A Spiraling Curriculum Project on Factoring in Algebra I Aligned with the New York State Common Core State Standards

Katie Elkins
The College at Brockport, kelki1@brockport.edu

Follow this and additional works at: http://digitalcommons.brockport.edu/ehd_theses
Part of the Education Commons

To learn more about our programs visit: http://www.brockport.edu/ehd/

Repository Citation
http://digitalcommons.brockport.edu/ehd_theses/623

This Thesis is brought to you for free and open access by the Education and Human Development at Digital Commons @Brockport. It has been accepted for inclusion in Education and Human Development Master’s Theses by an authorized administrator of Digital Commons @Brockport. For more information, please contact kmyers@brockport.edu.
A Spiraling Curriculum Project on Factoring in Algebra I Aligned with the New York State Common Core State Standards

Katie Elkins

Fall 2015

A thesis project submitted to the Department of Education and Human Development of the State University of New York College at Brockport

In partial fulfillment of the requirements for the degree of Master of Education – Adolescent Mathematics
# Table of Contents

Chapter 1: Introduction .................................................................3

Chapter 2: Literature Review ..........................................................4
  The Paradigm shift from NCTM to CCSS ....................................4
  Cognitive Load Theory ...............................................................5
  Spiral Curriculum ......................................................................6

Chapter 3: Curriculum Project .........................................................8
  Introduction ..............................................................................8
  Unit Plan ................................................................................10

Chapter 4: Validity of Curriculum Project .......................................34

Chapter 5: Conclusion ..................................................................37

References ..................................................................................39

Appendix: Unit Plan Answer Keys ..................................................41
Chapter One: Introduction

It is a common trend that while studying for a cumulative exam, students often forget the material they were taught in the beginning of the school year. They then struggle to relearn the content all over again in a short amount of time in order to perform well on the exam. What if there was a way to teach students well enough the first time so that this relearning process did not need to occur at the end of the year?

Cognitive Load Theory, a theory that explains how learning occurs, states that the definition of learning is a permanent change in long term memory (Van Merrienboer 2005). If this permanent change in a student’s long term memory does not occur before they move onto a new unit or mathematical topic, the student will have to repeat the learning process all over again. This makes learning a new topic incredibly difficult for a student when they do not have the necessary prior knowledge stored in their long term memory. Instead the student must now waste time to relearn old information while trying to learn new information.

A Spiraling Curriculum is a curriculum designed to help this permanent change in a student’s long term memory occur. By activating students’ relevant prior knowledge and continuously returning to topics at a progressively deeper level, connections can be taught throughout topics or units. The Common Core State Standards also have a goal of student learning that they intend to achieve by narrowing and deepening the curriculum, while providing opportunities to tie topics together. A Spiraling Curriculum will use these standards and provide a structure so that students can see and learn connections from topic to topic. The Common Core State Standards (CCSS), Cognitive Load Theory (CLT), and a Spiraling Curriculum all share this common goal of student learning.
Chapter Two: Literature Review

The Paradigm shift from NCTM to CCSS

The United States (US) education system is in the midst of a paradigm shift with the change from state standards (or the National Council Teachers of Mathematics (NCTM) Standards) to the 2010 Common Core State Standards (CCSS) (Porter, 2011). The new standards were released five years ago, but adapting to changes in standards takes time, and educators are still adapting their curriculum materials. A national curriculum offers benefits such as: shared expectations, focus, efficiency and quality of assessments. (Porter, 2011, p.103). More specifically, Engage New York (EngageNY), materials developed by New York State (NYS) to support the shift to the CCSS, lists six Instructional Shifts for mathematics: focus, coherence, fluency, deeper understanding, application and dual intensity. These instructional shifts require teachers to focus on fewer topics to support deeper student understandings, gain greater skill and fluency, and more robustly apply what is learned. Before the CCSS, most states used the NCTM Standards, which were first released in 1980 with the most recent version being from 1989. Porter, McMaken, Hwang, and Yang (2011) compared the CCSS to the NCTM Standards and revealed an overall modest shift toward higher levels of cognitive demand. More specifically, the CCSS emphasized understanding more than placing emphasis on memorizing and performing procedures (Porter, 2011). This is an important change for educators to understand since it will impact the way they teach and assess their students in preparation for the Common Core assessments. In the end, the goal is that the CCSS will help students make more connections within the curriculum. In high school the challenge teachers faced with the NCTM Standards was too many separately memorized techniques with little structure to tie them
altogether. So, narrowing and deepening the curriculum is not so much a matter of eliminating topics, as seeing the structure that ties them together. (Noh, 2012).

**Cognitive Load Theory**

Cognitive Load Theory (CLT) is one of the many theories that explains how learning occurs. Van Merrienboer (2005) stated that CLT used “interactions between information structures and knowledge of human cognition to determine instructional design” (p. 147). CLT works with balancing three types of cognitive load: extraneous, intrinsic and germane. Extraneous load is not necessary for learning and usually results from poor instruction. Intrinsic load depends on the expertise of the learner and the amount of element interactivity present in the task (see Van Merrienboer & Sweller (2005) for most recent work with element interactivity). Finally, germane load directly relates to the learning and processing of the task and the construction of the learners’ cognitive system (Merrienboer, 2005). Furthermore, Ayres (2006) argued that “the three sources of cognitive load are additive and the total cognitive load is given by the sum of the three” (p.288). In general, it is ideal to reduce extraneous and intrinsic cognitive load so that germane load can be maximized. However, each student can handle different amounts of intrinsic load based on their prior knowledge and expertise of mathematics.

CLT also explains the relationship between the working memory and long term memory. Van Merrienboer (2005) stated that working memory stores about seven elements but operates on just two to four elements at a time for no more than a few seconds unless refreshed by rehearsal. Furthermore, CLT defines learning as a permanent change in long term memory. CLT presents one way for teachers to consider how their students learn, and it aligns nicely with the idea of a spiraling curriculum.
Spiral Curriculum

CLT is important for teachers to consider when teaching mathematics because mathematics builds or spirals off of previously learned topics. When you learn a mathematical topic, the information from one unit carries onto the next unit. For example, after learning how to factor, students then learn how to reduce rational expressions, which requires knowing how to factor. Therefore, it is beneficial for students to keep practicing previously learned material so that it is easier to recall when they need it to solve new problems. Students should be able to transfer key mathematical concepts into their long term memory or else it will be difficult to recall previously learned information. The sense of repetition is what helps store information into the long term memory. Van Merrienboer (2005) explains that automation can free working memory capacity for other activities because an automated schema can directly steer behavior without needing to be processed in the working memory. (p.149). Therefore, back to our factoring example, if the students have practiced factoring long enough, this knowledge is then stored into their long term memory and then will free up space in their working memory to deal with the new concept of reducing rational expressions.

