a = 6 + \sqrt{6} \text{ and } a = 6 - \sqrt{6}\]
Independent Problems:

**Part #1:** Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \( x^2 = 81 \)

\[ x = 9 \text{ and } x = -9 \]

(b) \( b^2 - 4 = 0 \)

\[ b = 2 \text{ and } b = -2 \]

(c) \( z^2 = 3 \)

\[ z = \sqrt{3} \text{ and } z = -\sqrt{3} \]

(d) \( m^2 - 11 = 14 \)

\[ m = 5 \text{ and } m = -5 \]

(e) \( 3x^2 - 27 = 0 \)

\[ x = 3 \text{ and } x = -3 \]

(f) \( 2n^2 - 25 = 25 \)

\[ n = 5 \text{ and } n = -5 \]

(g) \( x^2 = 9b^2 \); for \( x \)

\[ x = 3b \text{ and } x = -3b \]

(h) \( x^2 = 9b^2 \); for \( b \)

\[ b = \frac{x}{3} \text{ and } b = -\frac{x}{3} \]

(i) \( k^2 = p^2q^2r^2 \); for \( k \)

\[ k = pqr \text{ and } k = -pqr \]

(j) \( 2y^2 = 2a^2n^2 \); for \( y \)

\[ y = an \text{ and } y = -an \]

(k) \( x^2 - z^2 = 0 \); for \( x \)

\[ x = z \text{ and } x = -z \]

(l) \( 5a^2 - 10b^2 = 0 \); for \( a \)

\[ a = \sqrt{2}b \text{ and } a = -\sqrt{2}b \]
Part #2: Solve the following quadratic equations using the Square Root Method (The method of extraction of roots).

(a) \((x - 1)^2 = 4\)

\[ x = -1 \text{ and } x = 3 \]

(b) \((x - 2)^2 = 9\)

\[ x = -1 \text{ and } x = 5 \]

(c) \((x + 5)^2 - 12 = 5\)

\[ x = -5 + \sqrt{17} \text{ and } x = -5 - \sqrt{17} \]

(d) \((x + 3)^2 = 5\)

\[ x = -3 + \sqrt{5} \text{ and } x = -3 - \sqrt{5} \]

(e) \((x - 3)^2 = -10\)

no real solutions

(f) \((x - 7)^2 = 45\)

\[ x = 7 + 3\sqrt{5} \text{ and } x = 7 - 3\sqrt{5} \]

(g) \((x + 1)^2 = a \); for \(x\)

\[ x = -1 + \sqrt{a} \text{ and } x = -1 - \sqrt{a} \]

(h) \((t - 5)^2 = b \); for \(t\)

\[ t = 5 + \sqrt{b} \text{ and } t = 5 - \sqrt{b} \]

(i) \((s^2 + 1)^2 = a^2 \); for \(s\)

\[ s^2 = a - 1 \text{ and } s^2 = -(a + 1) \]

\[ s = \pm \sqrt{a - 1} \text{ and } s \text{ is no real solution} \]

\[ x = \pm \sqrt{a - 1} \]

(j) \((x + c)^2 = z^6 \); for \(x\)

\[ x = \pm \sqrt{z^3 - c} \]

(k) \((m + c)^2 = j \); for \(m\)

\[ m = \sqrt{j} - c \text{ and } m = -(\sqrt{j} + c) \]

\[ m = \pm \sqrt{j} - c \text{ and } m \text{ is no real} \]

\[ m^2 = a^3 - n^4 \text{ and } m^2 = -(a^3 + n^4) \]

\[ m = \pm \sqrt{a^3 - n^4} \text{ and } m \text{ is no real} \]

(l) \((x^2 + n^4)^2 = a^6 \); for \(x\)

\[ m = a^3 - n^4 \text{ and } m = -(a^3 + n^4) \]

\[ x = \pm \sqrt{a^3 - n^4} \text{ and } x \text{ is no real} \]
4. Apply your knowledge.

Students must get 5 correct in a row. Each question is worth 5 points. They lose three points for hints used. If students use hints they receive only 40% for the question. Students get 15 minutes to complete. Whatever score out of 5 questions is recorded at the top of the screen are the points they receive.

Green check = 5 points
light bulb (hints) = 2 points
x mark = 2 points
total = 25 points

Part 2: Factor the GCF from each polynomial.

(1) $22g^6 + 44g^8$

$11g^6(2 + 4g^2)$

(2) $48x^{31} - 54x^{41}$

$6x^{31}(8 - 9x^{10})$

(3) $2y^3 - 10y^2 - 18y$

$2y(y^2 - 5y - 9)$

(4) $12a^2b^4 + 8a^2b$

$4ba^2(3ab^2 - 2)$

Part 3: Factor the Difference of Perfect squares .

(1) $144 - p^2$

$(12 - p)(12 + p)$

(2) $36 - 25x^2$

$(6 - 5x)(6 + 5x)$

(3) $4z^2 - 8$

$4(z^2 - 2)$

(4) $25c^2 - 81$

$(5c - 9)(5c + 9)$

(5) $w^2 + 100$

$(w - 10)(w + 10)$

(6) $9n^2 - 196$

$(3n - 13)(3n + 13)$
5. Solving quadratic equations by factoring.

Often books refer to solving a quadratic equation by factoring to mean the same thing as finding the roots. While finding the roots of a quadratic is an important skill, it does not encompass the entirety of what a quadratic is modeling. (Remember: the roots are where the quadratic intersects the x-axis) Quadratic equations can be solved by factoring, by completing the square, by using the quadratic formula, trig solutions, Vieta's solutions, Bramagupta's methods, or by graphing. In this lesson we will focus on specific quadratics that can be factored (not all quadratic equations can be solved in this method)

Begin Direct Instruction.

<table>
<thead>
<tr>
<th>FOIL</th>
<th>FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 7)(x – 2) = 0</td>
<td>1) Make parenthesis</td>
</tr>
<tr>
<td>x^2 + 5x - 14 = 0</td>
<td>2) Find square root of the first term (when 1)</td>
</tr>
<tr>
<td>( ) ( ) = 0</td>
<td>3) Choose the signs</td>
</tr>
<tr>
<td>14 = 1, 2, 7, 14</td>
<td>4) Find factors of last term</td>
</tr>
<tr>
<td>Sign</td>
<td>Sign * +</td>
</tr>
<tr>
<td>+ + + +</td>
<td>5) Choose factors that add to middle term</td>
</tr>
<tr>
<td>- - + -</td>
<td>6) solve for x (twice)</td>
</tr>
<tr>
<td>+ + + +</td>
<td>7) MAKE CONNECTIONS</td>
</tr>
<tr>
<td>(x – 2) (x + 3)</td>
<td>x = 2 , x = -3</td>
</tr>
</tbody>
</table>

Exercise #1: Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law). Sometimes you will be instructed to solve by factoring, find the roots, or find the zeros.

The zero product law’s importance to mathematics cannot be overstated. It finally allows us, in certain situations, to solve equations that are higher-order polynomials than just linear. Of course, for it to work, we must have two conditions met: (1) we must have the equation set equal to zero and (2) we must be able to factor the expression equal to zero.

End Direct Instruction. Begin modeling, rehearsal and feedback.

<table>
<thead>
<tr>
<th>I do</th>
<th>We do</th>
<th>You do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor: x^2 + x – 6 = 0</td>
<td>Find the roots: x^2 + x – 12 = 0</td>
<td>Find the zeros: x^2 + 6x + 8 = 0</td>
</tr>
<tr>
<td>1) ()</td>
<td>(x - 3) (x + 4)</td>
<td>(x + 2) (x + 4)</td>
</tr>
<tr>
<td>2) (x ) (x )</td>
<td>x = 3 , x = -4</td>
<td>x = -2 , x = -4</td>
</tr>
<tr>
<td>3) (x - ) (x + )</td>
<td>4) 6: 1,2,3,6</td>
<td></td>
</tr>
<tr>
<td>5) (x - 2) (x + 3)</td>
<td>6) x = 2 , x = -3</td>
<td></td>
</tr>
<tr>
<td>(a) x^2 + 5x + 6 = 0</td>
<td>(b) x^2 – x – 12 = 0</td>
<td></td>
</tr>
<tr>
<td>(x + 2) (x + 3)</td>
<td>(x - 4) (x + 3)</td>
<td></td>
</tr>
<tr>
<td>x = -2 , x = -3</td>
<td>x = 4 , x = -3</td>
<td></td>
</tr>
</tbody>
</table>
End modeling, rehearsal and feedback. Begin cooperative learning.

**Exercise #2:** Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law).

(a) $x^2 - 11x = -24$
   
   $(x - 3)(x - 8)$
   
   $x = 3, x = 8$

(b) $10x^2 + x - 21 = 0$ (not 1)
   
   $(5x - 7)(2x + 3)$
   
   $x = 7/5, x = -3/2$

(c) $x^2 + 6x + 9 = 0$
   
   $(x + 3)(x + 3)$
   
   $x = -3$

(d) $3x^2 + 90 - 39x = 0$
   
   $3(x - 3)(x - 10)$
   
   $x = 3, x = 10$

(e) $x^3 + 6x^2 - 16x = 0$
   
   $x(x + 8)(x - 2)$
   
   $x = -8, x = 2$

(f) $-20x + 2x^2 = -32$
   
   $2(x - 8)(x - 2)$
   
   $x = 8, x = 2$

(g) $3x^2 - 9x + 6 = 0$
   
   $3(x - 2)(x - 1)$
   
   $x = 2, x = 1$

(h) $4x^3 + 16x^2 + 64 = 4x^3 + x^2 + 10x^2 + 56$
   
   $(x + 2)(x + 4)$
   
   $x = -2, x = -4$

End cooperative learning. Begin sequential prompting.