Ernest (1996) states that the main idea of a Spiraling Curriculum in mathematics is that by returning to a topic at progressively more difficult levels again and again throughout the years of schooling, integration and continuity in learning can be achieved (p.7). Furthermore, by continually integrating key math concepts into the curriculum students will have an easier time being able to recall needed information. If students can use their prior knowledge without having to take up space in their working memory then there will be more space to learn a new topic. Teachers need to correctly assist students in retrieving the most relevant, correct prior knowledge and not prior knowledge that will confuse students.
SPIRALING CURRICULUM PROJECT ON FACTORING

The concept of a spiral curriculum was first described by Jerome Bruner in 1960. Bruner was struck by the fact that successful efforts to teach mathematics took the form of a metamorphic spiral. First, a set of ideas or operations were introduced in a rather intuitive way and once mastered in that spirit, were then revisited and reconstructed in a more formal or operational way. Then these ideas were connected with other knowledge, causing the mastery at this stage to be carried one step higher broadening the level of abstraction and comprehensiveness. The end goal of this process was eventually mastery of the topic as a whole. (Harden, 1999, p.141).

The spiral design also allows for continuous review throughout the academic year. Review built into the curriculum will help students to understand that what they are learning will continuously build off of previously learned topics. Snider (2004) describes effective review as being sufficient enough to promote fluency, being distributed over time, being cumulative with new information integrated into more complex skills, and being varied enough to facilitate generalization (p.34). Therefore, a Spiral Curriculum for mathematics is supported by the CLT because students will not be overwhelmed with information if it is taught in a spiraling design. This is as an effective way for students to learn and retain information in a mathematics classroom.
Chapter Three: Curriculum

Introduction

Each lesson is designed using The Spiral Curriculum Design and begins with a warm up section titled, “Remember when…” It is in this section where students will activate relevant prior knowledge that is necessary to revisit before learning that day’s new topic. While activating this prior knowledge, students are returning to topics with a deeper understanding and are also connecting topics to new material. By doing this, it is intended that students’ extraneous cognitive load will be reduced, allowing their working memories to free space to learn new topics. Furthermore, revisiting previous learned material is necessary repetition that will aid students in making a permanent change in their long term memory. Also each lesson is concluded with a homework that will allow students to continuously build off of that day’s lesson while including material from previous lessons. This will again, aid in the process of making that permanent change in students’ long term memory as well as help to build connections from various topics.

This unit plan aligns with The New York State Common Core Module 4, Lessons 1 through 4. In Module 4, students continue to: interpret expressions; create equations; rewrite equations and functions in different but equivalent forms; and graph and interpret functions. In previous modules, students worked with linear and exponential functions, but they will now focus on polynomial functions, specifically quadratic functions, as well as square root and cube root functions (Engage NY). However, in lessons one through four the main focus is on factoring polynomials, which is important for students to understand before they start graphing, solving and interpreting quadratics. Being able to factor an expression is essential to finding solutions and graphing the function. Therefore, it is important that students master the skill of factoring in order to be successful in the rest of the unit. Please note that the worksheets in this unit plan do
SPIRALING CURRICULUM PROJECT ON FACTORING

not follow APA format, for example, the margins and font may vary. The reason for this is because these lessons are intended to be used in a classroom setting.
Lesson One GCF/Difference of Two Perfect Squares

Introduction: This lesson is designed for an Algebra I class with 45 minute periods. This is the first day of the factoring unit. This lesson will introduce the greatest common factor and the difference of two perfect squares factoring methods.

How Spiraling is incorporated in this lesson:

The Spiraling Curriculum is designed to revisit topics to develop deeper understanding. Before students learn the greatest common factor and the difference of two perfect squares it is important that they remember how to distribute and multiply binomials. Practicing these skills in the beginning of the lesson is important to recall prior knowledge. The goal is for students to connect that distributing and factoring out the greatest common factor are inverses of each other. As well as, multiplying binomials and factoring are inverses of each other. The goal is to reduce students’ extraneous cognitive load, which will free working memory space to learn the new factoring skills.

Common Core State Standards

A-SSE Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Learning Objectives: Students will be able to: (a) interpret parts of an expression, such as terms, factors, and coefficients; (b) use the structure of an expression to identify ways to rewrite it; (c) identify the greatest common factor in any given expression; (d) write the greatest
SPIRALING CURRICULUM PROJECT ON FACTORING

common factor of an expression as the product of two factors; (e) recognize when a binomial can be factored using the difference of two perfect squares; (f) recognize that a perfect square is more than just a number but also a variable with an even exponent with a coefficient that is a perfect square; (g) write the expression as a product of two binomial factors.
Lesson 1
GCF, Difference of Two Perfect Squares

Remember when…

- Distribute –

Examples:

1. $3(2x^2 - 4x + 7)$

2. $2x(x^3 + 2x^2 + 3x + 1)$

3. $5ab(3a + 4)$

- F.O.I.L. –

Examples:

1. $(x + 5)(x - 5)$

2. $(3a - b)(3a + b)$

3. $(y^3 + y^2)(y^3 - y^2)$

4. $(x^2 + y^2)(x^2 + y^2)$
SPIRALING CURRICULUM PROJECT ON FACTORING

What do you notice about the first three examples? Why does the fourth example not work out the same?

Now let’s FACTOR!

- Greatest Common Factor –

To factor using the GCF:

1. Find the GCF
2. Divide each term by the GCF
3. Write in factored form – (think of un-distributing)

Find the GCF and then write in factored form:

Examples:

1. \(10x + 10\)

2. \(4x^3 + 6x^2 + 2x\)

3. \(4x^3 y^2 + 12x^2 y + 20xy^2\)

How are factoring out the GCF and distributing related?
Difference of Two Perfect Squares

Looking back at our first three F.O.I.L. examples, what do you notice about your final answers?

Let’s recall our Perfect squares:

If we call the first term $a^2$ and the second term $b^2$ can we come up with a formula to help us factor $a^2 - b^2$:

Note: this does not work for the sum of two perfect squares!

Examples:

1. $x^2 - y^2$

2. $25a^2 - 100b^4$

3. $36x^2y^2 - 64x^4y^4$

4. $x^4 - y^4$

WAIT! What do we notice about this last example…?

Later in the unit we will be working on examples with several steps of factoring!
Name: ______________________

Homework – Lesson 1

1. Factor: $6a^2b - 12ab^2$

2. Factor: $25t^2 - 49$

3. Factor: $38x^5 - 22x^3$

4. What is the product of $2d - 7$ and $2d + 7$

5. Factor: $36g^4 - 81h^6$

6. Subtract $3x^2 - 4x + 5$ from $6x^2 - x + 5$. Write your answer in factored form.

7. Find the solution set: $4x - 2(x + 3) \geq 18 + 8x$

8. Write your answer to number 7 in interval notation:
Lesson Two Trinomials ($a = 1$)

Introduction: This lesson is designed for an Algebra I class with 45 minute periods. This is the second day of the factoring unit. This lesson will introduce factoring a trinomial where the leading coefficient is equal to one.

How Spiraling is incorporated in this lesson:

It is part of the Spiraling Curriculum design to revisit topics to develop deeper understanding. Students will be revisiting the concept of multiplying binomials with a new understanding of how this process can be reversed. Before students learn this type of factoring, it is important that they are confident in how to multiply binomials. Practicing this skill in the beginning of the lesson is essential in order to trigger this relevant prior knowledge. By doing this, it is intended that students will understand that multiplying binomials and factoring are inverses of each other. Also, students will reaffirm the results of multiplying negative and positive numbers. This may seem like a simple skill that students should not need to review, but having this knowledge fresh in their minds will be necessary to help them factor. This is intended to also reduce students’ extraneous cognitive load, which will free working memory space to learn these new factoring skills.

Common Core State Standards

A-SSE Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

Learning Objectives: Students will be able to: (a) interpret parts of an expression, such as terms, factors, and coefficients; (b) use the structure of an expression to identify ways to rewrite
it; (c) factor a trinomial that has a leading coefficient of one; (d) represent their answer as a product of factors, in this case a product of two binomials.
Lesson 2
Trinomials ($a = 1$)

Multiply the following:

1. $(x + 6)(x + 3)$
2. $(x + 7)(x - 2)$
3. $(x - 3)(x - 4)$
4. $(x - 8)(x + 5)$

Let’s take a look at our signs…

We know that:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>A positive times a positive</td>
<td>A positive</td>
</tr>
<tr>
<td>A negative times a negative</td>
<td>A negative</td>
</tr>
<tr>
<td>A positive times a negative</td>
<td>A negative</td>
</tr>
</tbody>
</table>

We can keep this in mind when we factor!

Again, how is F.O.I.L. related to factoring?

So, we will be working backwards from our examples above.

When factoring a trinomial: $ax^2 + bx + c$ when $a = 1$:

1. List the factors of $ax^2$ (In this case where $a = 1$, your factors will always be $x \times x$.
2. List the factors of $c$
3. Pick two factors that add to $b$
4. Pay attention to the signs!!
   - If $c$ is positive, you will have two positive or two negative factors
   - If $c$ is negative, you will have one positive and one negative factor
   - To further determine, check if $b$ is positive or negative
5. Check your answer using F.O.I.L.
Now, let’s FACTOR!

Examples:

1. \(x^2 - 8x + 15\)
   - Factors of 15: 1, 15
   - 3, 5
   Since \(c = 15\) is positive, we know we want our factors to both be positive or both be negative
   Since \(b = -8\) is negative, we will choose two negative factors
   \[-3 \times -5 = 15\]
   \[-3 - 5 = -8\]
   So, \(x^2 - 8x + 15\) factors to: \((x - 3)(x - 5)\)
   Check:
   \((x - 3)(x - 5) = x^2 - 5x - 3x + 15\)
   \[= x^2 - 8x + 15\]

2. \(x^2 + 12x + 20\)
   - \(c\) is _____________, \(b\) is _____________, so my factors will be _____________ and _____________
   - Factors of 20:
   - Factors that add to 12:
   So, \(x^2 + 12x + 20\) factors to:
   Check:

3. \(b^2 + 2b - 35\)
   - \(c\) is _____________, \(b\) is _____________, so my factors will be _____________ and _____________
   - Factors of 35:
   - Factors that add to 2:
   So, \(b^2 + 2b - 35\) factors to:
   Check:
SPIRALING CURRICULUM PROJECT ON FACTORING

Factor the following with a partner using the steps above. Don’t forget to check your answer!

4. \( b^2 - 6b - 16 \)

5. \( x^4 - 7x^2 - 30 \)

6. \( a^6 + 5a^3 + 6 \)

7. \( x^2 - 6xy - 16y^2 \)

BONUS: (Notice your \( a \) term is not equal to 1 this time. Try some guessing and checking using F.O.I.L. still)

8. \( 3x^2 - x - 10 \)
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________
Homework – Lesson 2

Factor the following:

1. \( a^2 + 10a + 21 \)

2. \( y^2 - 7y - 30 \)

3. \( 24a^2b + 30ab \)

4. \( g^2 - 10g \)

5. \( p^2 - 9p + 8 \)

6. \( x^4 + 12x^2 + 32 \)

7. \( c^2 - 17c - 84 \)

8. \( u^6 - 13u^3 - 30 \)

9. \( 16c^2 - 121 \)

10. \( x^2 - 12x + 32 \)

11. \( h^2 + 14h + 40 \)

12. \( t^2 - 10t - 39 \)

13. \( 225 - x^4 \)

14. \( 3y + 12y^3 \)

15. \( w^2 - 22w + 57 \)

16. \( \frac{x^2}{4} - \frac{y^2}{9} \)
Lesson Three: Grouping

Introduction: This lesson is designed for an Algebra I class with 45 minute periods. This is the third day of the factoring unit. This lesson will introduce factoring a polynomial with four terms.

How Spiraling is incorporated in this lesson:

It is part of the Spiraling Curriculum design to revisit topics to develop deeper understanding. Students will be revisiting the concept of multiplying binomials and finding the greatest common factor with a new understanding of how this process is related to factoring by grouping. Before students learn this type of factoring, they will practice finding the greatest common factor again and multiplying binomials. However, these practice problems will be a little different from the previous days so that they are related to factoring by grouping. For example, students will recognize that the greatest common factor can look different than just one term, it can be a group of terms. Also when multiplying binomials it is possible for your answer to consist of four terms instead of three. Practicing these skills in the beginning of the lesson is essential in order to trigger this relevant prior knowledge. This is intended to also reduce students’ extraneous cognitive load, which will free working memory space to learn these new factoring skills.

Common Core State Standards

A-SSE Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

Learning Objectives: Students will be able to: (a) interpret parts of an expression, such as terms, factors, and coefficients; (b) use the structure of an expression to identify ways to rewrite it; (c) factor a polynomial with four terms and write their answer as a product of factors.
SPIRALING CURRICULUM PROJECT ON FACTORING

Lesson 3
Grouping

More practice with GCF and F.O.I.L.!

Examples:
Find the greatest common factor and factor:

1. \( x^3y + 2x^2y^2 + 3xy^3 \)  
2. \( 3x^2(x + 4) - 2(x + 4) \)

3. \( y(x^2 + y) + x(x^2 + y) \)  
4. \( 24a^2b^2 - 8a \)

F.O.I.L. the following:

1. \( (x^3 + 3)(x - 2) \)  
2. \( (2a - 3)(3b + 7) \)

3. \( (x^2 - 5)(x + 6) \)  
4. \( (x^2 + 4)(y - 3) \)

What do you notice that is different from your F.O.I.L. solutions compared to the previous days? Why do you think this is occurring?
Now, let’s FACTOR!

We will learn a factoring method called Grouping to factor expressions with four terms.

Steps to factor by Grouping:
1.
2.
3.
4.

Examples:
1. \(3x^3 + 6x^2 - 5x - 10\)

2. \(2x^4 + 10x^3 + 3x + 15\)

3. \(x^2y - 3x^2 - 2y + 6\)

4. \(8x^3 - 20x^2 + 2x - 5\)

5. \(30ab - 42a + 25b - 35\)
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________
Homework – Lesson 3

Factor the following:

1. \(2x^4 - 3x^3 - 8x + 12\) 

2. \(100a^2 - x^2\) 

3. \(6x^4 - x^3 + 30x - 5\) 

4. \(-x^2 - 10x - 25\) 

5. \(x^3 + 4x^2 + 22x\) 

6. \(36c^4d^4 - 49f^6\) 

7. \(28u^2 + 7u\) 

8. \(x^2 - 12x - 45\) 

9. \(3x^3 + 18x^2 + 2x + 12\) 

10. \(4ab^2 - 12a^2b\) 

11. \(3x^7 - 5x^5 + 3x^2 - 5\) 

12. \(12x^3 - 16x^2 - 9x + 12\) 

13. \(x^2 - 7x - 44\) 

14. \(a^2 + 22a + 72\)
Lesson Four: Trinomials ($a \neq 1$)

Introduction: This lesson is designed for an Algebra I class with 45 minute periods. This is the fourth day of the factoring unit. We will learn a method for factoring a trinomial with a leading coefficient that is not equal to one. We will call this new method “The ‘X’ Method.”