**Exercise #3:** Complete a checklist that will help you solve these equations.

**Exercise #4:** Find the zeroes of the quadratic function $y = 3x^2 - 6x - 24$ algebraically. Sketch a graph of the zeroes and $y$-intercept on the graph.

$3 (x - 4)(x + 2)$

$x = 4, x = -2$
Independent Problems:

Part #2: Find the roots (solutions) to each of the following equations by using the zero factor property also called the (zero product law).

(a) \(2x^2 + 12x + 18 = 0\)
\[2(x + 3)(x + 3)\]
\[x = 3\]

(b) \(x^3 - 8x^2 + 16x = 0\)
\[x(x - 4)(x - 4)\]
\[x = 0, x = 4\]

(c) \(x^2 + 23x = 50\)
\[(x + 25)(x - 2)\]
\[x = 3\]

(d) \(-8x^2 + 20 + x^2 = 0\)
\[(x + 10)(x - 2)\]
\[x = -10, x = 2\]

(e) \(x^2 - 18x + 32 = 0\)
\[(x - 16)(x - 2)\]
\[x = 3\]

(f) \(0 = x^2 + 3x - 10\)
\[(x - 2)(x + 5)\]
\[x = 2, x = -5\]

(g) \(x^2 - x - 56 = 0\)
\[(x + 8)(x - 9)\]
\[x = -8, x = 9\]

(h) \(8 = x^2 - x + 2\)
\[(x - 3)(x + 2)\]
\[x = 3, x = -2\]

(i) \(x^4 = 3x^3 + 10x^2\)
\[x^2(x - 5)(x + 2)\]
\[x = 0, x = 5, x = -2\]

(j) \(2x^2 - 22x - 24 = 0\)
\[(x - 12)(x + 1)\]
\[x = 12, x = -1\]
Part #4: History: The ancient civilizations of Babylonia, Egypt, Greece, China, and India used the method for finding the roots in order to calculate land taxes, finances, and solve problems relating the areas and sides of rectangles, this holds true today in finance, path and trajectory, changes in temperature, or structure, and area formations.

5 minute Presentation: Research and find an example of quadratics in use throughout history and explain the significance of the example you have found.

Multiple Choice: The roots of \( x^2 - 6x - 16 = 0 \) can be found by factoring as

(1) \{-16, 6\}  
(2) \{-8, 2\}  
(3) \{-2, 8\}  
(4) \{6, 16\}

Multiple Choice: The equation \((2x - 3)(x + 7) = 0\) has a solution set of

(1) \{-7, \frac{1}{2}\}  
(2) \{3, 7\}  
(3) \{-7, 3\}  
(4) \{\frac{1}{2}, -3\}

Part #5: Find the roots of each of the following equations by factoring:

(a) \( x^2 - 36 = 0 \)  
\( (x + 6)(x - 6) \)  
\( x = -6, x = 6 \)

(b) \( x^2 + 12x + 27 = 0 \)  
\( (x + 3)(x + 9) \)  
\( x = -3, x = -9 \)

(c) \( 3x^2 + 5x - 2 = 0 \)  
\( (3x - 1)(x + 2) \)  
\( x = 1/3, x = -2 \)

(d) \( 20x^2 - 10x = 0 \)  
\( 10x(2x - 1) \)  
\( x = 0, x = 1/2 \)

(e) \( 10x^2 + x - 21 = 0 \)  
\( (2x + 3)(5x - 7) \)  
\( x = -3/2, x = 7/5 \)

(f) \( 4x^2 - 16x - 84 = 0 \)  
\( 4(x - 7)(x + 3) \)  
\( x = 7, x = -3 \)
Part #6: A baking soda rocket is fired upwards with an initial speed of 80 feet per second. Its height, h, above the ground in feet can be modeled using the equation:
\[ h(t) = 16t^2 + 80t \]
where \( t \) is the time since launch in seconds.
At what time, \( t > 0 \), does the rocket hit the ground? Find algebraically using factoring.
\( T = 0, t = 5 \)


Factoring with other leading coefficients than 1

Part #6: Find the zeroes of the quadratic function
\[ y = x^2 - 4x - 5 \]
 algebraically. Sketch a graph of the zeroes and \textit{y-intercept} on the graph.

\[(x - 5)(x + 1)\]
\(x = 5, x = -1\)

Part #7: The two quadratic equations below have the same solutions. Can you determine why? Completely factor both to see what they have in common.

\[ x^2 - 7x + 12 = 0 \]
\[ 3x^2 - 21x + 36 = 0 \]
\( x = 4, x = 3 \)
\( \text{multiple of 3} \)

Part #8: ONLINE: http://www.ixl.com/math/algebra-1/factor-quadratics-special-cases

Factoring with special cases
7. Completing the square with coefficient 1.

Begin Direct Instruction.

Suppose we wish to solve the quadratic equation $x^2 - 3x - 1 = 0$: Since the equation is not of the form $x^2 = a$; we cannot use extraction of roots. Next, we try factoring, but after a few trials we see that $x^2 - 3x - 1 = 0$ is not factorable. We need another method for solving quadratic equations.

The method we shall study is based on perfect square trinomials and extraction of roots. The method is called solving quadratic equations by completing the square. Consider the equation $x^2 + 6x + 5 = 0$: This quadratic equation could be solved by factoring, but we'll use the method of completing the square. We will explain the method in detail after we look at this example. First we'll rewrite the equation as

\[
x^2 + 6x + 5 = 0
\]

\[
x^2 + 6x = -5
\]

Then, we'll add 9 to each side. We get

\[
x^2 + 6x + 9 = -5 + 9
\]

The left side factors as a perfect square trinomial.

\[
(x + 3)^2 = 4
\]

In this form the quadratic function is written in its shifted or vertex form as

\[
y = (x + 3)^2 - 4
\]

We can solve this by extraction of roots.

\[
x + 3 = \pm\sqrt{4}
\]

\[
x + 3 = 2 \quad x = 2 - 3 \quad x = -1
\]

\[
x + 3 = -2 \quad x = -2 - 3 \quad x = -5
\]

Notice that when the roots are rational numbers, the equation is factorable. The big question is, How did we know to add 9 to each side of the equation? We can convert any quadratic trinomial appearing in an equation into a perfect square trinomial if we know what number to add to both sides. We can determine that particular number by observing the following situation: Consider the square of the binomial and the resulting perfect square trinomial

\[
(x + p)^2 = x^2 + 2px + p^2
\]

Notice that the constant term (the number we are looking for) can be obtained from the linear term 2px:

If we take one half the coefficient of $x = \frac{2p}{2} p$; and square it, we get the constant term $p^2$: This is true for every perfect square trinomial with leading coefficient 1.

Visual representation: ONLINE: http://www.youtube.com/watch?v=Ax2mlah7bkQ 5:53
**Exercise #1**: Given the function \( y = (x - 3)^2 + 2 \) do the following:
(a) Give the coordinates of the turning point.
(b) Determine the range by drawing a rough sketch.

(3, 2) \([2, \infty)\)

Taking the standard form \( y = ax^2 + bx + c \) where \( a, b \) and \( c \) are real numbers and \( a \neq 0 \) and putting it into its vertex (shifted) form relies on several skills.

**End Direct Instruction. Begin previewing.**

**Review #1**: Write each of the following as an equivalent trinomial.

(a) \((x + 5)^2\) 
\(x^2 + 10x + 25\)

(b) \((x - 1)^2\) 
\(x^2 - 2x + 1\)

(c) \((x + 4)^2\) 
\(x^2 + 8x + 16\)

**End previewing. Begin sequential prompting.**

**Exercise #2**: Given each trinomial in Review #1 of the form \( ax^2 + bx + c \), what is true about the relationship between the value of \( b \) and the value of \( c \)? Illustrate.

C can be made from \( \frac{1}{2} b^2 \)

**End sequential prompting. Begin previewing.**

**Review #2**: Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.

(a) \( x^2 + 20x + 100 \) 
\((x + 10)(x + 10)\)

(b) \( x^2 - 6x + 9 \) 
\((x - 3)(x – 3)\)

(c) \( x^2 + 2x + 1 \) 
\((x + 1)(x + 1)\)

**End previewing. Begin sequential prompting.**

**Exercise #3**: If we graph the parabola \( y = x^2 - 4x - 1 \) we see the vertex is at \((2,-5)\), and \( h = 2 \), \( k = -5 \) which means the vertex form is \( y = (x - 2)^2 - 5 \).

so if we have \( x^2 - 4x - 1 = 0 \) or \( x^2 - 4x = 1 \)
then we add \( x^2 - 4x + \_4 \) = \( 1 + \_4 \)
to get \( (x - 2)^2 = 5 \) or \( (x - 2)^2 + 5 = 0 \)

**Completing the square algorithm**

Given: \( y = ax^2 + bx + c \) where \( a = 1 \)

1. find \( \frac{1}{2} \) the value of \( b \); \((b/2)\).
2. square this term; \((b/2)^2\)
3. Add or subtract it

**End sequential prompting. Begin Modeling, rehearsal and feedback and cooperative learning.**
Exercise #4: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s turning point and solve this by extraction of roots. The last two problems will involve fractions. Stick with it!