How Spiraling is incorporated in this lesson:

It is part of the Spiraling Curriculum design to revisit topics with deeper meaning. Previous skills in this unit: multiplying binomials, factoring trinomials and factoring by grouping, are revisited in this lesson to show how they can be used to factor trinomials where the leading coefficient is not equal to one. Before students learn this type of factoring, they will do some more practice with multiplying binomials, factoring trinomials and factoring by grouping. These three skills will be needed to understand how to factor a trinomial where the leading coefficient is not equal to one. Practicing these skills in the beginning of the lesson is essential in order to trigger this relevant prior knowledge. By doing this, it is intended that students will understand that multiplying binomials and factoring are inverses of each other. Also, once students learn “The X Method” of factoring they will need to be confident in how to factor by grouping. Or, if students choose to continue factoring by trial and error it is essential to think of multiplying binomials in reverse and to understand that these two processes are in fact inverses of each other. This is intended to also reduce students’ extraneous cognitive load, which will free working memory space to learn these new factoring skills

Common Core State Standards

A-SSE Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients.
**Learning Objectives:** Students will be able to: (a) interpret parts of an expression, such as terms, factors, and coefficients: (b) use the structure of an expression to identify ways to rewrite it; (c) factor a trinomial where the leading coefficient does not equal one.
Lesson 4
Trinomials \((a \neq 1)\)

Examples:
F.O.I.L. the following:

1. \((2x + 3)(x - 7)\)  
2. \((4x - 5)(2x - 5)\)

Factor the following:

1. \(x^2 - 3x - 18\)  
2. \(x^2 - 10x + 25\)

3. \(2x^3 + 8x^2 - 3x + 4\)  
4. \(12a^2 - 4a + 3ab - b\)

The Bonus factoring problem from Lesson 2…

You were asked to factor this problem by guessing and checking and thinking of doing F.O.I.L. in reverse.

\(3x^2 - x - 10\)

Some more tips for guessing and checking…

- As before, the first space in each parenthesis will be possible ________ of your ___________
- The second space in each parenthesis will be possible ________ of your ___________
- However, this time these factors will not directly add to your ___________, instead you will need to check if your factors work by using _________ and seeing if you get your original expression
SPIRALING CURRICULUM PROJECT ON FACTORING

Now, let’s FACTOR!

Try factoring the following using the guess and check method. Don’t forget to check your answer!

1. \(12b^2 - b - 20\)  
2. \(5b^2 - 9b - 18\)

3. \(4x^2 + 12x - 27\)  
4. \(3x^2 + x - 14\)

Here’s a trick that will always work if you are having a hard time with guessing and checking!

We can use a method called “The X Method” that involves grouping to factor these types of trinomials.

Then replace the middle term with the two new terms on the side of the “X” and set up a grouping problem.
SPIRALING CURRICULUM PROJECT ON FACTORING

Examples:
Factor the following using the “X-method”:

1. $6x^2 - 13x + 5$

Middle coefficient:
Multiply first and last coefficient:
Two numbers that multiply to _____ and add to ______

So, now we replace ______ with ____________ and factor by grouping:

Check:

2. $5x^2 + 16x + 3$

3. $10x^2 + 37x + 7$

4. $3x^2 + x - 10$

5. $3x^2 - 17x + 20$
Name: ______________________
Homework – Lesson 4

Factor the following:

1. \(3x^2 - 10x - 8\)

2. \(6a^2 - 13a - 5\)

3. \(x^3 - 5x^2 + 2x - 10\)

4. \(2k^2 - 7k + 6\)

5. \(a^2b^2c^2 - 4\)

6. \(x^3 + 4x^2 - 3x - 12\)

7. \(11x^2 - 51x - 20\)

8. \(x^2 - 8xy - 48y^2\)

9. \(4x^4 - 81y^2\)

10. \(2x^2 - 2x - 18\)

11. \(g^2 + 17g + 52\)

12. \(12d^2 + 17d - 40\)

13. \(w^2 - v^2\)

14. \(4x^4 - 16y\)
Lesson Five: Factoring Completely

Introduction: This lesson is designed for an Algebra I class with 45 minute periods. This is the fifth day of the factoring unit. This lesson will be putting together all the types of factoring that we have learned. The problems that we will be working on today require more than one step and more than one type of factoring.

How Spiraling is incorporated in this lesson:

It is part of the Spiraling Curriculum design to revisit topics to develop deeper understanding. In this lesson, all five types of factoring will be revisited, but this time the different types of factoring will be used together to solve problems. Now that the students have learned five different types of factoring, they will be solving problems that need to be factored completely. This means that there will be several steps with several different types of factoring required for each problem. Therefore, students need to be confident in all five types of factoring. They also need to be able to recognize which type of factoring to use. Finally, students need to be able to determine when an expression cannot be factored any further. To help students to be able to solve these types of problems they will be creating a “factoring foldable” which will include examples and steps for how to use each type of factoring. By creating this, students will simultaneously be reviewing these skills. This is intended to reduce students’ extraneous cognitive load, which will free working memory space to solve these new factoring problems.

Common Core State Standards

A-SSE Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients.
SPIRALING CURRICULUM PROJECT ON FACTORING

Learning Objectives: Students will be able to: (a) interpret parts of an expression, such as terms, factors, and coefficients; (b) use the structure of an expression to identify ways to rewrite it; (c) factor completely.
Lesson 5
Factor Completely

Now that we have learned five different methods of factoring, we will be factoring expressions that involve more than one type of factoring!

For our warm-up activity today we will be creating a “Factoring Foldable” to review our five different methods!

Materials:
- 3 sheets of paper (different colors if possible)
- Stapler
- Writing utensil
- Factoring notes

Steps to create your Factoring Foldable:
(look at attached pictures for guidance)

1. Align your three sheets of paper so that each paper is about an inch above the previous paper (this will allow you to have one inch tabs)

2. Fold the top half of the three pages over to create two more one inch tabs above the first three

3. Stable the top of the fold to keep the foldable together

4. So now you have a cover – (Label it Factoring Foldable), and five tabs for each of the five types of factoring – (Label in the following order: 1. GCF, 2. Difference of Two Perfect Squares, 3. Trinomials \((a = 1)\), 4. Trinomials \((a \neq 1)\), 5. Grouping)

5. Open each tab to write helpful hints to recognize each type of factoring and several examples of each (Again, look at attached pictures for guidance, or ask teacher and classmates for help)
Now, let’s FACTOR – completely!

Use your Factoring Foldable for guidance!

When asked to factor completely, ask yourself the following questions:

1.
2.
3.

Examples:

1. \(x^3 - 7x^2 + 10x\)

2. \(y^4 - w^4\)

3. \(a^4 - 6a^2 - 27\)

4. \(6a^3 - 22a^2 - 8a\)

5. \(x^8 - 1\)

6. \(5x^3 - 45x\)

7. \(x^4 - 4x^3 + 5x^2 - 20x\)

8. \(m^4 - 5m^2 + 4\)
Factoring Foldable:
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________

Homework – Lesson 5

Factor Completely!

1. \(2d^3 - 50d\) 
2. \(x^4 - 16\) 
3. \(x^3 - 4x^2 - 9x + 36\) 
4. \(a^2b^2 + 8ab + 12\) 
5. \(81x^4 - y^4\) 
6. \(4x^3 + 12x^2 - 9x - 27\) 
7. \(8m^2 - 16m + 8\) 
8. \(a^4 - 5a^2 - 36\) 
9. \(x^5 + 7x^4 + 10x^3\) 
10. \(3x^2 - 16x + 5\) 
11. \(2ax^2 - 2ax - 12a\) 
12. \(-x^2 + 11x + 26\)
Chapter Four: Validity of Curriculum Project

This curriculum project was reviewed by a veteran Algebra I teacher at a suburban school district in Western New York. The knowledge and the experience of this teacher is what will validate the author’s curriculum as well as its content validity. Feedback was given to the author about the curriculum design as a whole and broken down by each lesson. The teacher’s overall feedback is as follows:

- The unit plan is well constructed and logical.
- The curriculum design provides scaffolding from year to year as well as lesson to lesson.*
- The difficulty level throughout the unit and throughout each lesson increases at an appropriate rate.*
- In addition to the warm-up section of each lesson, I would add material from the previous day as well as the prior knowledge content, unless you plan on reviewing the homework in class.
- This is exactly what The Common Core is looking for.*

The second portion of feedback is broken down by each lesson:

Lesson 1:

- The Common Core is trying to steer away from using mnemonics such as F.O.I.L. because it is believed that students do not understand the meaning behind them. Instead of using the term F.O.I.L. I would use the term “double distribute” since students are already familiar with distributing.
- Show an example of why the sum of two perfect squares cannot be factored like the difference of two perfect squares.*
SPIRALING CURRICULUM PROJECT ON FACTORING

Lesson 2:

- Before giving the students the steps on how to factor a trinomial, have them try by trial and error first so that they can try and discover the process on their own. Then give them the steps. *

- I like the bonus problem at the end of the lesson where you ask the students to try and factor a trinomial where the leading term is not equal to one. This is a great preview to more difficult types of factoring. *

Lesson 3:

- This lesson is where I would add some more factoring examples from the previous day in the warm-up section. It is difficult for students to learn a new type of factoring every day, so extra review is appropriate at this time. *

Lesson 4:

- I like that students have to guess and check before learning the “X-method” so that they have a better understanding of factoring these types of trinomials. *

- I have never seen the “X-method” before and I was blown away by it! I plan on using this in my classroom now!

Lesson 5:

- This is an excellent way to pull it all together by having the students review all the types of factoring but also with more difficult examples. *

- The foldable is a nice way to organize the material, however, I prefer my students to make flash cards instead.
The comments with an asterisk after them are comments that strongly align with the Spiraling Curriculum design. In the teacher’s overall feedback he stated that the curriculum provided scaffolding and increased in difficulty at an appropriate rate throughout the unit and each lesson. These are both necessary components in the Spiraling Curriculum design because the idea is that while students are revisiting topics at increasingly difficult levels they will be trying to make connections of the content on their own. The comments with asterisks in the lesson section of the feedback all refer to the power of choosing relevant and appropriate examples and the importance of trial and error. These are also critical features of the Spiraling Curriculum. For example, in Lesson 1 where he suggests to add an example to show that the sum of two perfect squares cannot be factored like the difference of two perfect squares. This is important for students to understand, especially when they are beginning to realize that factoring and multiplying binomials are inverses of each other. Also, the example at the end of Lesson 2 where students are previewing a more difficult type of factoring, is a nice way to get students to make connections between what they have learned and what they will learn. Finally, by asking students to solve problems by trial and error before providing them with the process allows them to apply what they know to solve something new. By doing this, students will be able to make ties between topics throughout the unit. It is important for students to be making these ties and connections because this is what will help aid in the reduction of their extraneous cognitive load. Hence, students will be able to free up space in their working memory to continue learning new content while storing information in their long term memory.
Chapter Five: Conclusion

The paradigm shift from The NCTM Standards to The Common Core State Standards has created a lack of resources for teachers. This curriculum project is presented to be a resource for teachers to use in their classroom. This factoring unit plan was designed to not only support the CCSS but to help students learn by making a permanent change in their long term memory. The Spiraling Curriculum design is meant to aid students in making ties and connections between topics throughout the unit and throughout their years of schooling in mathematics. The idea is that this will also reduce students’ extraneous cognitive load, making it easier for them to learn new topics.

After taking the feedback into consideration, there is future work that can be done on this curriculum project. It will be important to see how students respond to the Spiraling Curriculum Design. The students’ response can be measured by their assessment performance throughout the year as well as at the end of the year. Teachers can use the feedback from their students’ assessments to re-evaluate what prior knowledge is essential for students to build off of and what information is not. The warm-up section is flexible so that each teacher can decide what they feel is necessary for their students to review. This can be modified day by day or even year by year depending on the response from students.

Teachers should differentiate these lessons as they see fit for their own classroom. For example, all students may not need to review the same concepts in the “Warm Up” section of the lessons. So, teachers can prepare more challenging examples for these students or even have them preview that day’s new content. Every classroom is unique, so the ties and connections made throughout the unit and throughout the year will be unique as well. Every teacher should base these connections on feedback from their students and how they have performed with previous content.
SPIRALING CURRICULUM PROJECT ON FACTORING

In conclusion, the goal of this Spiraling Curriculum project on factoring was to provide Algebra I teachers with a unit plan to use in their classroom as a resource while they are experiencing this paradigm shift from the NCTM *standards* to the CCSS. The constructive feedback that was received from a veteran Algebra I teacher reveals that this goal was achieved.
SPIRALING CURRICULUM PROJECT ON FACTORING

References


SPIRALING CURRICULUM PROJECT ON FACTORING


Appendix: Unit Plan Answer Keys

Lesson 1
GCF, Difference of Two Perfect Squares

Examples:

3. \(3(2x^2 - 4x + 7) = 6x^2 - 12x + 21\)

4. \(2x(x^3 + 2x^2 + 3x + 1) = 2x^4 + 4x^3 + 6x^2 + 2x\)

4. \(5ab(3a + 4) = 15a^2b + 20ab\)

Examples:

5. \((x + 5)(x - 5) = x^2 - 5x + 5x - 25 = x^2 - 25\)

6. \((3a - b)(3a + b) = 9a^2 + 3ab - 3ab - b^2 = 9a^2 - b^2\)

7. \((y^3 + y^2)(y^3 - y^2) = y^6 - y^5 + y^5 - y^4 = y^6 - y^4\)
SPIRALING CURRICULUM PROJECT ON FACTORING

8. 

\[(x^2 + y^2)(x^2 + y^2) = x^4 + x^2y^2 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4\]

What do you notice about the first three examples? Why does the fourth example not work out the same?

In the first three examples the middle terms cancel out. This does not happen in the fourth example because both \( y^2 \) terms are positive.

Now let’s FACTOR!

• Greatest Common Factor – The biggest factor that divides every term evenly.