(a) \( y = x^2 + 6x - 2 \)  
\( y = (x + 3)^2 - 11 \)  
vertex = (-3, -11)  
\[ x = -3 \pm \sqrt{11} \]

(b) \( y = x^2 - 2x + 11 \)  
\( y = (x - 1)^2 + 10 \)  
vertex = (1, 10)  
\[ x = 1 \pm \sqrt{10}i \]

(c) \( y = x^2 - 2x + 27 \)  
\( y = (x - 1)^2 + 26 \)  
vertex = (1, 26)  
\[ x = 1 \pm \sqrt{26}i \]

(d) \( y = x^2 + 8x \)  
\( y = (x + 4)^2 - 16 \)  
vertex = (-4, -16)  
\[ x = 0, x = 8 \]

(e) \( y = x^2 + 5x + 4 \)  
\( y = (x + 5/2)^2 - 9/4 \)  
vertex = (-5/2, -9/4)  
\[ x = -1, x = -4 \]

(f) \( y = x^2 - 9x - 2 \)  
\( y = (x - 9/2)^2 - 89/4 \)  
vertex = (9/2, -89/4)  
\[ x = 9 \pm \sqrt{89}/2 \]

(g) More complex solutions:  
\( y = x^2 - 3x - 1 \)  
\( y = (x - 3/2)^2 - 13/4 \)  
vertex = (3/2, -13/4)  
\[ x = 3 \pm \sqrt{13}/2 \]

(h) A factorable coefficient of a:  
\( y = 3x^2 - 36x - 39 \)  
\( y = (x - 6)^2 - 49 \)  
vertex = (6, -49)  
\[ x = -1, x = 13 \]

End Modeling, rehearsal and feedback. Begin sequential prompting.

Exercise #5: Complete a checklist that will help you complete the square.
Independent Problems:

Part #1: Find each of the following products in standard form.

(a) \((x + 4)^2\)
\(x^2 + 8x + 16\)

(b) \((x - 5)^2\)
\(x^2 - 10x + 25\)

(c) \((x + 8)^2\)
\(x^2 + 16x + 64\)

(d) \((x - 7)^2\)
\(x^2 - 14x + 49\)

(e) \((x + 2)^2\)
\(x^2 + 4x + 4\)

(f) \((x - 10)^2\)
\(x^2 - 20x + 100\)

Part #2: Each of the following trinomials is a perfect square. Write it in factored form, i.e., \((x + a)^2\) or \((x - a)^2\)

(a) \(x^2 + 6x + 9\)
\((x + 3)^2\)

(b) \(x^2 - 22x + 121\)
\((x - 11)^2\)

(c) \(x^2 + 10x + 25\)
\((x + 5)^2\)

(d) \(x^2 + 30x + 225\)
\((x + 15)^2\)

(e) \(x^2 - 2x + 1\)
\((x - 1)^2\)

(f) \(x^2 - 18x + 81\)
\((x - 9)^2\)


Completing the square with leading coefficient of 1

Part #4: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/e/completing_the_square_in_quadratic_expressions

Completing the square with leading coefficient of 1
Part #5: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s turning point and solve this by extraction of roots.

(a) \( y = x^2 - 12x + 40 \)  
\( y = (x - 6)^2 + 4 \)  
vertex = (6, 4)  
x = \(-6 \pm 2i\)

(b) \( y = x^2 + 4x + 14 \)  
\( y = (x + 2)^2 + 10 \)  
vertex = (-2, 10)  
x = \(-2 \pm \sqrt{10}i\)

(c) \( y = x^2 - 24x + 146 \)  
\( y = (x - 12)^2 + 2 \)  
vertex = (12, 2)  
x = \(12 \pm \sqrt{2}i\)

(d) \( y = x^2 - 2x - 48 \)  
\( y = (x - 1)^2 - 49 \)  
vertex = (1, -49)  
x = -6, x = 8

(e) \( y = x^2 + 3x - 5 \)  
\( y = (x + 3/2)^2 - 29/4 \)  
vertex = (-3/2, -29/4)  
x = \(-\frac{3 \pm \sqrt{29}}{2}\)

(f) \( y = x^2 + 4x + 7 \)  
\( y = (x + 2)^2 + 3 \)  
vertex = (-2, 3)  
x = \(-2 \pm \sqrt{3}i\)

(a) \( y = x^2 - 10x \)  
\( y = (x - 5)^2 - 25 \)  
vertex = (5, -25)  
x = 0, x = 10

(b) \( y = x^2 + 14x + 13 \)  
\( y = (x + 7)^2 - 36 \)  
vertex = (-7, -36)  
x = -1, x = -13

(c) \( y = x^2 + 7x + 12 \)  
\( y = (x - 7/2)^2 - 1/4 \)  
vertex = (7/2, -1/4)  
x = -3, x = -4

(d) \( y = x^2 - 6x \)  
\( y = (x - 3)^2 - 9 \)  
vertex = (3, -9)  
x = 6, x = 0

(e) \( y = x^2 - 2x - 24 \)  
\( y = (x - 1)^2 - 25 \)  
vertex = (1, -25)  
x = 6, x = -4

(f) \( y = x^2 - 5x - 6 \)  
\( y = (x - 5/2)^2 - 49/4 \)  
vertex = (5/2, -49/4)  
x = 6, x = -1
Part #6: Answer the following questions based on the information provided.

A cable is attached at the same height from two poles and hangs between them such that its height above the ground, y, in inches, can be modeled using the equation: 

\[ y = x^2 - 16x + 67 \]

where x represents the horizontal distance from the left pole, in feet.

(a) What height is point A above the ground? Show your work and use proper units.

\[ (0)^2 - 16(0) + 67 = 67 \text{ inches} \]

(b) Write the equation in vertex form.

\[ y = (x - 8)^2 + 3 \]

(c) What is the difference in the heights of points A and B? Show your analysis and include units.

\[(8, 3) \]

\[ \text{difference } 67 - 3 = 64 \text{ inches} \]

(d) What is the horizontal distance that separates points A and B? Explain your reasoning.

64 inches

Part #7: Use the vertex form of each of the following quadratic functions to determine which has the lowest y-value.

\[ y = x^2 - 8x + 6 \]

\[ y = (x - 4)^2 - 10 \text{ lowest} \]

\[ y = x^2 + 6x + 1 \]

\[ y = (x + 3)^2 - 8 \]
Integrated Algebra

Name: ____________________________ Date: __________________

9. Completing the square with coefficient other than 1.

Begin Direct instruction.

Method 1:

Consider the quadratic equation: 7x^2 - 5x - 1 = 0.

By the previous method we would move the constant to the right side of the equation to get:

7x^2 - 5x = 1

Then we would add \( \left( \frac{b}{2} \right)^2 \) to both sides which would be + \( \frac{25}{4} \)

However 7x^2 - 5x + \( \frac{25}{4} \) can not be written as (ax - \( \frac{5}{2} \))^2 since we need a * a to be 7. Impossible!

so we modify.

1. Write the equation so that the constant term appears on the right side of equation.

7x^2 - 5x = 1

2. If the leading coefficient is different from 1, divide each term of the equation by that coefficient.

\[ x^2 - \frac{5}{7} x = \frac{1}{7} \]

3. Take one half of the coefficient of the linear term, square it, then add it to both sides of the equation.

\[ \left( \frac{5}{14} \right)^2 = \frac{25}{196} \]

4. Square it.

\[ \left( \frac{5}{14} \right)^2 = \frac{25}{196} \]

5. Add it to both sides of the equation.

\[ x^2 - \frac{5}{7} x + \frac{25}{196} = \frac{1}{7} + \frac{25}{196} \]

4. The trinomial on the left is now a perfect square trinomial and can be factored as ( )^2: The rest term in the parentheses is the square root of the quadratic term. The last term in the parentheses is one-half the coefficient of the linear term.

\[ (x - \frac{5}{14})^2 = \frac{53}{196} \]

5. Solve this equation by extraction of roots.

\[ x = \frac{5}{14} \pm \frac{\sqrt{53}}{\sqrt{196}} \]

6. Since there is a square root in the denominator, you must rationalize the denominator.

\[ x = \frac{5}{14} \pm \frac{\sqrt{53}}{14} \]

7. Check to determine if you can simplify the square root. In this case we can not.

8. Write your two solutions.

\[ X = \frac{5 + \sqrt{53}}{14} \approx 0.877 \] and \[ x = \frac{5 - \sqrt{53}}{14} \approx -0.163 \]

SPECIAL CASE: consider 2x^2 + x + 4 = 0

When we perform steps 1- 4 from above we get:

\[ (x - \frac{1}{4})^2 = -2 + \left( \frac{1}{4} \right)^2 \]

OR \[ -2 + \left( \frac{1}{16} \right) \approx -1.938 \]

so RHS = (-)

Since we know that the square of any number is positive, this equation has no real number solution.

End direct instruction. Begin cooperative learning.
Exercise #1: Solve each of the following quadratic equations using the method of completing the square

(a) \( y = 2x^2 - 12x + 11 \)
\[
y = (x - 3)^2 - \frac{7}{2}
\]
vertex = (3, -\(\frac{7}{2}\))
\[
x = 3 \pm \sqrt{\left(\frac{7}{2}\right)}
\]

(b) \( y = 5x^2 + 20x + 23 \)
\[
y = (x + 2)^2 + \frac{3}{5}
\]
vertex = (-2, 3/5)
\[
x = -2 \pm \sqrt{\left(\frac{3}{5}\right)}i
\]

(c) \( y = -2x^2 + 4x + 7 \)
\[
y = (x - 1)^2 - \frac{9}{2}
\]
vertex = (1, -9/2)
\[
x = -1 \pm \frac{3\sqrt{2}}{2}
\]

(d) \( y = 6x^2 - 24x + 14 \)
\[
y = (x - 2)^2 - \frac{5}{3}
\]
vertex = (-2, -5/3)
\[
x = 2 \pm \sqrt{\left(\frac{5}{3}\right)}
\]

(e) \( y = -x^2 - 12x - 33 \)
\[
y = (x + 6)^2 - 3
\]
vertex = (-6, -3)
\[
x = -6 \pm \sqrt{3}
\]

(f) \( y = 5x^2 - 2x - 24 \)
\[
y = (x - 1/5)^2 - 5
\]
vertex = (1/5, -5)
\[
x = 1/5 \pm \sqrt{5}
\]

End cooperative learning. Begin direct instruction.
Method 2:
The difficulty with the method above is understanding the form of \( y = a (x – h)^2 + k \).
from above we get \((x - \frac{5}{14})^2 = \frac{53}{196}\) or alternately \( y = (x - \frac{5}{14})^2 - \frac{53}{196}\)
so \( a \) will always be 1. However we can simplify this by multiplying both sides by the factor of 7 (the number we factor) to get: \( 7(x - \frac{5}{14})^2 = \frac{53}{28}\) or alternately \( y = 7(x - \frac{5}{14})^2 - \frac{53}{28}\)
which is the form we want. Since \( a=7 \) is positive, the parabola opens up from the vertex, and the vertex is a minimum.

Exercise #2: Edit your last checklist to complete the square when the leading coefficient is not 1.

**Given** \( y = a (x – h)^2 + k \)
If \( a \geq 0 \) then the vertex is a minimum. If \( a < 0 \) then the vertex is a maximum

End direct instruction. Begin modeling, rehearsal and feedback.

Exercise #3: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s vertex (turning point), state whether the vertex is a maximum or minimum and solve this by extraction of roots.

(a) \( y = 5x^2 – 2x - 4 \)
\( y = 5(x - 1/5)^2 - 21/5 \)
vertex = ( -1/5 , -21/5 )
\( x = \frac{1 \pm \sqrt{21}}{5} \)
Max or Min

(b) \( y = 4x^2 + 5x + 1 \)
\( y = 4(x + 5/8)^2 - 9/16 \)
vertex = ( -5/8 , -9/16 )
\( x = -1/4, x = -1 \)
Max or Min

(c) \( y = 2x^2 + 2x - 1 \)
\( y = 2(x + 1/2)^2 - 3/2 \)
vertex = ( -1/2 , -3/2 )
\( x = \frac{-1 \pm \sqrt{3}}{2} \)
Max or Min

(d) \( y = 4x^2 - 8x - 16 \)
\( y = 4(x + 1)^2 - 20 \)
vertex = ( -1 , -20 )
\( x = -1 \pm \sqrt{3} \)
Max or Min
Independent Problems:

Part #1: ONLINE: https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/e/completing_the_square_2

Completing the square with leading coefficient other than 1.

Part #2: Write each quadratic in vertex form by Completing the Square, identify the quadratic’s vertex (turning point), state whether the vertex is a maximum or minimum and solve this by extraction of roots.

(a) \( y = 9x^2 + 12x - 5 \)

\[ y = 9\left( x + \frac{2}{3} \right)^2 - 9 \]

vertex = ( -2/3 , -9 )

x = 5/3, x = -1/3

Max or Min

(b) \( y = 16x^2 - 8x - 3 \)

\[ y = 16\left( x - \frac{1}{4} \right)^2 - 4 \]

vertex = ( 1/4 , -4 )

x = -1/4, x = 3/4

Max or Min

(c) \( y = 2x^2 + 5x - 4 \)

\[ y = 2\left( x + \frac{5}{4} \right)^2 - \frac{57}{8} \]

vertex = ( -5/4 , -57/8 )

x = \( -\frac{5\pm\sqrt{57}}{4} \)

Max or Min

(d) \( y = 3x^2 + 2x - 18 \)

\[ y = 3\left( x + \frac{1}{3} \right)^2 - \frac{82}{3} \]

vertex = ( -1/3 , -82/3 )

x = \( -\frac{1\pm\sqrt{82}}{3} \)

Max or Min

Part #3: Use the method completing the square to write each of the following quadratic functions in the form \( y = a(x - h)^2 + k \) Then, identify the turning point and whether it is a maximum or minimum.

(a) \( y = 3x^2 - 12x + 17 \)

\[ y = 3\left( x - 2 \right)^2 + 5 \]

vertex = ( 2 , 5 )

Max or Min

(b) \( y = -5x^2 + 40x - 70 \)

\[ y = 5\left( x - 4 \right)^2 - 10 \]

vertex = ( 4 , -10 )

Max or Min
Multiple choice: Which of the following equations models the graph shown at the right? Explain how you made your choice?

(1) \( y = (x - 1)^2 - 5 \)

(2) \( y = -3(x + 1)^2 - 5 \)

(3) \( y = (x + 1)^2 - 5 \)

(4) \( y = 2(x - 1)^2 - 5 \)

Explanation:
calculate the y intercept where \( x=0 \), then \( y = -3 \)

Part 4: Answer the following questions based on the information given.

The vertical height of projectiles above level ground can be modeled by equations in the form:

\[ h(t) = -16(t - t_{\text{max}})^2 + h_{\text{max}} \]

where \( h_{\text{max}} \) is the maximum height in feet and \( t_{\text{max}} \) is the time, in seconds, when it occurs.

(a) A given projectile has a height function given by
\[ h(t) = -16(t - 8)^2 + 156 \] What is its maximum height and at what time, \( t \), does it occur?
156 feet at 8 seconds (vertex)

(b) A projectile has a height function given by
\[ h(t) = -16t^2 + 160t + 120 \] Write this in vertex form (from a)
\[ h(t) = -16(t - 5)^2 + 520 \]

(c) What is the maximum height and at what time does it occur for the projectile from (b)?
520 feet at 5 seconds

(d) At what height does the projectile in (b) start above the ground? Show the work that leads to your answer.
.97 seconds

(e) Using your calculator, sketch a graph of the height on the axes below for the projectile from (b). Mark your answers from (c) and (d) on the graph.
10. Solving Quadratic Equations Using the Quadratic Formula

Begin previewing.

Recall that a quadratic is in the standard form of  \(0 = ax^2 + bx + c\) where \(a \neq 0\) and \(a\), \(b\) and \(c\) are real numbers.

**Exercise 1:** Identify the real numbers for coefficients \(a\) and \(b\), and the constant term \(c\).

(a) \(0 = 2x^2 - 12x + 11\)
\[
a = 2 \\
b = -12 \\
c = 11
\]

(b) \(0 = 3x^2 + 5x + 2\)
\[
a = 3 \\
b = 5 \\
c = 2
\]

(c) \(0 = 1x^2 + 3x - 1\)
\[
a = 1 \\
b = 3 \\
c = -1
\]

(d) \(0 = 6x^2 - 2\)
\[
a = 6 \\
b = 0 \\
c = -2
\]

(e) \(0 = 3x^2 + 7x\)
\[
a = 3 \\
b = 7 \\
c = 0
\]

(f) \(12 = x^2 + 10x\)
\[
a = 1 \\
b = 10 \\
c = -12
\]

When a quadratic equation is written in standard form so that the values \(a\), \(b\) and \(c\) are readily determined, the equation can be solved using the quadratic formula. The values that satisfy the equation are found by substituting the values \(a\); \(b\); and \(c\) into the formula.

Quadratic Formula: Given \(y = ax^2 + bx + c\) where \(a \neq 0\,

the zeroes can be found by \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

Keep in mind that the plus or minus symbol, is just a shorthand way of denoting two possibilities.

\[
x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]

End previewing. Begin modeling, rehearsal and feedback.

**Exercise 2:** Solve the quadratic equations using the quadratic formula and express your answers in simplest radical form.