To factor using the GCF:

4. Find the GCF
5. Divide each term by the GCF
6. Write in factored form – (think of un-distributing)

Find the GCF and then write in factored form:

Examples:

4. \( 10x + 10 \)

\[ GCF = 10 \]

\[ 10(x + 1) \]

5. \( 4x^3 + 6x^2 + 2x \)

\[ GCF = 2x \]

\[ 2x(2x^2 + 3x + 1) \]

6. \( 4x^3y^2 + 12x^2y + 20xy^2 \)

\[ GCF = 4xy \]

\[ 4xy(x^2y + 3x + 5y) \]

How are factoring out the GCF and distributing related?

They are inverses of each other.
SPIRALING CURRICULUM PROJECT ON FACTORING

Difference of Two Perfect Squares

Looking back at our first three F.O.I.L. examples, what do you notice about your final answers?

- There are two terms
- The second term is always being subtracted
- Both terms are perfect squares

Let’s recall our Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100…

: $x^2, x^4, x^6, x^8$ ...

If we call the first term $a^2$ and the second term $b^2$ can we come up with a formula to help us factor $a^2 - b^2$:

$$a^2 - b^2 = (a + b)(a - b)$$

Note: this does not work for the sum of two perfect squares!

Examples:

5. $x^2 - y^2 = (x + y)(x - y)$

6. $25a^2 - 100b^4 = (5a + 10b^2)(5a - 10b^2)$

7. $36x^2y^2 - 64x^4y^4 = (6xy + 8x^2y^2)(6xy - 8x^2y^2)$

8. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$

WAIT! What do we notice about this last example…?

The second part can be factored again using difference of two perfect squares! And even though the first part has two perfect square terms, because of the addition sign it cannot be factored any more.

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$$

$$= (x^2 + y^2)(x + y)(x - y)$$

Later in the unit we will be working on examples with several steps of factoring!
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________
Homework – Lesson 1

1. Factor: \(6a^2b - 12ab^2 = 6ab(a - 2b)\)

2. Factor: \(25t^2 - 49 = (5t + 7)(5t - 7)\)

3. Factor: \(38x^5 - 22x^3 = 2x^3(19x^2 - 11)\)

4. What is the product of \(2d - 7\) and \(2d + 7\)

\[4d^2 - 49\]

5. Factor: \(36g^4 - 81h^6 = (6g^2 - 9h^3)(6g^2 + 9h^3)\)

6. Subtract \(3x^2 - 4x + 5\) from \(6x^2 - x + 5\). Write your answer in factored form.

\[3x^2 + 3x = 3x(x + 1)\]

7. Find the solution set: \(4x - 2(x + 3) \geq 18 + 8x\)

\[x \leq 4\]

8. Write your answer to number 7 in interval notation: \((-\infty, 4]\)
Lesson 2
Trinomials ($a = 1$)

Remember when…

- F.O.I.L. – First, Outer, Inner, Last
  - Used to multiply two binomials

More examples:

1. $(x + 6)(x + 3) = x^2 + 9x + 18$

2. $(x + 7)(x - 2) = x^2 + 5x - 14$

3. $(x - 3)(x - 4) = x^2 - 7x + 12$

4. $(x - 8)(x + 5) = x^2 - 3x - 40$

Let’s take a look at our signs…

We know that:

- A positive times a positive gives us a positive
- A negative times a negative gives us a positive
- A positive times a negative gives us a negative

We can keep this in mind when we factor!

Again, how is F.O.I.L. related to factoring?

They are inverses of each other!

So, we will be working backwards from our examples above.

When factoring a trinomial: $ax^2 + bx + c$ when $a = 1$:

1. List the factors of $ax^2$ (In this case where $a = 1$, your factors will always be $x \times x$.)
2. List the factors of $c$
3. Pick two factors that add to $b$
4. Pay attention to the signs!!
   - If $c$ is positive, you will have two positive or two negative factors
   - If $c$ is negative, you will have one positive and one negative factor
   - To further determine, check if $b$ is positive or negative
5. Check your answer using F.O.I.L.
Now, let’s FACTOR!

Examples:

1. \( x^2 - 8x + 15 \)
   Factors of 15: 1, 15
   3, 5
   Since \( c = 15 \) is positive, we know we want our factors to both be positive or both be negative
   Since \( b = -8 \) is negative, we will choose two negative factors
   \[ -3 \times -5 = 15 \]
   \[ -3 - 5 = -8 \]
   So, \( x^2 - 8x + 15 \) factors to: \( (x - 3)(x - 5) \)
   Check:
   \( (x - 3)(x - 5) = x^2 - 5x - 3x + 15 \)
   \[ = x^2 - 8x + 15 \]

2. \( x^2 + 12x + 20 \)
   - \( c \) is positive, \( b \) is positive, so my factors will be positive and positive
   - Factors of 20: (1, 20), (2, 10), (4, 5)
   - Factors that add to 12: (2, 10)
   \[ 2 \times 10 = 20 \]
   \[ 2 + 10 = 12 \]
   So, \( x^2 + 12x + 20 \) factors to: \( (x + 2)(x + 10) \)
   Check: \( (x + 2)(x + 10) = x^2 + 10x + 2x + 20 \)
   \[ = x^2 + 12x + 20 \]

3. \( b^2 + 2b - 35 \)
   - \( c \) is negative, \( b \) is positive, so my factors will be positive and negative
   - Factors of 35: (1, 35), (5, 7)
   - Factors that add to 2: (-5, 7)
   \[ -5 \times 7 = -35 \]
   \[ -5 + 7 = 2 \]
   So, \( b^2 + 2b - 35 \) factors to: \( (b - 5)(b + 7) \)
   Check: \( (b - 5)(b + 7) = b^2 - 5b + 7b - 35 \)
   \[ = b^2 + 2b - 35 \]
SPIRALING CURRICULUM PROJECT ON FACTORING

Factor the following with a partner using the steps above. Don’t forget to check your answer!

4. \( b^2 - 6b - 16 \)
   \((b + 2)(b - 8)\)

5. \( x^4 - 7x^2 - 30 \)
   \((x^2 + 3)(x^2 - 10)\)

6. \( a^6 + 5a^3 + 6 \)
   \((a^3 + 2)(a^3 + 3)\)

7. \( x^2 - 6xy - 16y^2 \)
   \((x + 2y)(x - 8y)\)

BONUS: (Notice your a term is not equal to 1 this time. Try some guessing and checking using F.O.I.L. still)

8. \( 3x^2 - x - 10 \)
   \(= (3x + 5)(x - 2)\)
Factor the following:

1. \(a^2 + 10a + 21\)
   \(= (a + 3)(a + 7)\)

2. \(y^2 - 7y - 30\)
   \(= (y + 3)(y - 10)\)

3. \(24a^2b + 30ab\)
   \(= 6ab(4a + 5)\)

4. \(g^2 - 10g\)
   \(= g(g - 10)\)

5. \(p^2 - 9p + 8\)
   \(= (p - 1)(p - 8)\)

6. \(x^4 + 12x^2 + 32\)
   \(= (x^2 + 4)(x^2 + 8)\)

7. \(c^2 - 17c - 84\)
   \(= (c + 4)(c - 21)\)

8. \(u^6 - 13u^3 - 30\)
   \(= (u^3 + 2)(u^3 - 15)\)

9. \(16c^2 - 121\)
   \(= (4c + 11)(4c - 11)\)

10. \(x^2 - 12x + 32\)
    \(= (x - 4)(x - 8)\)

11. \(h^2 + 14h + 40\)
    \(= (h + 4)(h + 10)\)

12. \(t^2 - 10t - 39\)
    \(= (t + 3)(t - 13)\)

13. \(225 - x^4\)
    \(= (15 + x^2)(15 - x^2)\)

14. \(3y + 12y^3\)
    \(= 3y(1 + 4y^2)\)

15. \(w^2 - 22w + 57\)
    \(= (w - 3)(w - 19)\)

16. \(\frac{x^2}{4} - \frac{y^2}{9}\)
    \(= \left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)\)
Lesson 3
Grouping

More practice with GCF and F.O.I.L.!

Examples:
Find the greatest common factor and factor:

1. \(x^3y + 2x^2y^2 + 3xy^3\)
   \[\text{GCF} = xy\]
   \[\text{GCF} = (x^2 + 2xy + 3y^2)\]

2. \(3x^2(x + 4) - 2(x + 4)\)
   \[\text{GCF} = (x + 4)\]
   \[(x + 4)(3x^2 - 2)\]

3. \(y(x^2 + y) + x(x^2 + y)\)
   \[\text{GCF} = (x^2 + y)\]
   \[(x^2 + y)(x + y)\]

4. \(24a^2b^2 - 8a\)
   \[\text{GCF} = 8a\]
   \[8a(3ab^2 - 1)\]

F.O.I.L. the following:

1. \((x^3 + 3)(x - 2)\)
   \[= x^4 - 2x^3 + 3x - 6\]

2. \((2a - 3)(3b + 7)\)
   \[= 6ab + 14a - 9b - 21\]

3. \((x^2 - 5)(x + 6)\)
   \[= x^3 + 6x^2 - 5x - 30\]

4. \((x^2 + 4)(y - 3)\)
   \[= x^2y - 3x^2 + 4y - 12\]

What do you notice that is different from your F.O.I.L. solutions compared to the previous days? Why do you think this is occurring?

- The solutions have four terms instead of three
- There are no like terms to combine
Now, let’s FACTOR!

We will learn a factoring method called Grouping to factor expressions with four terms.

Steps to factor by Grouping:
1. Split the expression down the middle.
2. Take out the GCF of both sides
3. We WANT the same left over in the parenthesis
4. To write your final answer: Factor out the common term from both sides to go in one set of parenthesis and then combine the remaining terms in the second set of parenthesis.

Examples:

1. \(3x^3 + 6x^2 - 5x - 10\)
   \[
   3x^2(x + 2) | -5(x + 2)
   \]
   \(\frac{\text{Notice that if you factored out a } +5, \text{ you would be left with } (-x - 2), \text{ which is NOT what we want!}}{\}

2. \(2x^4 + 10x^3 + 3x + 15\)
   \[
   2x^3(x + 5) + 3(x + 5)
   
   (2x^3 + 3)(x + 5)
   \]

3. \(x^2y - 3x^2 - 2y + 6\)
   \[
   x^2(y - 3) - 2(y - 3)
   
   (x^2 - 2)(y - 3)
   \]

4. \(8x^3 - 20x^2 + 2x - 5\)
   \[
   4x^2(2x - 5) | +1(2x - 5)
   
   (4x^2 + 1)(2x - 5)
   \]
   \(\text{Notice that if there is not GCF or you already have what you want left over, factor out a } +1\)

5. \(30ab - 42a + 25b - 35\)
   \[
   6a(5b - 7) | +5(5b - 7)
   
   (6a + 5)(5b - 7)\)
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________
Homework – Lesson 3

Factor the following:

1. \[2x^4 - 3x^3 - 8x + 12 = (x^3 - 4)(2x - 3)\]
2. \[100a^2 - x^2 = (10a - x)(10a + x)\]
3. \[6x^4 - x^3 + 30x - 5 = (x^3 + 5)(6x - 1)\]
4. \[-x^2 - 10x - 25 = -(x + 5)(x + 5)\]
5. \[x^3 + 4x^2 + 22x = x(x^2 + 4x + 22)\]
6. \[36c^4d^4 - 49f^6 = (6c^2d^2 - 7f^3)(6c^2d^2 + 7f^3)\]
7. \[28u^2 + 7u = 7u(4u + 1)\]
8. \[x^2 - 12x - 45 = (x + 3)(x - 15)\]
9. \[3x^3 + 18x^2 + 2x + 12 = (3x^2 + 2)(x + 6)\]
10. \[4ab^2 - 12a^2b = 4ab(b - 3a)\]
11. \[3x^7 - 5x^5 + 3x^2 - 5 = (x^5 + 1)(3x^2 - 5)\]
12. \[12x^3 - 16x^2 - 9x + 12 = (4x^2 - 3)(3x - 4)\]
13. \[x^2 - 7x - 44 = (x + 4)(x - 11)\]
14. \[a^2 + 22a + 72 = (a + 4)(a + 18)\]
Lesson 4
Trinomials ($a \neq 1$)

Remember when…

Some more review from the past few days…

Examples:
F.O.I.L. the following:

1. $(2x + 3)(x - 7)$
   \[= 2x^2 - 14x + 3x - 21\]
   \[= 2x^2 - 11x - 21\]

2. $(4x - 5)(2x - 5)$
   \[= 8x^2 - 20x - 10x + 25\]
   \[= 8x^2 - 30x + 25\]

Factor the following:

2. $x^2 - 3x - 18$
   \[= (x + 3)(x - 6)\]

2. $x^2 - 10x + 25$
   \[= (x - 5)(x - 5)\]

4. $2x^3 + 8x^2 - 3x + 4$
   \[= (2x^2 - 3)(x + 4)\]

4. $12a^2 - 4a + 3ab - b$
   \[= (4a + b)(3a - 1)\]

The Bonus factoring problem from Lesson 2…

You were asked to factor this problem by guessing and checking and thinking of doing F.O.I.L. in reverse.

\[3x^2 - x - 10\]
\[= (3x + 5)(x - 2)\]

Some more tips for guessing and checking…

- As before, the first space in each parenthesis will be possible factors of your first term
- The second space in each parenthesis will be possible factors of your last term
- However, this time these factors will not directly add to your middle term, instead you will need to check if your factors work by using F.O.I.L. and seeing if you get your original expression
Now, let’s FACTOR!

Try factoring the following using the guess and check method. Don’t forget to check your answer!

2. \(12b^2 - b - 20\)  
   \[= (3b - 4)(4b + 5)\]

2. \(5b^2 - 9b - 18\)  
   \[= (5b + 6)(b - 3)\]

4. \(4x^2 + 12x - 27\)  
   \[= (2x + 9)(2x - 3)\]

4. \(3x^2 + x - 14\)  
   \[= (3x + 7)(x - 2)\]

Here’s a trick that will always work if you are having a hard time with guessing and checking!

We can use a method called “The X Method” that involves grouping to factor these types of trinomials.