<table>
<thead>
<tr>
<th>I do</th>
<th>We do</th>
<th>You do</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (0 = 12x^2 - 2x - 1)</td>
<td>(b) (0 = 2x^2 - 9x + 4)</td>
<td>(c) (0 = x^2 + 8x + 3)</td>
</tr>
<tr>
<td>(x = \frac{2 \pm \sqrt{4 - 4(12)(-1)}}{2(12)})</td>
<td>(x = \frac{9 \pm \sqrt{81 - 4(2)(4)}}{2(2)})</td>
<td>(x = \frac{-8 \pm \sqrt{64 - 4(1)(3)}}{2(1)})</td>
</tr>
<tr>
<td>(x = \frac{2 \pm \sqrt{52}}{24})</td>
<td>(x = \frac{9 \pm \sqrt{49}}{4})</td>
<td>(x = -\frac{8 \pm \sqrt{52}}{2})</td>
</tr>
<tr>
<td>(x = \frac{1 \pm \sqrt{13}}{12})</td>
<td>(x = \frac{9 \pm 7}{4} = 4, 1/2)</td>
<td>(x = -4 \pm \sqrt{13})</td>
</tr>
</tbody>
</table>
(d) $0 = x^2 + 6x - 9$
\[ x = -3 \pm 3\sqrt{2} \]

(e) $0 = 3x^2 + 4x - 1$
\[ x = \frac{-2 \pm \sqrt{7}}{3} \]

(f) $0 = -8x^2 + 11x$
\[ x = 0, x = 11/8 \]

End modeling, rehearsal and feedback Begin direct instruction.

**Proof of the Quadratic formula video:**


**Exercise 3:** Solve $ax^2 + bx + c = 0$ for $x$ by completing the square.

**Exercise 4:** Consider the quadratic equation $2x^2 - 7x - 15 = 0$

(a) Find the solutions to this equation either by factoring or completing the square.

\[ y = 2(x - 7/4)^2 + 169/8 \]

vertex = (7/4, 169/8)

\[ x = \frac{7 \pm \sqrt{49 - 4(2)(-15)}}{2(2)} \]

\[ x = \frac{9 \pm \sqrt{49}}{4} = \frac{7 \pm 13}{4} \]

\[ x = 5, -3/2 \]

(a) Find the solutions to this equation using the quadratic formula.

\[ x = \frac{-20 \pm \sqrt{400 - 4(-16)(60)}}{2(-16)} = \frac{-20 \pm \sqrt{4240}}{-32} = \frac{-20 \pm 4\sqrt{265}}{-32} = \frac{-5 \pm \sqrt{265}}{-8} = 2.7 \]

**Exercise 5:** A projectile is fired vertically from the top of a 60 foot tall building. It’s height in feet above the ground after $t$-seconds is given by the formula $h = -16t^2 + 20t + 60$

At what time, $t$, does the ball hit the ground? Solve by using the quadratic formula to the nearest tenth of a second.
Integrated Algebra

Name:___________________________________ Date:_________________

Independent Problems:

Part 2: Identify the real numbers for coefficients a and b, and the constant term c.

(a) \(0 = 4x^2 – 3x + 5\)  
\(a = 4\)  
\(b = -3\)  
\(c = 5\)

(b) \(-9 + 3x^2 = -9x\)  
\(a = 3\)  
\(b = 9\)  
\(c = -9\)

(b) \(1 = -2x^2 + 4x\)  
\(a = -2\)  
\(b = 4\)  
\(c = -1\)

(d) \(0 = x^2 – 2x\)  
\(a = 1\)  
\(b = -2\)  
\(c = 0\)

(e) \(5x – 3 = -3x^2\)  
\(a = -3\)  
\(b = -5\)  
\(c = 3\)

(f) \(0 = -3x^2 – 11x - 2\)  
\(a = -3\)  
\(b = -11\)  
\(c = -2\)

Part 3: Solve the quadratic equations using the quadratic formula and express your answers in simplest radical form

(a) \(0 = 2x^2 + 3x – 7\)  
\(x = \frac{-3 \pm \sqrt{65}}{4}\)

(b) \(0 = 5x^2 – 2x – 1\)  
\(x = \frac{1 \pm \sqrt{6}}{5}\)

(c) \(0 = -3x^2 + x\)  
\(x = 0, x = 1/3\)

(d) \(0 = x^2 - 2x – 3\)  
\(x = 3, x = -1\)

(e) \(0 = x^2 + 5x + 6\)  
\(x = -3, x = -2\)

(f) \(0 = -6x^2 – 1x + 2\)  
\(x = 2/3, x = -1/2\)
Multiple choice: If the quadratic formula is used to solve the equation $x^2 - 4x - 41 = 0$, the correct roots are:

(a) $4 \pm 3 \sqrt{10}$  
(b) $-4 \pm 3 \sqrt{10}$  
(c) $2 \pm 3 \sqrt{5}$  
(d) $-2 \pm 3 \sqrt{5}$

Part 5: Answer the following questions based on the information given.
The flow of oil in a pipe, in gallons per hour, can be modeled using the function $F(t) = -2t^2 + 20t + 11$

(a) Using the quadratic formula, find, to the nearest tenth of an hour, the time when the flow stops (is zero). Show your work.

$$x = \frac{-10 \pm \sqrt{122}}{-2} = 10.5$$

(b) Use the process of completing the square to write $F(t)$ in its vertex form. Then, identify the peak flow and the time at which it happens.

$$y = 2(x - 5)^2 + 61$$

61 gallons at 5 hrs

Part 6 History: Watch: http://www.youtube.com/watch?v=vy6vjQ-wlgE

Create a 5-10 minute presentation that showcases some part of the quadratic formula. Explain the significance of this development, how it became developed, or the uses (how to). Construct using: http://www.kizoa.com or other presentation builder. See Rubric for scoring.
12. Characteristics of Quadratic Functions

Begin previewing.
A parabola is: ____________________________________________________________________________________

Exercise #1: Answer the following from the graph of the quadratic function.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>vertex</td>
<td>(4, -6)</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td>(-∞, ∞)</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>[-6, ∞)</td>
<td></td>
</tr>
</tbody>
</table>

End previewing. Begin cooperative learning.

The y-intercept:

Exercise #2: What is the y-intercept given the quadratic function:

(a) \(7x^2 + 2x - 3 = y\)
-3

(b) \(-7 + 16x^2 + 2x + y = 0\)
7

(c) \(2y - 20x^2 = 8x - 4\)
-2

(d) \(4y = 40x^2\)
0

(e) \(y = (x + 4)^2\)
16

(f) \(y = (x - 5)^2\)
25

(g) \(y = (x + 1)^2 - 5\)
1

(h) \(y = 2(x - 1)^2 - 5\)
-3

End cooperative learning. Begin direct instruction.
The axis of symmetry:
The axis of symmetry is: The line of symmetry for a graph

**Exercise #3:** State the axis of symmetry given the graph:

<table>
<thead>
<tr>
<th>Graph 1</th>
<th>Graph 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
</tr>
<tr>
<td>Axis of symmetry: $x = 0$</td>
<td>Axis of symmetry: $x = -3$</td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph 3" /></td>
<td><img src="image4.png" alt="Graph 4" /></td>
</tr>
<tr>
<td>Axis of symmetry: $x = 0$</td>
<td>Axis of symmetry: $x = 3$</td>
</tr>
</tbody>
</table>

**Axis of symmetry:** Given $y = ax^2 + bx + c$ where $a \neq 0$, the axis of symmetry can be found by $x = \frac{-b}{2a}$

**Exercise #4:** State the axis of symmetry given the quadratic function:

<table>
<thead>
<tr>
<th>Quadratic Function</th>
<th>Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x^2 + 2x - 3 = y$</td>
<td>$x = -\frac{1}{7}$</td>
</tr>
<tr>
<td>$-7 + 16x^2 + 2x + y = 0$</td>
<td>$x = -\frac{1}{16}$</td>
</tr>
<tr>
<td>$2y - 20x^2 = 8x - 4$</td>
<td>$x = -\frac{1}{5}$</td>
</tr>
<tr>
<td>$4y = 40x^2$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$y = (x + 4)^2$</td>
<td>$x = -4$</td>
</tr>
<tr>
<td>$y = (x - 5)^2$</td>
<td>$x = 5$</td>
</tr>
<tr>
<td>$y = (x + 1)^2 - 5$</td>
<td>$x = -1$</td>
</tr>
<tr>
<td>$y = 2(x - 1)^2 - 5$</td>
<td>$x = 1$</td>
</tr>
</tbody>
</table>

End direct instruction. Begin cooperative learning.
Exercise #5: State the vertex of the parabola given the graph:

- Graph 1: Vertex: (0, 0)
- Graph 2: Vertex: (-3, -9)
- Graph 3: Vertex: (0, 4)
- Graph 4: Vertex: (3, -10)

Exercise #6: State the vertex given the quadratic function. Determine if function has a maximum or minimum:

(a) \(7x^2 + 2x - 3 = y\)
    Vertex: \((-1/7, -18/7)\) min

(b) \(-7 + 16x^2 + 2x + y = 0\)
    Vertex: \((-1/16, 109/16)\) max

(c) \(2y - 20x^2 = 8x - 4\)
    Vertex: \((-1/5, -12/5)\) min

(d) \(4y = 40x^2\)
    Vertex: \((0, 0)\) min

(e) \(y = (x + 4)^2\)
    Vertex: \((-4, 0)\) min

(f) \(y = (x - 5)^2\)
    Vertex: \((5, 0)\) min

(g) \(y = (x + 1)^2 - 5\)
    Vertex: \((-1, -5)\) min

(h) \(y = 2(x - 1)^2 - 5\)
    Vertex: \((1, -5)\) min

End cooperative learning. Begin modeling, rehearsal and feedback.
The discriminant:  Given \( y = ax^2 + bx + c \) where \( a \neq 0 \), the discriminant can be found by \( \sqrt{b^2 - 4ac} \)

Exercise #7: Calculate the discriminant from the quadratic function. State the number of roots:

(a) \( 7x^2 + 2x - 3 = y \)
\[ 2 \sqrt{22} \] 2 roots

(b) \(-7 + 16x^2 + 2x + y = 0 \)
\[ 8 \sqrt{7} \] 2 roots

(c) \( 2y - 20x^2 = 8x - 4 \)
\[ 4 \sqrt{6} \] 2 roots

(d) \( 4y = 40x^2 \)
0 1 root

(e) \( y = (x + 4)^2 \)
0 1 root

(f) \( y = (x + 1)^2 - 5 \)
2 \( \sqrt{5} \) 2 roots

(g) \( y = 2x^2 + 3x - 7 \)
\[ \sqrt{65} \] 2 roots

(h) \( y = 5x^2 - 2x - 1 \)
2 \( \sqrt{6} \) 2 roots

(i) \( y = -3x^2 + x \)
0 1 root

(j) \( 0 = x^2 - 2x - 3 \)
4 2 roots

(k) \( y = x^2 + 5x + 6 \)
1 1 root

(l) \( y = -6x^2 - 1x + 2 \)
7 2 roots
End modeling, rehearsal and feedback. Begin cooperative learning.

**Quadratic roots:**

**Exercise #8:** Determine the roots of the quadratic from the graph

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Graph</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise #8</td>
<td><img src="image1" alt="Graph 1" /></td>
<td>X = 0, 6</td>
</tr>
<tr>
<td></td>
<td><img src="image2" alt="Graph 2" /></td>
<td>X = 0</td>
</tr>
<tr>
<td></td>
<td><img src="image3" alt="Graph 3" /></td>
<td>X = -2, 2</td>
</tr>
</tbody>
</table>

**Exercise #8:** Determine the roots of the quadratic. Hint: you have already calculated the discriminant.

(a) \(7x^2 + 2x - 3 = y\)  
\[
-x = \frac{-2 \pm \sqrt{4 - 4 \cdot 7 \cdot (-3)}}{2 \cdot 7} = \frac{1 \pm \sqrt{4(7)}}{16}
\]

(b) \(-7 + 16x^2 + 2x + y = 0\)  
\[
x = \frac{-2 \pm \sqrt{4 + 4 \cdot 7}}{2 \cdot 7} = \frac{1 \pm \sqrt{4(7)}}{16}
\]

(c) \(2y - 20x^2 = 8x - 4\)  
\[
-x = \frac{-8 \pm \sqrt{64 + 4 \cdot 20 \cdot 4}}{2 \cdot 20} = \frac{1 \pm \sqrt{4(16)}}{5}
\]

(d) \(4y = 40x^2\)  
\[
x = \frac{-4 \pm \sqrt{16 + 4 \cdot 40}}{2 \cdot 40} = \frac{-1 \pm \sqrt{5}}{5}
\]

(e) \(y = (x + 4)^2\)  
\[
x = \frac{-4 \pm \sqrt{16 + 4 \cdot 1}}{2} = \frac{-1 \pm \sqrt{5}}{5}
\]

(f) \(y = (x + 1)^2 - 5\)  
\[
x = \frac{-1 \pm \sqrt{1 + 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{5}
\]

(g) \(y = 2x^2 + 3x - 7\)  
\[
x = \frac{-3 \pm \sqrt{9 + 4 \cdot 2 \cdot 7}}{4} = \frac{1 \pm \sqrt{6(65)}}{4}
\]

(h) \(y = 5x^2 - 2x - 1\)  
\[
x = \frac{-2 \pm \sqrt{4 + 4 \cdot 5 \cdot (-1)}}{2 \cdot 5} = \frac{2 \pm \sqrt{16(65)}}{4}
\]

(i) \(y = -3x^2 + x\)  
\[
x = \frac{1 \pm \sqrt{1 + 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{1 \pm \sqrt{65}}{4}
\]

(j) \(0 = x^2 - 2x - 3\)  
\[
x = \frac{2 \pm \sqrt{4 + 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{1 \pm \sqrt{65}}{4}
\]

(k) \(y = x^2 + 5x + 6\)  
\[
x = \frac{-5 \pm \sqrt{25 + 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{-5 \pm \sqrt{65}}{2}
\]

(l) \(y = -6x^2 - 1x + 2\)  
\[
x = \frac{1 \pm \sqrt{1 + 4 \cdot 6 \cdot 2}}{2 \cdot (-6)} = \frac{-2 \pm \sqrt{65}}{12}
\]
Integrated Algebra

Name:___________________________________
Date:_________________

Independent Problems:

**Part #1:** (Review) ONLINE: http://www.ixl.com/math/algebra-1/characteristics-of-quadratic-functions

**Part #2:** From the graph determine the y-intercept, the axis of symmetry, the vertex, if it is a maximum or minimum, tell if the discriminant is positive, negative or zero, and determine the roots:

<table>
<thead>
<tr>
<th>Graph</th>
<th>y-intercept</th>
<th>Axis of Symmetry</th>
<th>Vertex</th>
<th>Max / Min</th>
<th>Discriminant</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td>-12</td>
<td>x = -2</td>
<td>(-2, -16)</td>
<td>min</td>
<td>+</td>
<td>-6, 2</td>
</tr>
<tr>
<td><img src="image2.png" alt="Graph 2" /></td>
<td>-5</td>
<td>x = 2</td>
<td>(2, -1)</td>
<td>max</td>
<td>-</td>
<td>none</td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph 3" /></td>
<td>27</td>
<td>x = -6</td>
<td>(-6, -9)</td>
<td>min</td>
<td>+</td>
<td>-3, -9</td>
</tr>
<tr>
<td><img src="image4.png" alt="Graph 4" /></td>
<td>-30</td>
<td>x = 13/2</td>
<td>(13/2, -133/4)</td>
<td>min</td>
<td>+</td>
<td>-3, 10</td>
</tr>
<tr>
<td><img src="image5.png" alt="Graph 5" /></td>
<td>0</td>
<td>x = 0</td>
<td>(0, 0)</td>
<td>min</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><img src="image6.png" alt="Graph 6" /></td>
<td>2</td>
<td>x = 1</td>
<td>(1, ?)</td>
<td>min</td>
<td>-</td>
<td>none</td>
</tr>
</tbody>
</table>

**Part #3:** ONLINE: http://www.ixl.com/math/algebra-1/using-the-discriminant
Part #4: From the quadratic function determine the y-intercept, the axis of symmetry, the vertex, if it is a maximum or minimum, tell if the discriminant is positive, negative or zero, and determine the roots:

(a) \( 3x^2 + 5x - 2 = y \)

- y-intercept: -2
- axis of symmetry: -5/6
- vertex: (-5/6, -49/12)
- max / min: min
- \((+),(-),(0)\) discriminant: +
- roots: 1/3, -2

(b) \( y = x^2 - 4x - 5 \)

- y-intercept: -5
- axis of symmetry: 2
- vertex: (2, -9)
- max / min: min
- \((+),(-),(0)\) discriminant: +
- roots: 5, -1

(c) \( x^2 - 7x + 12 = y \)

- y-intercept: 12
- axis of symmetry: 7/2
- vertex: (7/2, )
- max / min: min
- \((+),(-),(0)\) discriminant: +
- roots: 4, 3

(d) \( 2x = x^2 - y + 3x - 10 \)

- y-intercept: -10
- axis of symmetry: -1/2
- vertex: (-1/2, )
- max / min: min
- \((+),(-),(0)\) discriminant: +
- roots: \(-1 \pm \sqrt{41} / 2\)

(e) \( y = (x + 5)^2 \)

- y-intercept: 25
- axis of symmetry: -5
- vertex: (-5, 0)
- max / min: min
- \((+),(-),(0)\) discriminant: 0
- roots: -5

(f) \( y = (x + 2)^2 - 3 \)

- y-intercept: 1
- axis of symmetry: -2
- vertex: (-2, -3)
- max / min: min
- \((+),(-),(0)\) discriminant: +
- roots: \(-2 \pm \sqrt{3}\)
16. Slope of a Quadratic Functions
Begin cooperative learning and peer tutoring.

Exercise #1: ONLINE:http://mathbitsnotebook.com/Algebra1/Quadratics/QDSlope.html

End cooperative learning and peer tutoring. Begin direct instruction.

Exercise #2: Consider the quadratic $y = x^2 + 4x + 4$

(a) Calculate the average rate of change

<table>
<thead>
<tr>
<th>points being used</th>
<th>average rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2, 0) (-1,1)</td>
<td>1</td>
</tr>
<tr>
<td>(-1,1) (0,4)</td>
<td>3</td>
</tr>
<tr>
<td>(0,4) (1, 9)</td>
<td>5</td>
</tr>
<tr>
<td>(1, 9) (2, 16)</td>
<td>7</td>
</tr>
<tr>
<td>(2, 16)(3 , 25)</td>
<td>9</td>
</tr>
</tbody>
</table>

(b) Graph the points on the graph provided.