- Middle coefficient
- Multiply first and last coefficient
- Two numbers that add to the top number and multiply to the bottom number

Then replace the middle term with the two new terms on the side of the “X” and set up a grouping problem.
SPIRALING CURRICULUM PROJECT ON FACTORING

Examples:
Factor the following using the “X-method”:

6. \(6x^2 - 13x + 5\)

\[
\begin{array}{c|c}
-13 & \ \\
-3 & -10 \\
6 \times 5 & = 30
\end{array}
\]

Middle coefficient: \(-13\)
Multiply first and last coefficient: \(6 \times 5 = 30\)
Two numbers that multiply to \(30\) and add to \(-13\): \(-3\) \(\text{and} -10\)

So, now we replace \(-13x\) with \(-3x - 10x\) and factor by grouping:

\[
6x^2 - 3x - 10x + 5 = 3x(2x - 1) - 5(2x - 1) = (3x - 5)(2x - 1)
\]

Check: \((3x - 5)(2x - 1) = 3x^2 - 3x - 10x + 5 = 3x^2 - 13x + 5\)

7. \(5x^2 + 16x + 3\)
   \[= (5x + 1)(x + 3)\]

8. \(10x^2 + 37x + 7\)
   \[= (5x + 1)(2x + 7)\]

9. \(3x^2 + x - 10\)
   \[= (3x - 5)(x + 2)\]

10. \(3x^2 - 17x + 20\)
    \[= (3x - 5)(x - 4)\]
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________

Homework – Lesson 4

Factor the following:

1. \(3x^2 - 10x - 8\) = \((3x + 2)(x - 4)\)

2. \(6a^2 - 13a - 5\) = \((3a + 1)(2a - 5)\)

3. \(x^3 - 5x^2 + 2x - 10\) = \((x^2 + 2)(x - 5)\)

4. \(2k^2 - 7k + 6\) = \((2k - 3)(k - 2)\)

5. \(a^2b^2c^2 - 4\) = \((abc + 2)(abc - 2)\)

6. \(x^3 + 4x^2 - 3x - 12\) = \((x^2 - 3)(x + 4)\)

7. \(11x^2 - 51x - 20\) = \((11x + 4)(x - 5)\)

8. \(x^2 - 8xy - 48y^2\) = \((x + 4y)(x - 12y)\)

9. \(4x^4 - 81y^2\) = \((2x^2 - 9y)(2x^2 + 9y)\)

10. \(2x^2 - 2x - 18\) = \(2(x^2 - x - 9)\)

11. \(g^2 + 17g + 52\) = \((g + 4)(g + 13)\)

12. \(12d^2 + 17d - 40\) = \((3d + 8)(4d - 5)\)

13. \(w^2 - v^2\) = \((w + v)(w - v)\)

14. \(4x^4 - 16y\) = \(4(x^4 - 4y)\)
Lesson 5
Factor Completely

Now that we have learned five different methods of factoring, we will be factoring expressions that involve more than one type of factoring!

For our warm-up activity today we will be creating a “Factoring Foldable” to review our five different methods!

Materials:
- 3 sheets of paper (different colors if possible)
- Stapler
- Writing utensil
- Factoring notes

Steps to create your Factoring Foldable:
(Look at attached pictures for guidance)

6. Align your three sheets of paper so that each paper is about an inch above the previous paper (this will allow you to have one inch tabs)

7. Fold the top half of the three pages over to create two more one inch tabs above the first three

8. Staple the top of the fold to keep the foldable together

9. So now you have a cover – (Label it Factoring Foldable), and five tabs for each of the five types of factoring – (Label in the following order: 1. GCF, 2. Difference of Two Perfect Squares, 3. Trinomials \((a = 1)\), 4. Trinomials \((a \neq 1)\), 5. Grouping)

10. Open each tab to write helpful hints to recognize each type of factoring and several examples of each (Again, look at attached pictures for guidance, or ask teacher and classmates for help)
Now, let’s FACTOR – completely!

Use your Factoring Foldable for guidance!

When asked to factor completely, ask yourself the following questions:

1. First, is there a GCF?
2. Then, what kind of factoring?
3. Finally, CAN I DO MORE?

Examples:

1. \( x^3 - 7x^2 + 10x \)
   \( x(x^2 - 7x + 10) \) - Take out GCF
   \( x(x - 2)(x - 5) \) – Factor

2. \( y^4 - w^4 \)
   \( (y^2 + w^2)(y^2 - w^2) \) - Diff. Perf. Sq.
   \( (y^2 + w^2)(y + w)(y - w) \) - Diff. Perf. Sq.

3. \( a^4 - 6a^2 - 27 \)
   \( (a^2 + 3)(a^2 - 9) \) - Factor
   \( (a^2 + 3)(a + 3)(a - 3) \) - Diff. Perf. Sq.

4. \( 6a^3 - 22a^2 - 8a \)
   \( 2a(3a^2 - 11a - 4) \) - GCF
   \( 2a(3a + 1)(a - 4) \) – Factor

5. \( x^8 - 1 \)
   \( (x^4 + 1)(x^4 - 1) \) - DPS
   \( (x^4 + 1)(x^2 + 1)(x^2 - 1) \) - DPS
   \( (x^4 + 1)(x^2 + 1)(x + 1)(x - 1) \) – DPS

6. \( 5x^3 - 45x \)
   \( 5x(x^2 - 9) \) - GCF
   \( (5x)(x + 9)(x - 9) \) – DPS

7. \( x^4 - 4x^3 + 5x^2 - 20x \)
   \( x(x^3 - 4x^2 + 5x - 20) \) - GCF
   \( x(x^2(x - 4) + 5(x - 4)) \) - Grouping
   \( x(x^2 + 5)(x - 4) \)

8. \( m^4 - 5m^2 + 4 \)
   \( (m^2 - 1)(m^2 - 4) \) – Factor
   \( (m + 1)(m - 1)(m + 2)(m - 2) \) - DPS twice!
SPIRALING CURRICULUM PROJECT ON FACTORING

Factoring Foldable:
SPIRALING CURRICULUM PROJECT ON FACTORING

Name: ______________________
Homework – Lesson 5

Factor Completely!

1. \(2d^3 - 50d\)
   \[= 2d(d + 5)(d - 5)\]

2. \(x^4 - 16\)
   \[= (x^2 + 4)(x + 2)(x - 2)\]

3. \(x^3 - 4x^2 - 9x + 36\)
   \[= (x + 3)(x - 3)(x - 4)\]

4. \(a^2b^2 + 8ab + 12\)
   \[= (ab + 2)(ab + 6)\]

5. \(81x^4 - y^4\)
   \[= (9x^2 + y^2)(3x + y)(3x - y)\]

6. \(4x^3 + 12x^2 - 9x - 27\)
   \[= (2x + 3)(2x - 3)(x + 3)\]

7. \(8m^2 - 16m + 8\)
   \[= 8(m - 1)(m - 1)\]

8. \(a^4 - 5a^2 - 36\)
   \[= (a^2 + 4)(a + 3)(a - 3)\]

9. \(x^5 + 7x^4 + 10x^3\)
   \[= x^3(x + 2)(x + 5)\]

10. \(3x^2 - 16x + 5\)
    \[= (3x - 1)(x - 5)\]

11. \(2ax^2 - 2ax - 12a\)
    \[= 2a(x + 2)(x - 3)\]

12. \(-x^2 + 11x + 26\)
    \[= -(x + 2)(x - 13)\]