(b) Is there a pattern developing regarding the average rate of change?
Yes as $x$ increases by 1 starting at -2, $y = 1, 3, 5, 7, 9, ...$

Exercise #2: Consider the quadratic whose graph is shown.

(a) Find the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

$(1, 0) (3, -4)$
avg = -2

The average rate of change finds the slope of the line through the points (1,0) and (3,-4) as shown.
Since the average rate of change is negative, we know that the function values for the quadratic function, $y = f(x)$ are mostly decreasing on the interval from $x = 1$ to $x=3$. 
Independent Problems:

Part #1: Consider the function \( f(x) = 3x^2 + 6x + 4 \).

(a) Find the average rate of change of the function from \( x = 1 \) to \( x = 4 \).

\[(1, 13) \quad (4, 70)\]
\[\text{avg} = \frac{-57}{-3} = 19\]

(b) Sketch a graph of \( f(x) \) and a graph of the line connecting the corresponding points at \( x = 1 \) and \( x = 4 \).

(c) What does the average rate of change of \( f(x) \) from \( x = 1 \) to \( x = 4 \) calculated in part (a) represent in your picture in part (b)?

increasing slope

Part #2: Given the function \( f(x) = x^2 + 4x - 1 \), find the average rate of change from \( x = 1 \) to \( x = 5 \) and interpret what the average rate of change tells you about the function.

\[(1, 4) \quad (5, 44)\] \[\text{avg} = \frac{-40}{-4} = 10\] increasing on this interval

Part #3: Partner Work:

With your elbow partner, find the average rate of change for the following functions over the given interval. Then, answer the conclusion question.

(a) \( f(x) = 3x + 5 \) from \( x = 2 \) to \( x = 3 \) increasing at slope = 3

(b) \( f(x) = x^2 + 4 \) from \( x = 2 \) to \( x = 3 \) increasing at slope = 5

(c) \( f(x) = 3^x \) from \( x = 2 \) to \( x = 3 \) increasing at slope = 18

d) Write a conclusion (with no less than 4 complete sentences) regarding how you can use the average rate of change to compare the three functions on the given interval.

Ans will vary
17. Applications of Quadratic Functions

Stations: In small groups or partners go around to the different stations and apply your knowledge.

Exercise #1: Consider a rectangle whose area is 45 square feet. If we know that the length is one less than twice the width, then we would like to find the dimensions of the rectangle.

(a) If we represent the width of the rectangle using the variable w, then write an expression for the length of the rectangle in terms of w.

\[ l = 2x - 1 \]

(b) Set up an equation that could be used to solve for the width, w, based on the area.

\[ l \times w = x (2x - 1) = 45 \]

(c) Solve the equation to find both dimensions. Why is one of the solutions for w not viable?

\[ \frac{1 \pm \sqrt{361}}{4} \]

One solution is not viable because it's negative – distance cannot be negative.

Exercise #2: A square has one side increased in length by two inches and an adjacent side decreased in length by two inches. If the resulting rectangle has an area of 60 square inches, what was the area of the original square? First, draw some possible squares and rectangles to see if you can solve by guess-and-check. Then, solve it algebraically.

\[ X = 4 \sqrt{15} \]
Exercise #3: There are two rational numbers that have the property that when the product of seven less than three times the number and one more than the number is found it is equal to two less than ten times the number. Find the two rational numbers that fit this description.

5 and -2/6

Exercise #4: Find all sets of consecutive integers such that their product is eight less than ten times the smaller integer.

1, 2
8, 9

Exercise #5: Brendon claims that the number five has the property that the product of three less than it with one more than it is the same as the three times one less than it. Show that Brendon’s claim is true and algebraically find the other number for which this is true.

It is true for 0 as well
The Five Step method for solving word problems

Step 1: Let $x$ (or some other letter) represent the unknown quantity.
Step 2: Translate the verbal expression to mathematical symbols and form an equation.
Step 3: Solve this equation.
Step 4: Check the solution by substituting the result into the equation found in step 2.
Step 5: Write a conclusion.

Exercise #6: A producer of personal computer mouse covers determines that the number $N$ of covers sold is related to the price $x$ of a cover by $N = 35x - x^2$: At what price should the producer price a mouse cover in order to sell 216 of them?

8 or 27

Exercise #7: It is estimated that $t$ years from now the population of a particular city will be $P = t^2 - 24t + 96,000$:

How many years from now will the population be 95,865?

9 and 15 years from now

Exercise #8: The length of a rectangle is 4 inches more than twice its width. The area is 30 square inches. Find the dimensions (length and width).

$x (2x + 4) = 30$

$x = -5$, 3 but can't be negative so $x=3$

length = 10
**Exercise #9:** The product of two consecutive integers is 156. Find them.

12, -13

**Exercise #10:** A box with no top and a square base is to be made by cutting out 2-inch squares from each corner and folding up the sides of a piece of a square cardboard. The volume of the box is to be 8 cubic inches. What size should the piece of cardboard be?

\[ X = 2, 6 \]

only 6 checks.
Exercise #11: A study of the air quality in a particular city by an environmental group suggests that t years from now the level of carbon monoxide, in parts per million, in the air will be

\[ A = 0.3t^2 + 0.1t + 4.2 \]

(a) What is the level, in parts per million, of carbon monoxide in the air now?

\[ A = 4.2 \]

(b) How many years from now will the level of carbon monoxide be at 8 parts per million?

\[ t = 3.4 \text{ and } -3.73 \]

About 3.4 years from now the carbon monoxide level will be 8

Exercise #12: A contractor is to pour a concrete walkway around a swimming pool that is 20 feet wide and 40 feet long. The area of the walkway is to be 544 square feet. If the walkway is to be of uniform width, how wide should the contractor make it?

\[ x = 4 \text{ or } x = -34 \] has no physical meaning.

The contractor should make the walkway 4 feet wide.
Independent Problems:

**Part #1:** The product of two consecutive positive even integers is 14 more than their sum. Set up an equation that can used to find the two numbers and solve it.

\[2x(2x+2) = 2x+2x+2+14\]

2 and 4

**Part #2:** The length of a rectangle is 4 less than twice the width. The area of the rectangle is 70. Find the width, w, of the rectangle algebraically. Explain why one of the solutions for w is not viable.

5 and -7 (does not make sense)

**Part #3:** Two sets of three consecutive integers have a property that the product of the larger two is one less than seven times the smallest. Set up and solve an equation that can be used to find both sets of integers.

3 and 1
1,2,3
3,4,5
Part #4: A curious pattern occurs in a group of people who all shake hands with one another. It turns out that you can predict the number of handshakes that will occur if you know the number of people.

If we are in a room of 5 people, we can determine the number of handshakes by this line of reasoning:

The first person will shake 4 hands (she won’t shake her own).
The second person will shake 3 hands (he won’t shake his own of the hand of the first person, they already shook).
The third person will shake 2 hands (same reasoning).
The fourth person will shake 1 hand (that of the fifth person).
The fifth person will shake 0 hands. So there will be a total of 1+2+3+4=10 handshakes

(a) Determine the number of handshakes, h, that will occur for each number of people, n, in a particular room.

<table>
<thead>
<tr>
<th>n (people)</th>
<th>Calculation</th>
<th>h (handshakes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2+1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3+2+1</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>1 + 2 + 3 + 4 = 10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>5+4+3+2+1</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>6+5+4+3+2+1</td>
<td>21</td>
</tr>
</tbody>
</table>

(b) Prestel poses the formula \( \frac{n(n-1)}{2} \). Determine if the formula is correct in this instance.

Let \( n = 7 \), \( 7(6) = 42/2 = 21 \)

(c) Assuming Prestel’s formula is correct, algebraically determine number of people in a room if there are 66 handshakes that occur.

\[
66 = \frac{x(x-1)}{2} = 132 = x^2 + x
\]

\( x = 11 \) or \( x = -12 \) (doesn't make sense)
Part #5: A manufacturer of cloth personal computer dust covers notices that the number \( N \) of covers sold is related to the price of covers by \( N = 30x - x^2 \): At what price should the manufacturer price the covers in order to sell 216 of them?

12 or 18

Part #6: The length of a rectangle is 3 feet more than twice its width. The area is 14 square feet. Find the dimensions.

\[ x = 2 \text{ or } x = -7/2 \text{ (doesn't make sense)} \]

Part #7: The area of a triangle is 24 square meters. The base is 2 meters longer than the height. Find the base and height. The formula for the area of a triangle is \( A = 1/2 \cdot bh \).

\[ x = 6 \text{, or } x = -8 \text{ (doesn't make sense)} \]
**Part #8:** The product of two consecutive integers is 210. Find them

14 and -15 so either -15, 16 or 14 and 15

**Part #9:** Four is added to an integer and that sum is tripled. When this result is multiplied by the original integer, the product is –12. Find the integer.

-2

**Part #10:** A box with no top and a square base is to be made by cutting 3-inch squares from each corner and folding up the sides of a piece of cardboard. The volume of the box is to be 48 cubic inches. What size should the piece of cardboard be?

X = 2, and 10 ; 2 is not viable. So x = 10
Part #11: A study of the air quality in a particular city by an environmental group suggests that \( t \) years from now the level of carbon monoxide, in parts per million, in the air will be
\[
A = 0.2t^2 + 0.1t + 5.1
\]

(a) What is the level, in parts per million, now?

5.1

(b) How many years from now will the level of carbon monoxide be at 8 parts per million?
Round to the nearest tenth.

3.6 years

Part #12: A contractor is to pour a concrete walkway around a swimming pool that is 15 feet wide and 25 feet long. The area of the walkway is to be 276 square feet. If the walkway is to be of uniform width, how wide should the contractor make it?

\[ x = 3 \text{ and } x = -23 \]

The contractor should make the walkway 3 feet wide.
18. Project: Modeling with Quadratic Functions

Part #1: Goal: Students will research, model, create and sell a product for their school fund raising activities. They will develop the maximum profit through the use of quadratics and present their product to the sales committee explaining why this product will be viable and successful.

Watch how professionals market their products for financing:
http://www.youtube.com/watch?v=7C0LP1ehmVk

Directions: Research the product you want to sell for fund raising activities. Give your product a name. Make a guess as to what your customer would pay for this product. Find all the components you would need to make your product and complete the information below.

Example!!

My Product Name: flower pens

Estimated Product cost to consumer: $ 2 or 3 dollars

Directions: Create a spreadsheet labeled “components" to reflect the table below.

Components:
A: Enter the amount needed for one product under QTY
B: Enter the unit cost as exactly one component times the QTY
C: Enter the amount sold by manufacturer/retail in one package for PKG Qty
D: Enter the cost of the package from the manufacturer/retail under total $

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>QTY</th>
<th>Unit cost</th>
<th>PKG Qty</th>
<th>Total $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flowers</td>
<td>1</td>
<td>.20</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Pens</td>
<td>1</td>
<td>.06</td>
<td>50</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Tape</td>
<td>1</td>
<td>.04</td>
<td>30</td>
<td>1.20</td>
</tr>
</tbody>
</table>
**Integrated Algebra**

**Name:** ___________________________  
**Date:** ___________________________

**Part #2:** Goal: make estimations on your initial costs based on the product you want to sell.

**Estimations:** From the spreadsheet in part 1 calculate:

A: The minimum number of products to be constructed from PKG Qty: 5

   How many components would not be used? 70  Excess $ 2.7 and 1.0 = $3.7

B: The Maximum number of products to be constructed from PKG Qty: 50

   How many components would not be used? 10  Excess $.40

C: What is the total cost for one product? 30 cents  This is the cost per item.

**Initial Cost:**

Research a definition for initial cost and complete below:

**Definition:** costs incurred during the design and construction process

______________________________________________________________________________________

**Spread sheet application:**

Add a new tab on your spreadsheet to reflect initial costs:

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pizza party to make products</td>
<td>25.00</td>
</tr>
<tr>
<td></td>
<td>Losses</td>
<td>.30</td>
</tr>
</tbody>
</table>

Construct your product to use the lowest excess cost available. Add this amount to the Initial cost.

*Your Initial cost $ 25.30*
Part #3: Goal: Develop an understanding for demand curves and modeling these using graphing software.

Demand Curve:

An example:

$700,000 for manufacturing set-up costs, advertising, etc
$110 to make each bike

Based on similar bikes, you can expect sales to follow this "Demand Curve":

- Unit Sales = 70,000 - 200P

Where "P" is the price.

For example, if you set the price:

- at $0, you would just give away 70,000 bikes
- at $350, you would not sell any bikes at all.
- at $300 you might sell 70,000 - 200×300 = 10,000 bikes

Developing a demand curve usually entails understanding previous sales or estimating projection sales by market demands. The coordinate points would be an estimated range from (free item, max units sold) and (max price – no one would pay, min units sold – usually zero)

y=mx+b, b is the maximum units sold at zero, calculate m (slope) from the max,min point. (rise/run)

The linear function of the demand curve is NOT an absolute. It is a starting point to develop and model the ideal product sales. Unit sales will often be tweaked as other developments arise.

Graphing software application: Geogebra
Model your Demand curve

From this model present the linear equation of the unit sales.

What is your Unit Sales: ________________________________

Use “p” as the price.
Part #4: Goal: Understand vocabulary, and set up of various types of business applications.

Developing modeling skills.

Read the following information about Business Applications:

Many business contexts can be modeled with quadratic functions. This is because the expressions representing price (price per item), the cost (cost per item), and the quantity (number of items sold) are typically linear. The product of any two of those linear expressions will produce a quadratic expression that can be used as a model for the business context. The variables used in business applications are not as traditionally accepted as variables are in physics applications, but there are some obvious reasons to use c for cost, p for price, and q for quantity (all lowercase letters). For total production cost we often use C for the variable, R for total revenue, and P for total profit (all uppercase letters).

Business Application Vocabulary:

Unit Price (Price per Unit): The price per item a business sets to sell its product, sometimes represented as a linear expression.

Quantity: The number of items sold, sometimes represented as a linear expression.

Revenue: The total income based on sales (but without considering the cost of doing business).

Unit Cost (Cost per Unit) or Production Cost: The cost of producing one item, sometimes represented as a linear expression.

Profit: The amount of money a business makes on the sale of its product. Profit is determined by taking the total revenue (the quantity sold multiplied by the price per unit) and subtracting the total cost to produce the items (the quantity sold multiplied by the production cost per unit): Profit = Total Revenue − Total Production Cost.

The following business formulas will be used in this next examples:

Total Production Costs = (cost per unit)(quantity of items sold)

Total Revenue = (price per unit)(quantity of items sold)

Profit = Total Revenue − Total Production Costs

Quizlet application:

Create note cards for the vocabulary words on quizlet.com, then test your knowledge.
Part #5: Goal: Apply knowledge of business application to word problems.

Exercise #1: A theater decided to sell special event tickets to benefit a local charity at $10 per ticket. The theater can seat up to 1,000 people and they expect to be able to sell all 1,000 seats for the event. To maximize the revenue for this event, a research company volunteered to do a survey to find out if they could increase the ticket price without losing revenue. The results showed that for each $1 increase in ticket price, 20 fewer tickets will be sold.

a. Let \( x \) represent the number of $1.00 price-per-ticket increases. Write an expression to represent the expected price for each ticket:

b. Use the survey results to write an expression representing the possible number of tickets sold.

c. Using \( x \) as the number of $1-ticket price increases and the expression representing price per ticket, write the function, \( R(x) \), to represent the total revenue in terms of the number of $1-ticket price increases.

d. How many $1-ticket price increases will produce the maximum revenue? (i.e., what value for \( x \) produces the maximum \( R \)?)
e. What is the price of the ticket that will provide the maximum revenue?

f. What is the maximum revenue?

g. How many tickets will the theater sell to reach the maximum revenue?

h. How much more will the theater make for the charity by using the results of the survey to price the tickets than they would had they sold the tickets for their original $10 price?
Exercise 2: Amazing Photography Studio takes school pictures and charges $20 for each class picture. The company sells an average of 12 class pictures in each classroom. They would like to have a special sale that will help them sell more pictures and actually increase their revenue. They hired a business analyst to determine how to do that. The analyst determined that for every reduction of $2 in the cost of the class picture, there would be an additional 5 pictures sold per classroom.

a. Write a function to represent the revenue for each classroom for the special sale.

b. What should the special sale price be?

c. How much more will the studio make than they would have without the sale?
Group work Questions:

a. What is the relevance of the vertex in business applications?

b. Katrina developed an app (application) that she sells for $5 per download. She has free space on a website that will let her sell 500 downloads. According to some research she did, for each $1 increase in download price, 10 fewer apps are sold. Determine the price that will maximize her profit.
Part #6: Goal: Complete your product analysis for your presentation.

Directions: Complete the following information from the data you have gathered and by the definitions.

________________________________________________ initial cost.

________________________________________________ cost per item.

________________________________________________ unit sales.

Calculate:

________________________________________________ dollar sales = (unit sales)(price=p)

________________________________________________ cost = ( initial cost + (cost per item)(unit sales)

________________________________________________ profit = dollar sales – costs

Graphing software application: Geogebra
Model your profit

Profit is your quadratic. Graph the quadratic showing the roots, vertex, and line of symmetry from steps below:

Maximum profit (1):
(a) Find the roots of the quadratic using the quadratic equation:

(b) Find the average of the roots: (This is the sales price.)

(c) Find the axis of symmetry using x=-b/2a (this should be the same number as above)
Integrated Algebra

Name:___________________________________ Date:_________________

(d) Evaluate the sales price in the profit quadratic. (This is maximum profit)

(e) Evaluate the sales price in the cost. (This is minimum cost at maximum profit)

(f) Evaluate the sales price in the unit sales. (This is maximum sales at maximum profit)

(g) How many units need to be sold?  ____________________________________________________________

(h) How much will this cost?  _________________________________________________________________

(i) How much profit will be made?  ______________________________________________________________
Part #7: Goal: Create a presentation using the standards of the rubric.

Video: Product analysis 7:40

Create a presentation Using the product analysis, graphs and data from the software above. Describe your product, why this product is profitable, why it should be the next product made, and how you can validate this.

Presentation software application: kizoa.com
Use kizoa or other presenting software to showcase your product.